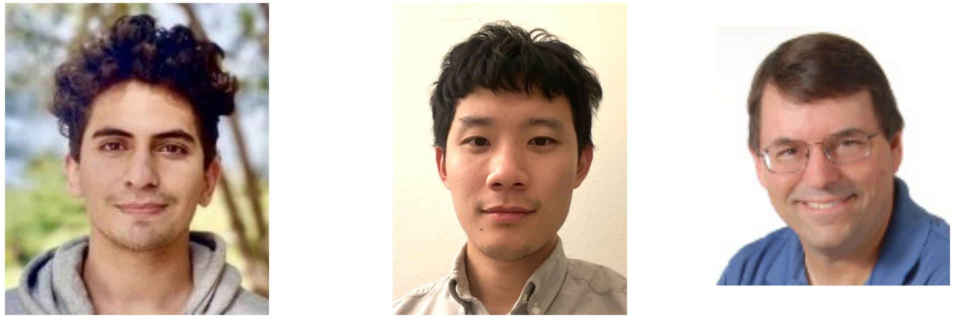
spin liquids and supersolids in anisotropic triangular-lattice model

spin liquids and supersolids in anisotropic triangular-lattice model



Cesar Gallegos

Shengtao Jiang

Steven White

UCIrvine
University of California, Irvine

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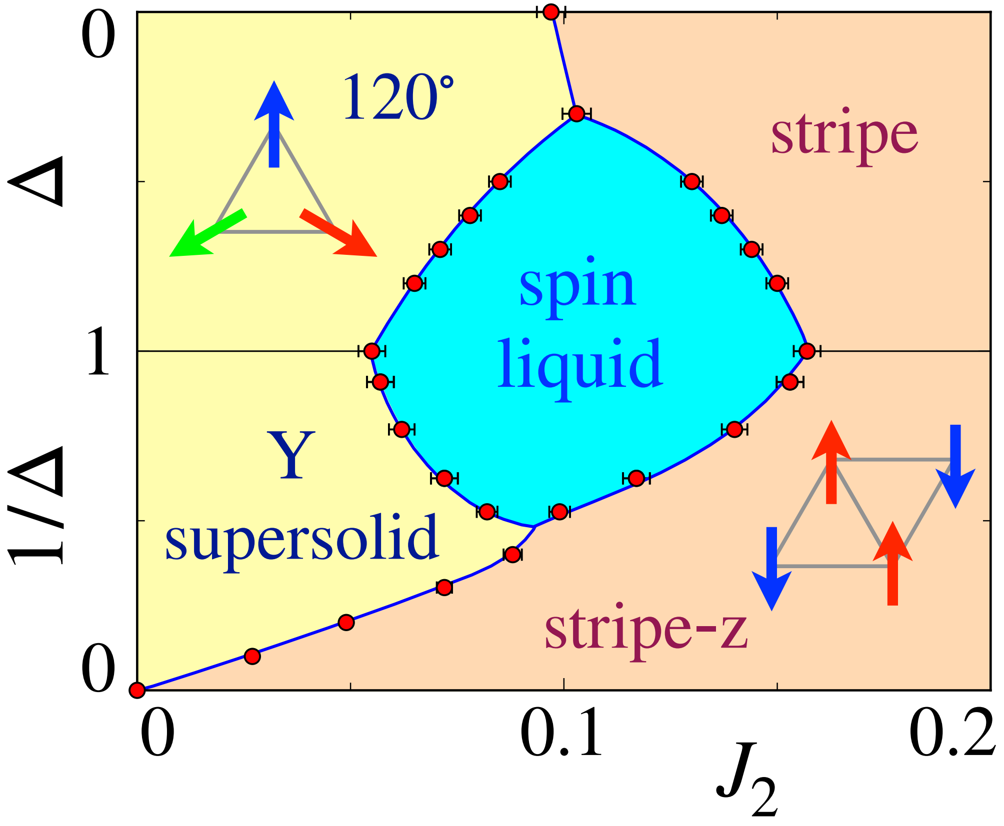


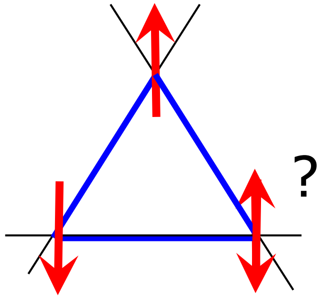
C. A. Gallegos, S. Jiang, S. R. White, and **SC**, PRL **134**, 196702 (2025).

U.S. DEPARTMENT OF ENERGY | Office of Science

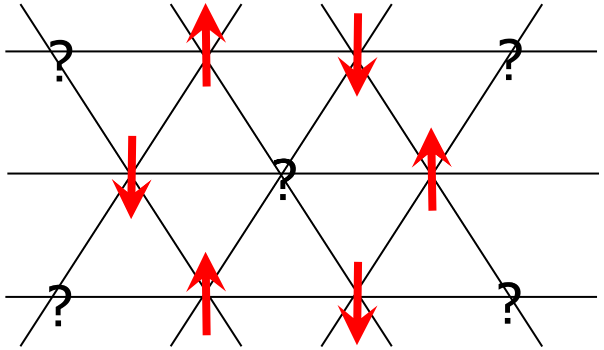
the Creed: triangular foundation

Google Scholar: 1897 cites

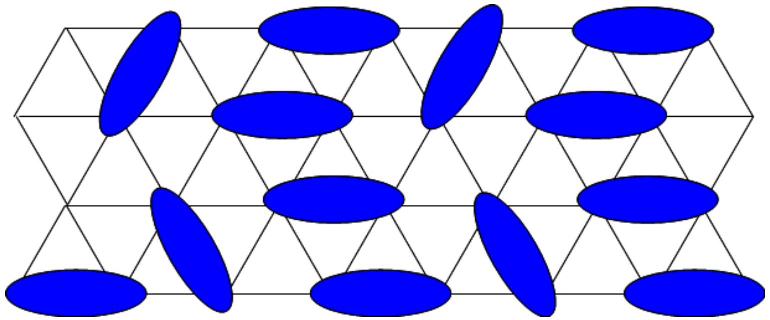
Google Scholar: 3714 cites



Ising



Heisenberg



frustration yields degeneracy!

no unique ground state exists

exotic states are possible

exotic state **may** exist

G. H. Wannier, *Phys. Rev.* **79**, 357 (1950).

P. W. Anderson, *Mat. Res. Bul.* **8**, 153 (1973).

a Hungarian connection…

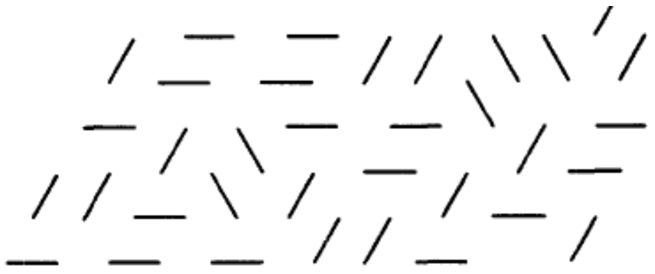
P. W. Anderson, *Mat. Res. Bul*. **8**, 153 (1973).

RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?*

**This paper was originally intended for the Pauling Festschrift, Volume 7,

Rumer (7) and Pauling (8) have given rules for determining which and how many of these configurations are independent: as we see from eqn. (9), in

b)



APRIL, 1933 JOURNAL OF CHEMICAL PHYSICS

The Calculation of Matrix Elements for Lewis Electronic Structures of Molecules

LINUS PAULING, *California Institute of Technology*

(Received February 14, 1933)

- **Valence Bond basis**

Starting from the discovery by Rumer that the eigenfunctions corresponding to different distributions of valence bonds in a molecule can be represented by plane diagrams which provide information regarding their mutual linear independence, a very simple graphical method is developed for calculating the coefficients of the integrals occurring in the matrix elements involved in Slater's treatment of the electronic structure of molecules.

7. G. Rumer, Nachr. d. Ges. d. Wiss. zu Gottingen, M. P. Klasse, 337 (1932).

8. L. Pauling, J. Chem. Phys. 1, 280 (1933).

who is Mr. Rumer??

Physics – Uspekhi **44** (10) 1075 – 1081 (2001)

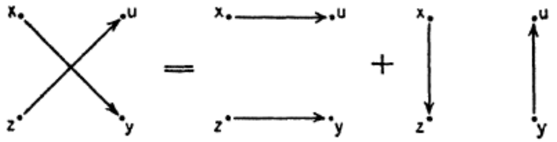
valence bonds

Zur Theorie der Spinvalenz.

Von

Georg Rumer in Moskau.

Vorgelegt von H. Weyl in der Sitzung am 22. Juli 1932.

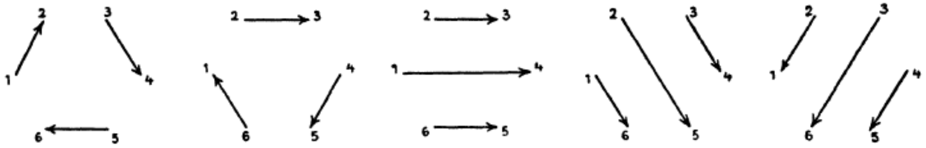


Eine für die Valenztheorie geeignete Basis der binären Vektorinvarianten.

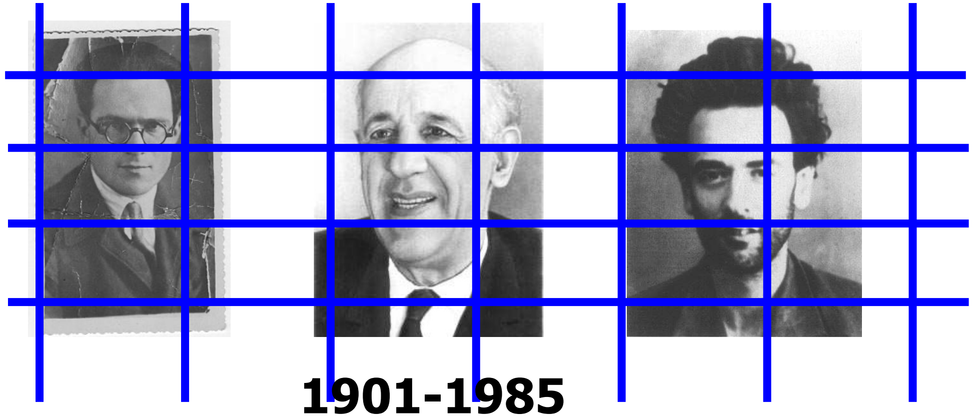
Von

G. Rumer (Moskau), **E. Teller** und **H. Weyl** (Göttingen).

Vorgelegt von H. Weyl in der Sitzung am 28 Oktober 1932.



Yurii (George) Rumer



1901-1985

- “sharashka” = research Gulag
- Karl Szilard (≠ but related to Leo Szilard)

7. G. Rumer, Nachr. d. Ges. d. Wiss. zu Gottingen, M. P. Klasse, 337 (1932).

ode to Hungarian
math-physics

triangular XXZ

$$\hat{\mathcal{H}} = J_1 \sum_{\langle ij \rangle_1} \left(S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z \right)$$

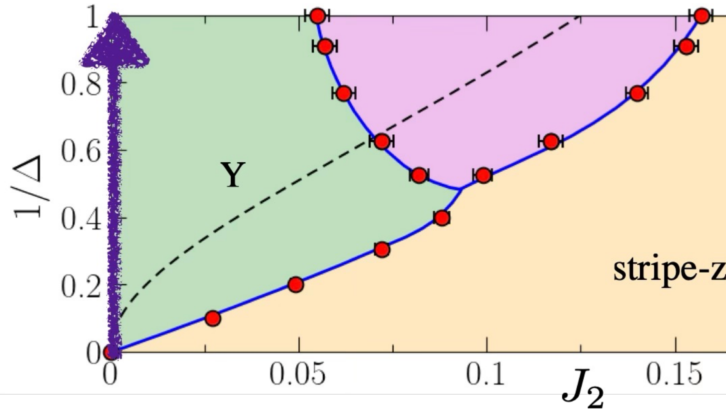
Google Scholar:
1038 cites

Patrick Fazekas

$$\frac{1}{N} E_{\rm cl} = -JS^2 \frac{1+\alpha+\alpha^2}{1+\alpha} \approx \frac{1}{N} E_{\rm Ising} - JS^2\alpha^2$$

In what follows we will be speaking only of the $S=\frac{1}{2}$ case. Our argument in favour of a non-Néel-type state rests on the recognition that it is possible to construct states whose energy depends linearly on α

$$\frac{1}{N} E_{\rm tr} = \frac{1}{N} E_{\rm Ising} - b\alpha, \qquad (5)$$



- extend Heisenberg RVB to Ising to tie it with Wannier degeneracy
- classically, near Ising, linear term in $\alpha = 1/\Delta$ is missing
- real-space perturbation theory restores linear term

B. Kleine, E. Muller-Hartmann, K. Frahm, and P. Fazekas, Z. Phys. B **87**, 103 (1992);
B. Kleine, P. Fazekas, and E. Muller-Hartmann, Z. Phys. B **86**, 405 (1992).

- what was discovered was an order-by-disorder effect, true for **any** spin, not just S=1/2

show that the original argument [14] in favour of a spin-liquid state was fallacious. The reason for this is the highly non-trivial,

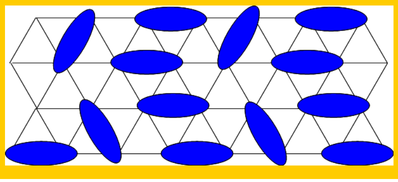
- is S=1/2 special? magnetization ≈ 0 (?) while classical value = NS/3 (up-up-down)

P. Fazekas and P. W. Anderson, *Phil Mag* **30**, 23 (1974).

Google Scholar:
29 and 11 cites

Anderson, *Mat. Res. Bul.* **8**, 153 (1973).

triangular foundation?



