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Redistribution of electron energies at the interface between laser-radiation-filled space and vacuum

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Abstract. Using a simple one-dimensional model, we show that at the interface between a half-space filled by laser radiation and vacuum a considerable redistribution of the energies of electrons, either scattered or ionized in the laser field, takes place. This indicates that electron spectra evaluated by using electromagnetic plane wave fields for the description of a laser pulse, cannot reliably be compared with data observed experimentally.

1. Introduction

Shortly after the invention of the laser, several papers appeared (in the early 1960s) which were devoted to the simplest nonlinear process induced by a powerful laser beam, namely Compton scattering by a free electron. In these investigations an intensity-dependent frequency shift of the scattered laser light on account of the momentum and energy picked up by the electron from the laser beam was predicted by Brown and Kibble [1], Gol'dman [2] and somewhat earlier by Sengupta [3]. Soon after that a controversy arose about the existence or non-existence of this frequency shift and it was realized that this shift essentially depends on the proper choice of the scattering boundary conditions, as discussed in the review by Eberly [4] and elaborated in more detail by Kibble [5] and by Neville and Rohrlich [6]. The boundary conditions required to describe the passage of an electron through a laser beam of finite dimensions and the nonlinear scattering of electrons accompanied by this process were considered in some detail by Bergou *et al* [7] some time ago. Boundary effects became moreover of interest in connection with the interpretation of the experimental data on above-threshold ionization (ATI), as discussed in classical terms in great detail in the work of Muller *et al* [8] and Bucksbaum *et al* [9] and as surveyed in the reviews by Freeman and Bucksbaum [10] and by one of the present authors [11]. In these works it was shown that the energy of the ionized ATI electrons depends in an essential way on the laser pulse duration. Namely, the electrons are either subjected after ionization to the field gradient force at the boundary of the laser beam to its full extent or not. The only more recent paper that we know of, in which boundary effects at the interface between radiation filled space and vacuum are discussed, is the work by Ho *et al* [12]. In this paper the change of the electron energy during entrance into and exit from a laser beam is discussed in classical terms. In most of the above papers, however, a quantum-mechanical treatment of the boundary effects is not presented, except the consideration to introduce

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electron wavepackets in order to fulfil the appropriate boundary conditions, as discussed by Kibble [5] and Neville and Rohrlich [6].

It is the purpose of this paper to show by means of a simple, one-dimensional model calculation that there is a considerable energy redistribution of the electrons which become either scattered by an atom inside the laser beam and are emerging from it, or are created by ionization inside the beam and leave at the boundary of the beam. In order to simplify matters, we only consider the case in which the electrons pass the boundary of the laser beam along the direction of the laser polarization which is taken to point perpendicular to the interface between radiation-filled space and vacuum, since usually the ionization probability is largest for electrons emitted parallel to the laser polarization, except for very high laser powers in the plateau region of the electron energy spectrum. Thus, assuming that the electrons escape the laser field perpendicular to the laser polarization is of less interest. For simplicity, the above interface is approximated by a plane of infinite extent which is tangential to the laser beam and we also replace this interface by a sharp step such that the problem becomes largely tractable by analytic means. We are well aware that this assumption is not a realistic approximation. Moreover, we treat the laser field in the dipole approximation which is permissible for laser powers considerably less than the critical intensity of some 10^{18} W cm^{-2} and not too short laser pulses such that the space dependence of the radiation field in the direction of propagation can be neglected and the assumption of adiabatic switching on and off of the laser field is justified. We are also aware that a radiation field having a sharp interface with vacuum, does in fact not fulfil Maxwell's equations. This investigation is carried out in view of the recent experiments by Moore *et al* [13] and Meyerhofer *et al* [14] who were able to experimentally demonstrate the existence of the mass and momentum shift of an electron in a powerful laser beam, mentioned at the beginning.