András Sütő

$$-\frac{1}{J}E_{\triangle}{}^{\text{sing}} = 0{\cdot}527 \pm 0{\cdot}004.$$

Anderson

$$-\frac{1}{J}E_{\triangle}{}^{\text{sing}} = 0{\cdot}482,$$

Sütő

$$-\frac{1}{J}E_{\triangle}{}^{\text{sw}} = \cancel{0{\cdot}463 \pm 0{\cdot}007}.$$

Néel, from Anderson

incorrect

In this paper we return to the original RVB problem of pure spin systems. In 1973 Anderson [3] conjectured that the isotropic, nearest-neighbour (*nn*) Heisenberg-model on the triangular lattice does not have LRO but instead the ground state should be envisaged as resonating between different arrangements of singlet pair bonds. Subsequent investigations [4] soon indicated that the original energetical arguments in favour of an RVB state are not conclusive, and recent work using variational [5],

Hence, the original expectations of Anderson (1973) about the excitation spectrum of the triangular antiferromagnet are only partially borne out : the singlet gap indeed is zero, but so, contrary to his conjecture, is the triplet gap. Therefore a conclusion about the impossibility of Néel-type symmetry-breaking cannot readily be made : there *are* low-lying excitations other than singlets. What really seems to matter is whether the singlet gap is zero or

J. Phys. Soc. Jpn. 52 (1983) Suppl. p. 183–186

THE SPIN WAVE THEORY IN ANTIFERROMAGNETIC LAYERED TRIANGULAR LATTICE

T. Oguchi

railroad-trestle	-0.54
spin wave	-0.539

correct

$$E_{gs}/N = -\frac{3}{2}JS^2\left[1 + \frac{0.436\,824}{2S} + \frac{0.021\,41}{(2S)^2}\right]$$

- singlet and triplet gaps both scale (1/N) to zero ⇛ not RVB
- 1/N scaling of E_{GS} in RVB paper is too generous, correct one ≈ Néel

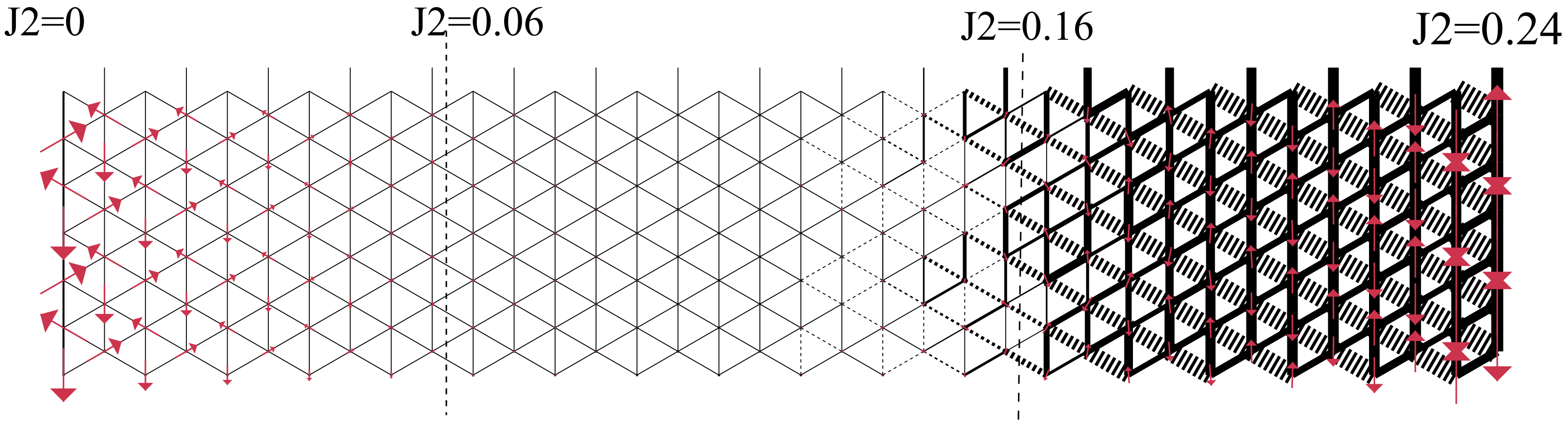
Google Scholar: 16 cites

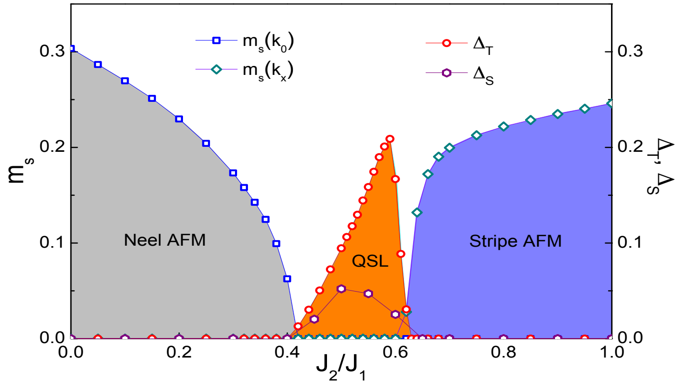
B. Kleine *etal.*, Z. Phys. B **87**, 103 (1992)

[4] András Sütő and Patrick Fazekas, Philos. Mag. **35**, 623 (1977)

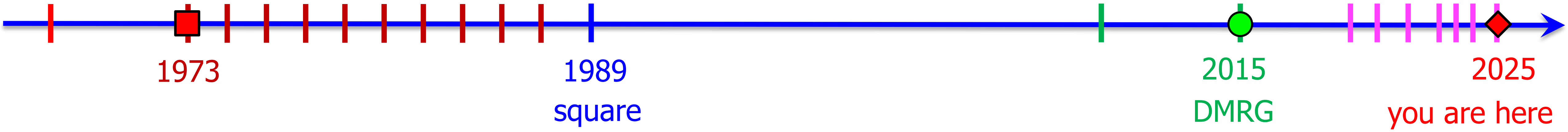
$J_1 - J_2$ timeline

- why did it take so long to study triangular case?
- classically: square ⇛ degenerate at J_2=0.5, triangular at J_2=1/8 ⇛ is not





- there were earlier claims about SL in J_1–J_2 triangular model …
- but nobody gave disznófing (much thought) about them, before DMRG



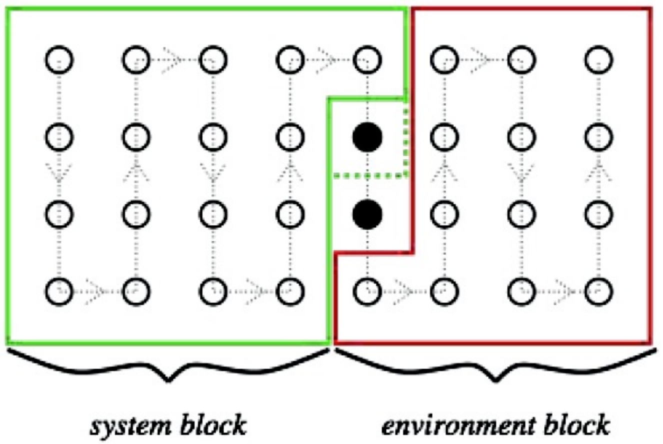
Z. Zhu and S. R. White, *Spin liquid phase of the S=1/2 J_1–J_2 Heisenberg model on the triangular lattice*, PRB **92**, 041105(R) (2015).

ode to DMRG

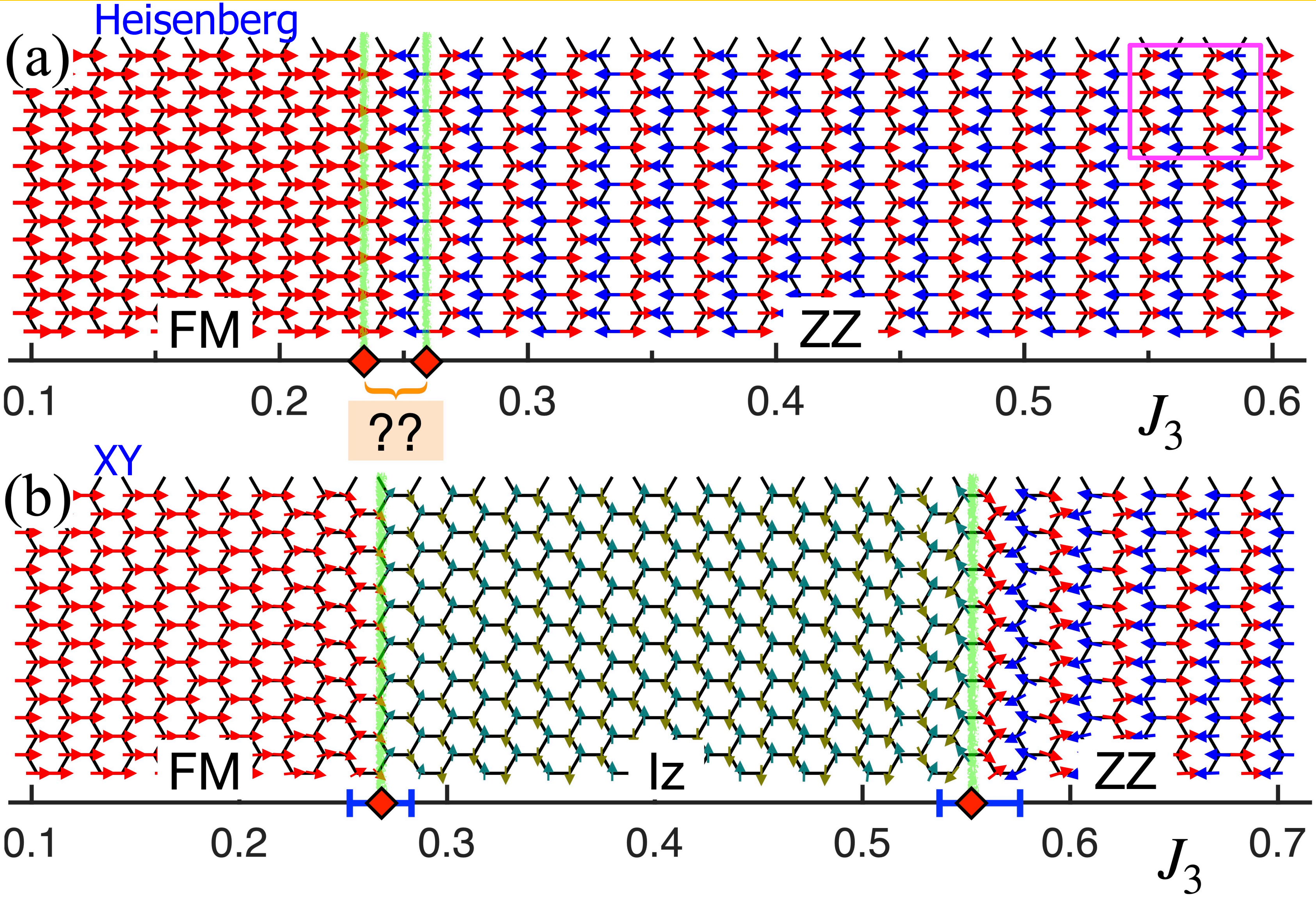
ode to DMRG

- it is a variational method in a generic sense,
  selecting the ground state from the low-energy sector

- it **wants** to find the low-entanglement (typically = ordered) state,
  when it cannot ⇛ this finding is (very) robust



sheer power of DMRG



⇛ **DMRG scans**

⇛ great exploration tool!

"direct" look at orders,

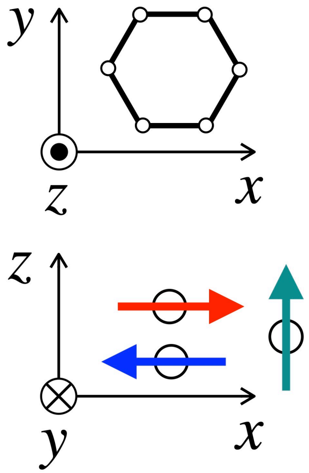
focus on suspicious regions

⇛ open-periodic

⇛ $m \sim 3000$, error $\mathcal{O}(10^{-5})$

⇛ further "zooms"