2. Theory

Let us consider the following set-up. An interface in the (y, z)-plane separates the homogeneously radiation filled-region for $x \leqslant 0$ and vacuum for $x \geqslant 0$. The unit vector ε of linear polarization of the radiation field, taken in the dipole approximation, points along the x-direction. Due to the translational symmetry in the (y, z)-directions our problem is one dimensional and thus consideration of the electron momentum components in the y-and z-directions can be neglected, since these momenta will be conserved quantities during the transition from the laser-field-filled half-space to vacuum. We thus consider the Schrödinger equation of an electron moving in either of the two half-spaces and passing the interface in the positive or negative x-directions

$$\left[\frac{p^2}{2m} + V(x, t)\right]\Psi = \mathrm{i}\hbar\partial_t\Psi \tag{1}$$

where

$$V(x, t) = \begin{cases} exF\sin\omega t & x \leqslant 0 \\ 0 & x \geqslant 0. \end{cases} \tag{2}$$

In (1) $p = (\hbar/\mathrm{i})\partial_x$ and in (2) e is the electron charge, F is the field strength and ω is the frequency of the laser beam. In order to solve our one-dimensional scattering problem at the oscillating potential step between the interior and exterior region, we have to require that both Ψ and $\partial_x\Psi$ are continuous at $x = 0$, such that $\rho = \Psi^*\Psi$ and $j_x = (\hbar/m)\mathrm{Im}\,(\Psi^*\partial_x\Psi)$ are continuous all over the spacetime. We assume the laser field to be stationary, which means the switching on and off of the field is taken to be adiabatic. In other words, the time envelope function of the laser field strength $F = F_0 f(t)$ must have a characteristic time much larger than any time parameters appearing in the process under study, as pointed out in the introduction.

The fundamental solutions of (1) inside the laser beam, $(x \leqslant 0)$, are the well known Gordon–Volkov states [15, 16] which read in the electric field gauge

$$\phi = \exp\left(\frac{\mathrm{i}}{\hbar}\frac{exF}{\omega}\cos\omega t\right)\exp\left[\frac{\mathrm{i}}{\hbar}(qx - Et) - \mathrm{i}(a\sin\omega t + b\sin 2\omega t)\right] \qquad (3)$$

where

$$a = \frac{eF}{\hbar m\omega^2}q, \qquad b = \frac{1}{8m\hbar\omega}\left(\frac{eF}{\omega}\right)^2 = \frac{U_p}{2\hbar\omega} \qquad (4)$$

$$U_p = 2mc^2\frac{\mu^2}{8}, \qquad \mu = \frac{eF}{mc\omega} = 10^{-9}\frac{\sqrt{s}}{v} \qquad (5)$$

where μ is the usual intensity parameter, s is the laser intensity in units of W cm^{-2} and v is the photon energy measured in eV. In (5) we have introduced the ponderomotive energy U_p. The momentum q in (4) is defined through the energy E by means of the mass-shell relation

$$q = +\sqrt{2m(E - U_p)}. \qquad (6)$$

Clearly, an electron having a kinetic energy less than the ponderomotive energy U_p, cannot propagate in the region occupied by the field. Only if $E > U_p$ can the wavenumber $q/\hbar$ be real, corresponding to free propagation.

Outside the laser field, for $x \geqslant 0$, ordinary free particle plane wave solutions have to be employed

$$\chi = \exp\left[\frac{\mathrm{i}}{\hbar}(px - Et)\right], \; p = \sqrt{2mE}. \qquad (7)$$

One particular solution ϕ and χ of the form (3) and (7), respectively, cannot satisfy the boundary conditions $\phi(x = 0, t) = \chi(x = 0, t)$ and $\partial_x\phi(x = 0, t) = \partial_x\chi(x = 0, t)$ at every instant of time. On the other hand, the momenta q and p are not good quantum numbers, since the Hamiltonian in (1) is not translationally invariant. Moreover, in this case, the energy E is also not a good quantum number, since the Hamiltonian in (1) is time dependent, but E is the parameter of interest, since it acts as the 'quasi-energy' of the system. Hence, we are looking for solutions of the Floquet form $\Psi(x, t) = \exp(-\mathrm{i}Et/\hbar)u(x, t)$ where $u(x, t)$ is a periodic function of time, i.e. $u(x, t + 2\pi/\omega) = u(x, t)$ [17, 18]. Thus we choose as the wavefunction inside the radiation field $(x \leqslant 0)$ a superposition of an incoming Gordon–Volkov wave $\phi_0^{(+)}$ with the initial energy E and a sum of reflected waves $\phi_n^{(-)}$ of similar Gordon–Volkov type belonging to the energies $E + n\hbar\omega$. Similarly, in the radiation-free half-space, $(x \geqslant 0)$, we take as a solution χ, a superposition of transmitted waves $\chi_n^{(+)}$ of the type (7) belonging to the energies $E + n\hbar\omega$. Thus we obtain