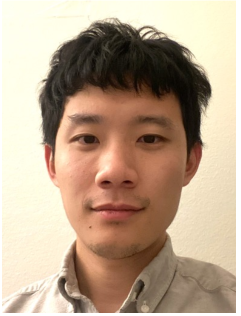
⇛ non-scans, FS scaling, …



S. Jiang, S. R. White, and **SC**, PRB **108**, L180406 (2023). J_1-J_3 F-AF XXZ model

our story …

Cesar Gallegos



Shengtao Jiang

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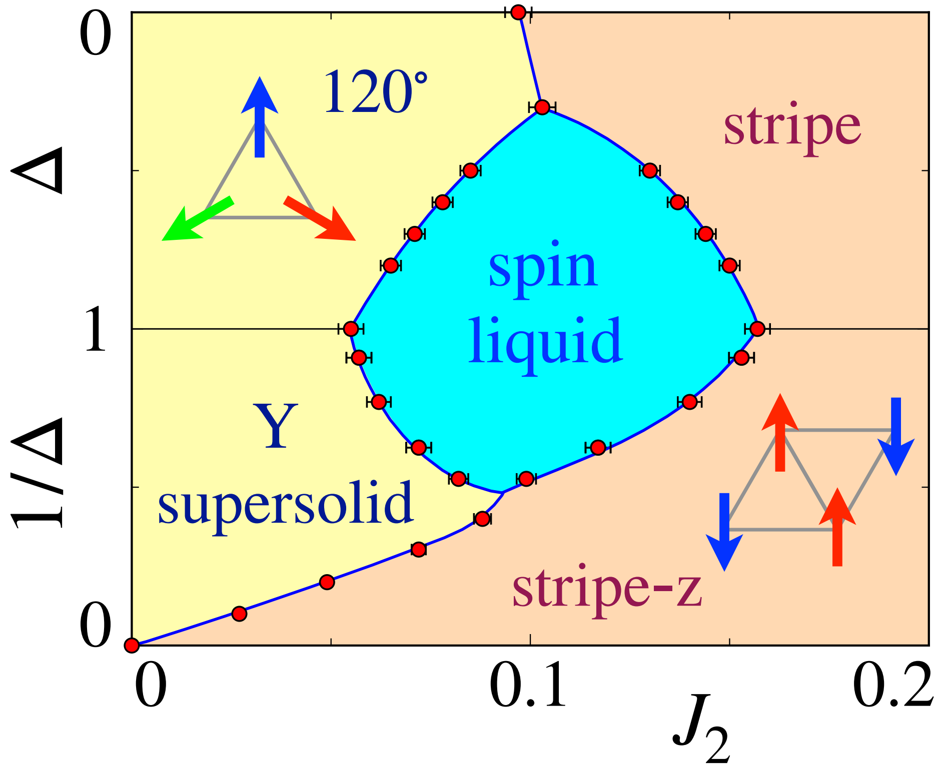
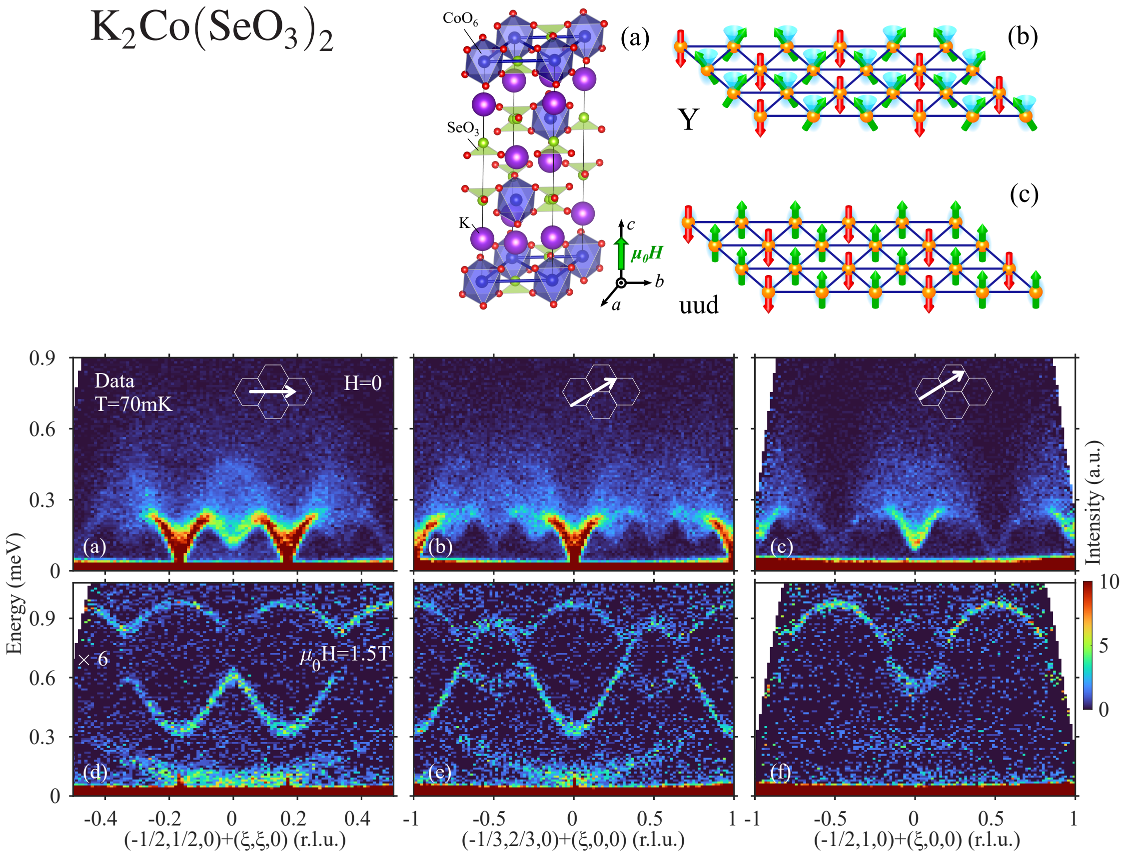
University of California, Irvine

Steven White

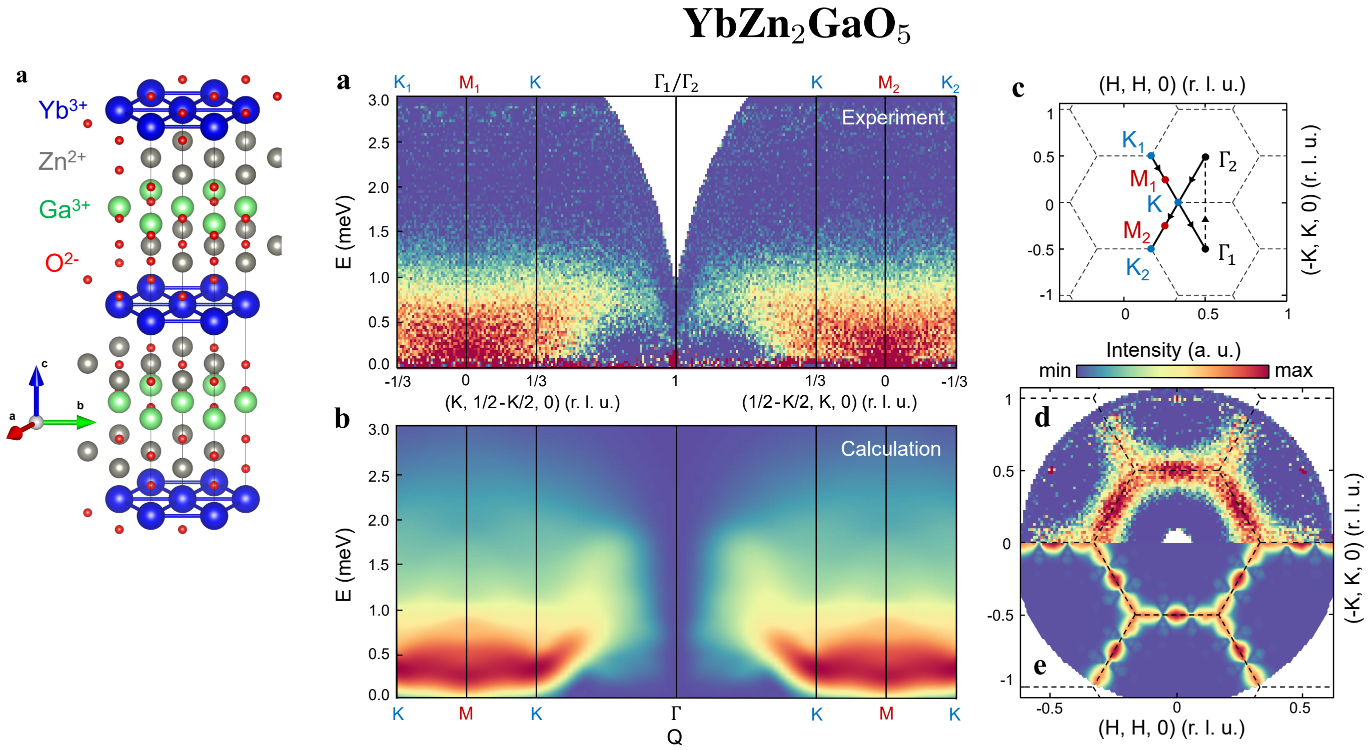


C. A. Gallegos, S. Jiang, S. R. White, and **SC**, PRL **134**, 196702 (2025).

KITP 2023, etc.: easy-axis models



- **easy-axis** triangular lattice magnets: supersolids, rich dynamical properties



- was proposed (and modeled as) an **easy-axis**, ***J_1-J_2 XXZ***, triangular-lattice magnet (model)
- what is its (quantum, S =1/2) phase diagram??