$$\phi = \phi_0^{(+)} + \sum_n R_n\phi_n^{(-)}, \qquad x \leqslant 0 \qquad (8)$$

$$\chi = \sum_n T_n\chi_n^{(+)}, \qquad x \geqslant 0 \qquad (9)$$

where

$$\phi_n^{(\pm)} = \exp\left[\frac{\mathrm{i}}{\hbar}\frac{exF}{\omega}\cos\omega t\right]\exp\left[\frac{\mathrm{i}}{\hbar}(\pm q_n x - (E + n\hbar\omega)t)\right]\exp(\mp \mathrm{i} a_n \sin\omega t - \mathrm{i} b \sin 2\omega t) \tag{10}$$

$$a_n = \frac{eF}{\hbar m\omega^2}q_n, \qquad q_n = +\sqrt{2m(E + n\hbar\omega - U_p)} \tag{11}$$

and

$$\chi_n^{(\pm)} = \exp\left[\frac{\mathrm{i}}{\hbar}(\pm p_n x - (E + n\hbar\omega)t)\right] \tag{12}$$

$$p_n = +\sqrt{2m(E + n\hbar\omega)}. \tag{13}$$

By evaluating the reflection coefficients R_n and the transmission coefficients T_n one can calculate the time-averaged multiphoton current components $j_r(n)$ and $j_t(n)$ of reflection and transmission, respectively, normalized to the incoming current, namely

$$j_r(n) = \frac{q_n}{q_0}|R_n|^2, \qquad j_t(n) = \frac{p_n}{q_0}|T_n|^2. \tag{14}$$

Since the total probability has to be conserved, these current components have to satisfy the sum rule

$$\sum_{n=n_0}^{\infty} j_r(n) + \sum_{n=n_1}^{\infty} j_t(n) = 1. \tag{15}$$

In (15) n_0 is the smallest integer for which $q_{n_0} = \sqrt{2m(E + n_0\hbar\omega - U_p)}$ is real, hence corresponding to free particle propagation. Similarly, n_1 is the smallest integer for which $p_{n_1} = \sqrt{2m(E + n\hbar\omega)}$ is real, such that $\exp(\frac{\mathrm{i}}{\hbar} p_{n_1} x)$ is a freely propagating wave in vacuum, whereas $\exp(\frac{\mathrm{i}}{\hbar} p_{n_1-1} x) = \exp(-\frac{1}{\hbar}|p_{n_1-1}|x)$ already corresponds to an evanescent wave bound to the boundary plane.

Applying the boundary conditions

$$\phi(x = 0, t) = \chi(x = 0, t) \tag{16}$$

and

$$\partial_x\phi(x = 0, t) = \partial_x\chi(x = 0, t) \tag{17}$$

to solutions (8)–(13), we obtain the following two infinite sets of linear algebraic equations for the unknown reflection coefficients R_n and transmission coefficients T_n. To find these equations, we have to use the generating function of the following generalized Bessel functions B_n,

$$\exp[-\mathrm{i}(a\sin\omega t + b\sin 2\omega t)] = \sum_{n=-\infty}^{+\infty} B_n(a, b)\exp(-\mathrm{i}n\omega t) \tag{18}$$

$$B_n(a, b) = \sum_{k=-\infty}^{+\infty} J_{n-2k}(a)J_k(b) \tag{19}$$

where the J_n are ordinary Bessel functions of the first kind of the integer order n.

From (16), (17) and (8)–(13) we thus obtain the two sets of equations for the R_n and T_n

$$B_n(a_0, b) + \sum_k B_{n-k}(-a_k, b)R_k = T_n \tag{20}$$

$$S_n = q_0 B_n(a_0, b) + \frac{eF}{2\omega}[B_{n-1}(a_0, b) + B_{n+1}(a_0, b)] \tag{21}$$

$$S_n + \sum_k \left\{ -q_k B_{n-k}(-a_k, b) + \frac{eF}{2\omega} [B_{n-1-k}(-a_k, b) + B_{n+1-k}(-a_k, b)] \right\} R_k = p_n T_n. \quad (22)$$

Unfortunately, (20)–(22) cannot be solved analytically. Hence, we have to find numerical solutions. Of course, the accuracy of the numerical solutions depends on the truncation of the infinite kernel matrices in (20)–(22). Moreover, the matrix elements contain the generalized Bessel functions $B_n(a, b)$ which are themselves infinite series. Although Leubner [19] has developed excellent numerical methods to calculate the generalized Bessel functions, we did not need these methods in our study. We have checked that the functions B_n could very well be approximated by a finite sum, $\sum_{l=-L}^{L} J_{n-2l}(a) J_l(b)$, where L is not larger than 40. We should mention that condition (15) of the conservation of probability can be used to check the accuracy of the numerical calculations.

3. Examples

By means of the following examples we show how at the transition from inside the laser beam to outside the laser beam an electron can pick up or release an additional photon such that the electron energy measured in the exterior region does not quite reflect the situation which prevailed inside the radiation field. We choose an Nd:YAG laser field for which $\hbar\omega = 1.17$ eV and we consider the range of moderate intensities between 10^{12} and 10^{13} W cm^{-2}. In this case the use of the dipole approximation is certainly justified, if we moreover assume a picosecond laser pulse duration so that the laser field can be considered to have a constant amplitude in time.

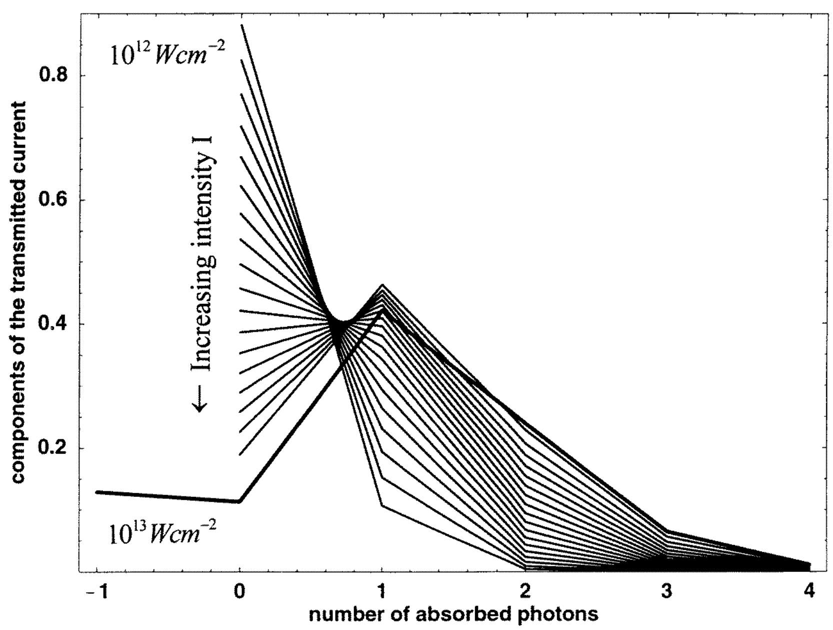

Figure 1. If the initial electron kinetic energy in the laser field is chosen as $E'_{\rm kin} = 0.28$ eV, the transmitted current components, $j_t(n)$, are shown as a function of the number of absorbed photons n. With increasing intensity I of the laser field, $j_t(0)$ is decreasing and, at the same time, $j_t(1)$ is gradually increasing, while at an intensity of 10^{13} W cm^{-2} the channel with $n = -1$ gets opened.