M. Zhu *etal.*, PRL **133**, 186704 (2024)

R. Bag *etal.*, PRL **133**, 266703 (2024)

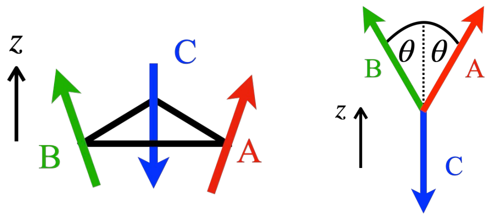
phase diagram

$$\hat{\mathcal{H}} = \sum_{n=1,2} \sum_{\langle ij \rangle_n} J_n \left(S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z \right)$$

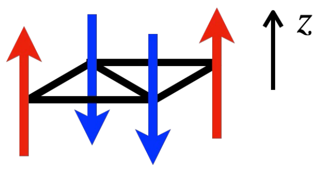
spin-1/2

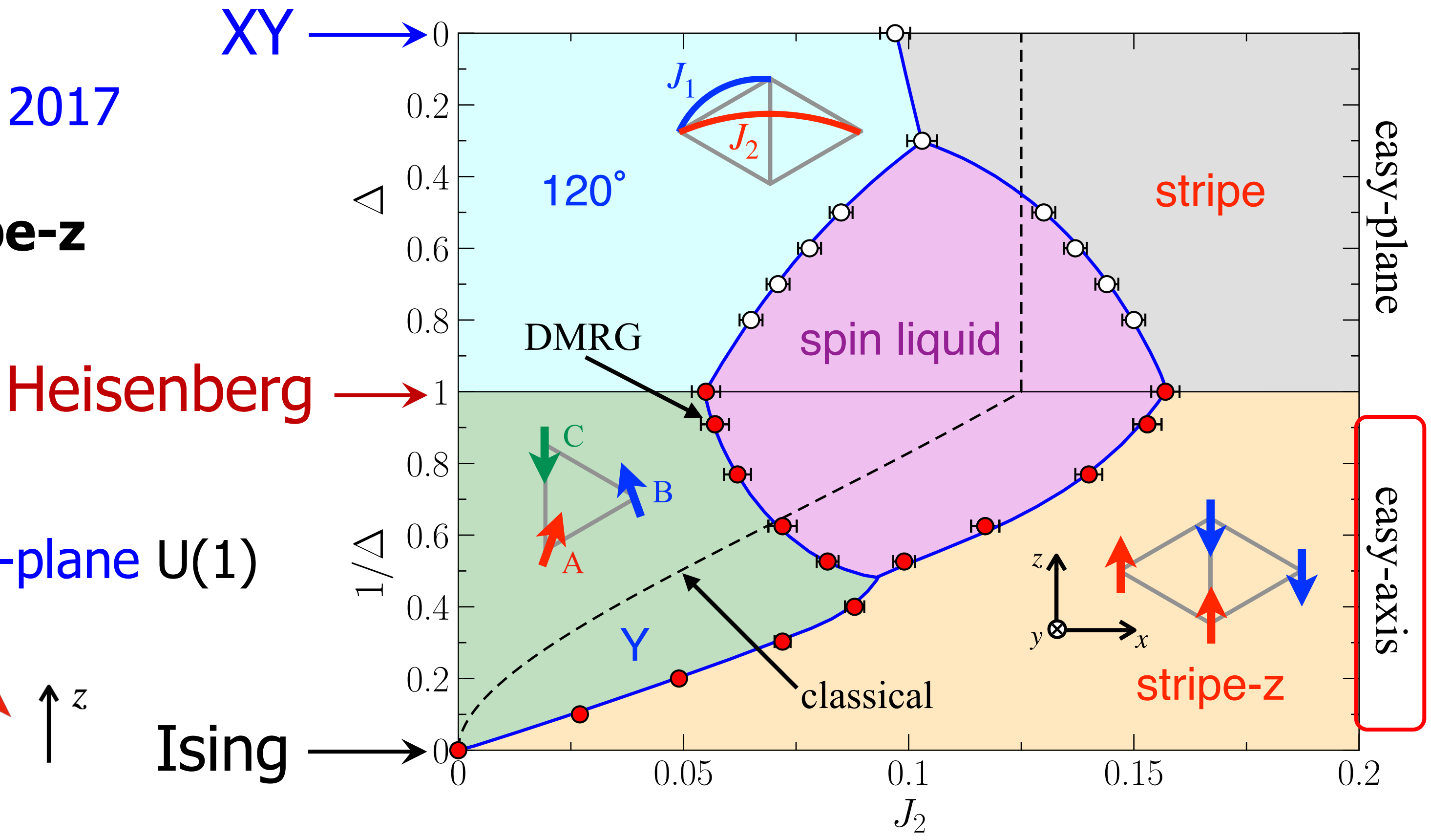
- Heisenberg ⇛ much studied
- easy-plane part ⇛ DMRG and VMC (*) 2017

- easy-axis, ordered phases: **Y** and **stripe-z**



- **Y** ⇛ supersolid: 3-sublattice solid + in-plane U(1)
- **stripe-z** ⇛ 4-sublattice, order along z