In the first example, depicted in figure 1, the initial kinetic energy of an electron in the laser beam is taken to be $E'_{\rm kin} = 0.28$ eV and the corresponding kinetic energy outside the beam will be $E_{\rm kin} = E'_{\rm kin} + U_p$, if no photons are emitted or absorbed during the transition. As is apparent from our figure, the probability current, $j_t(1)$, for absorption of one photon during the transition of an electron from the interior to the exterior region, increases with increasing intensity of the laser field, while at the same time the probability current, $j_t(0)$, for no photon exchange decreases. At a laser intensity of 10^{13} W cm^{-2} the ponderomotive energy U_p has increased by so much that the electron kinetic energy outside the radiation field has become $E_{\rm kin} = 1.193\,14$ eV. Thus it is sufficiently large so that an electron could moreover emit one photon during the exit from the radiation field, since the channel with $n = -1$ was opened. For the lower intensities, on the other hand, this channel remains closed.

In the second example, shown in figure 2, we consider the total transmitted and reflected currents, j_t and j_r respectively, as a function of the intensity. Here the initial kinetic energy of an electron in the laser beam was chosen $E'_{\rm kin} = 0.05$ eV. As we can see, at first the total probability current, j_t, of transmission decreases while the reflected current, j_r, increases, but upon approaching the intensity of 10^{13} W cm^{-2} the transmitted current rises again and there is a *cusp* at an intensity of 1.25×10^{13} W cm^{-2} which reflects the fact that at this intensity the ponderomotive energy U_p has become so large that the electron kinetic energy outside the radiation field reaches the value $E_{\rm kin} = 1.191\,43$ eV. Consequently, an electron can now emit one photon and one additional channel with $n = -1$ is opened. Hence, at that value of the intensity, the transmitted current, j_t, has a local maximum and, at the same time, the reflected current, j_r, a minimum, since the total probability has to remain constant, i.e. $j_t + j_r = 1$.

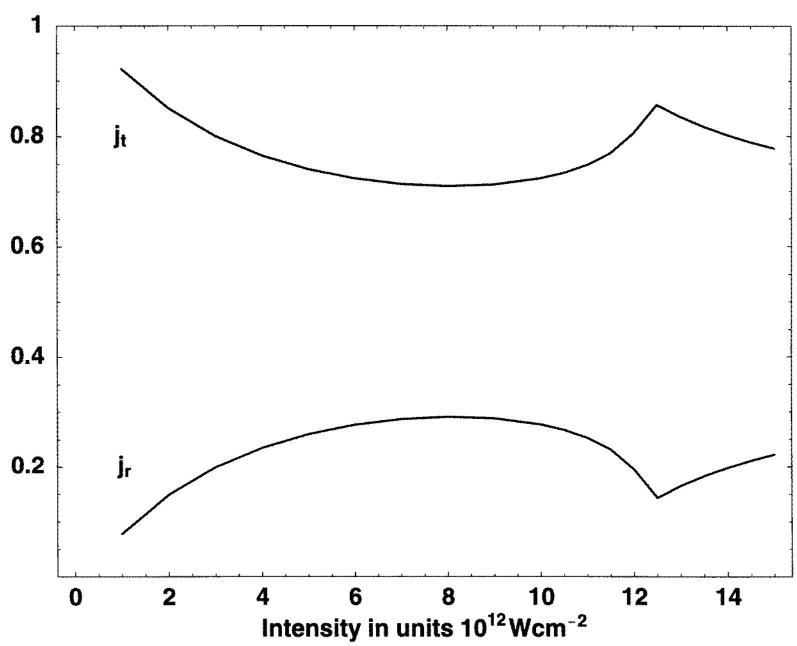

Figure 2. The total transmitted and reflected currents, j_t and j_r, respectively, as a function of the laser intensity I. If the initial kinetic energy of the electron in the laser field is chosen $E'_{\rm kin} = 0.05$ eV, then at the particular intensity of 1.25×10^{13} W cm^{-2} a *cusp* appears due to the opening of a further channel at $n = -1$.

4. Conclusions

Considering electrons which were either born from or scattered by an atom inside a laser beam, we were able to show by means of our simple model calculations that these electrons can suffer during the transition from the radiation-filled half-space into the radiation-free region a considerable redistribution of their energies, in particular, if the process takes place at sufficiently high laser field intensities such that additional transition channels can get opened. This finding can be of relevance for the detailed comparison of experimental data with theoretical predictions on laser-induced or laser-assisted processes in which the calculations were performed by describing the laser field by an electromagnetic plane wave of infinite extent.

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