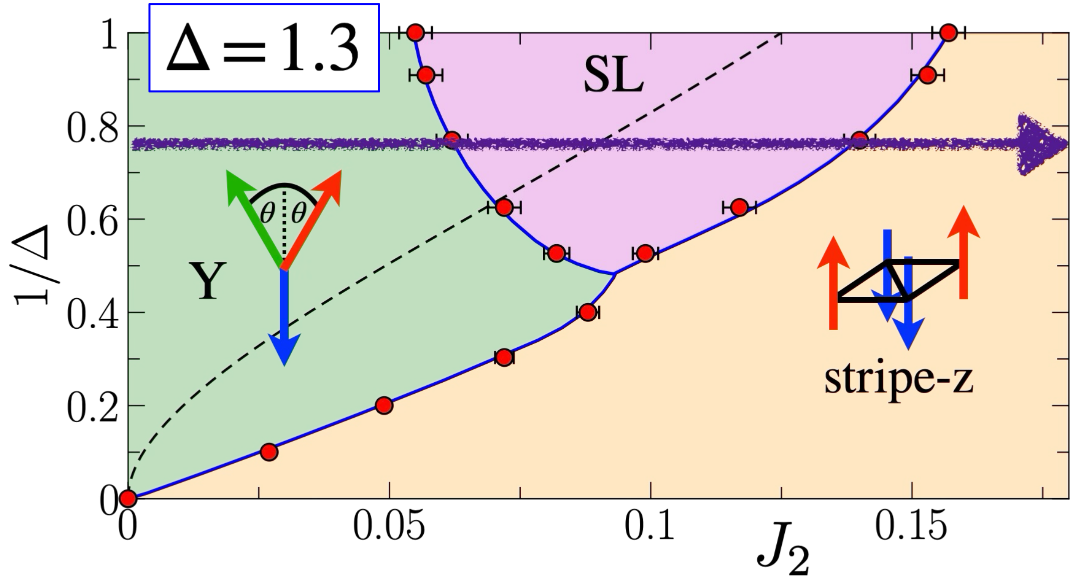


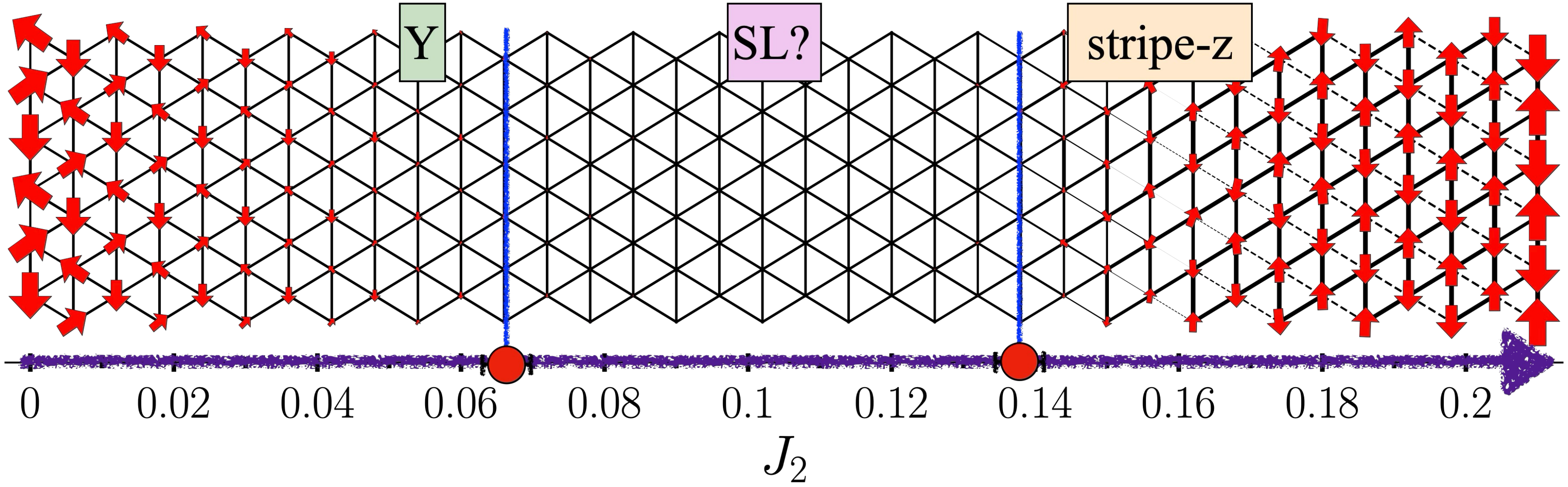
C. A. Gallegos *etal.*, PRL **134**, 196702 (2025).

(*) Z. Zhu, *etal*, PRL **119**, 157201 (2017); Iaconis *etal*, SciPost Phys. **4**, 3 (2018)

agnostic approach: DMRG scans, I

- J_2 ⇛ varied along the 6x36 cylinder, fixed Δ
- measure local order $\langle \boldsymbol{S}_i \rangle$
- ⇛ faithful visual extent of the phases

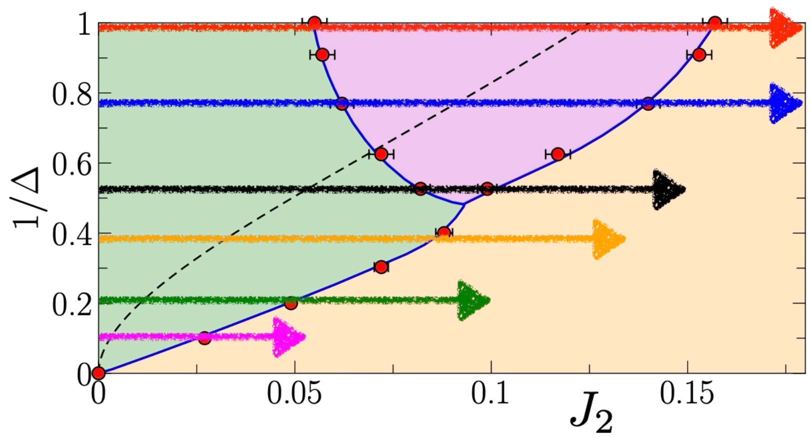


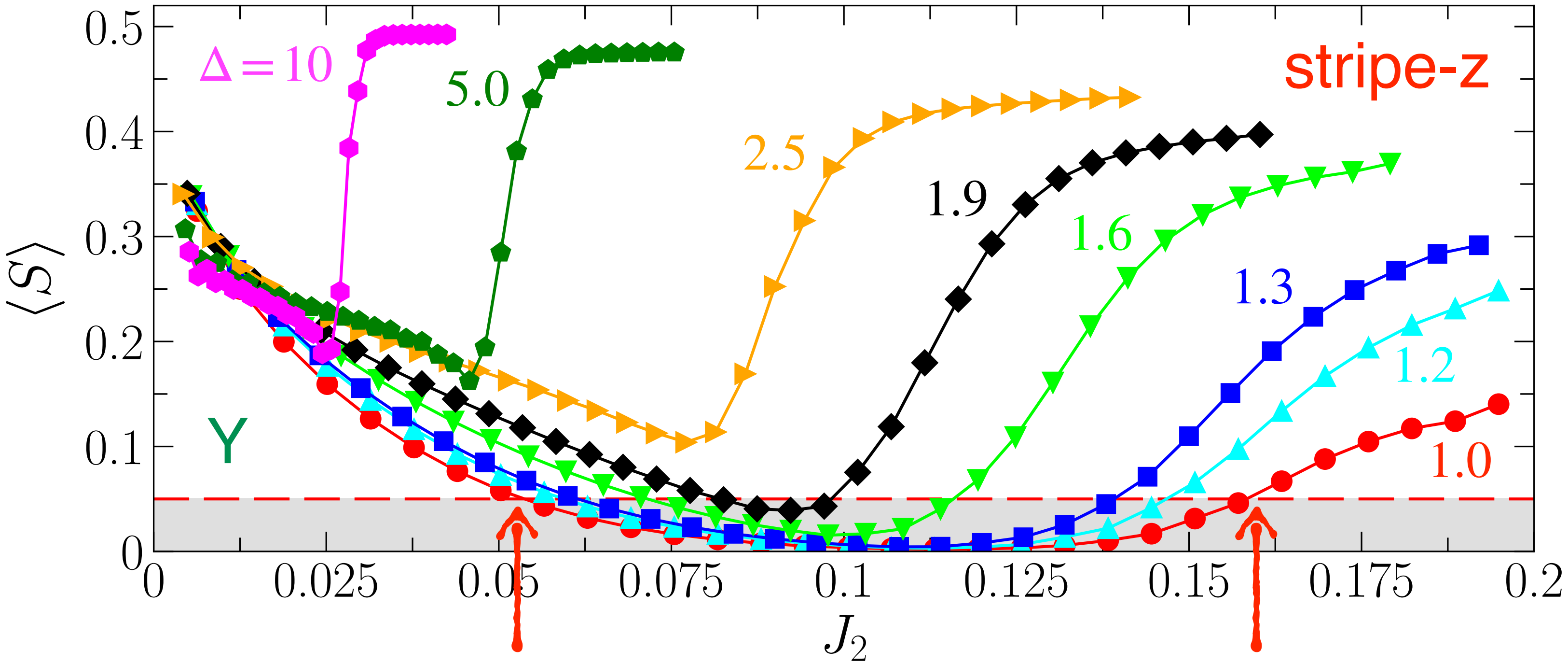


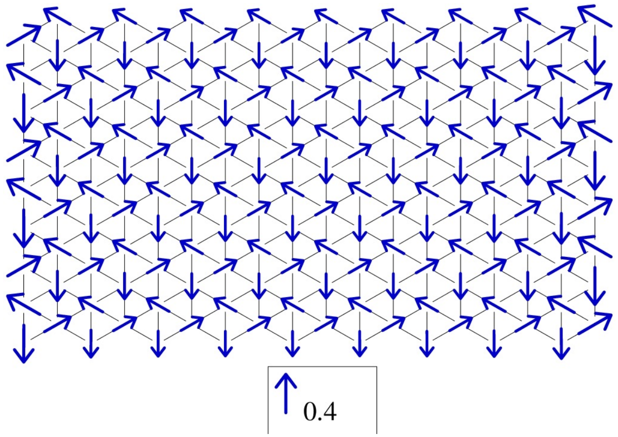
C. A. Gallegos, S. Jiang, S. R. White, and **SC**, PRL **134**, 196702 (2025).

agnostic approach: DMRG scans, II

- **SL** ⇛ matching Heisenberg boundaries, cutoff $\langle S \rangle \leq 0.05$
- **SL** ⇛ verified by **non-scans** (all parameters fixed) in up to 9x20 cylinders (up to 5000 states)



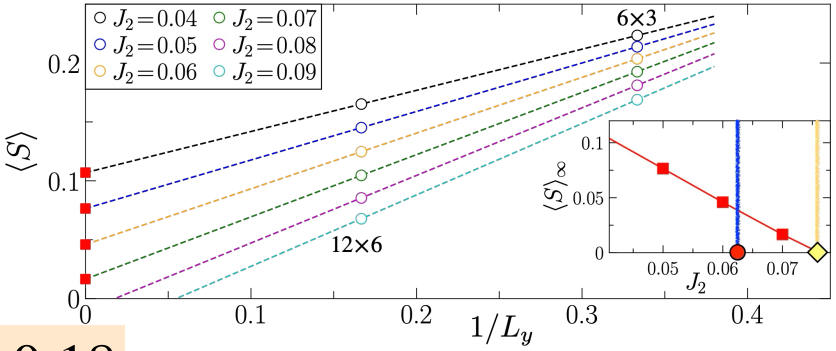


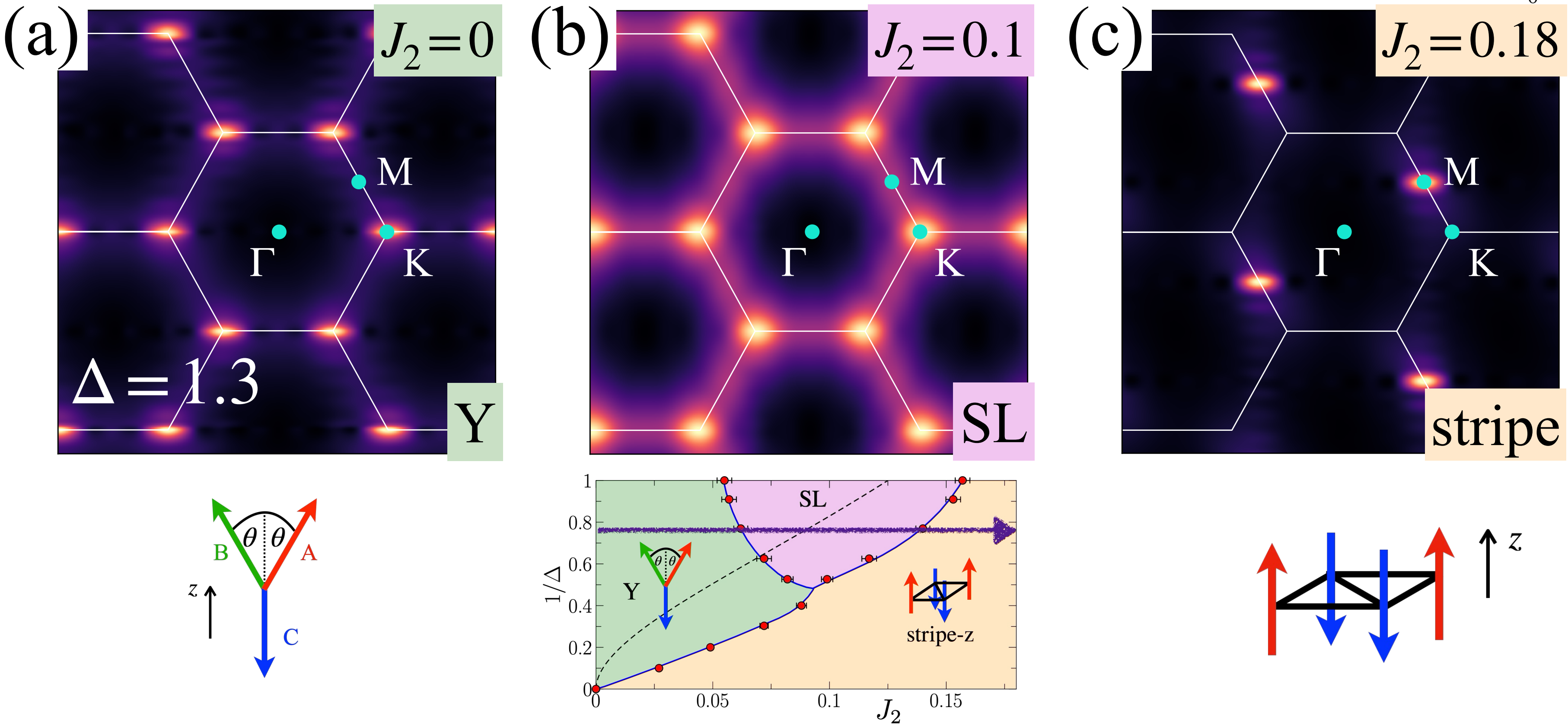


C. A. Gallegos, S. Jiang, S. R. White, and **SC**, PRL **134**, 196702 (2025).

non-scans: nice visuals …

- $S(\mathbf{q})$ ⇛ static structure factor
- **SL** ⇛ broadened K peaks ⇛ “molten” **Y** phase (soft transition)
- transition **SL-stripe-z** ⇛ 1st order





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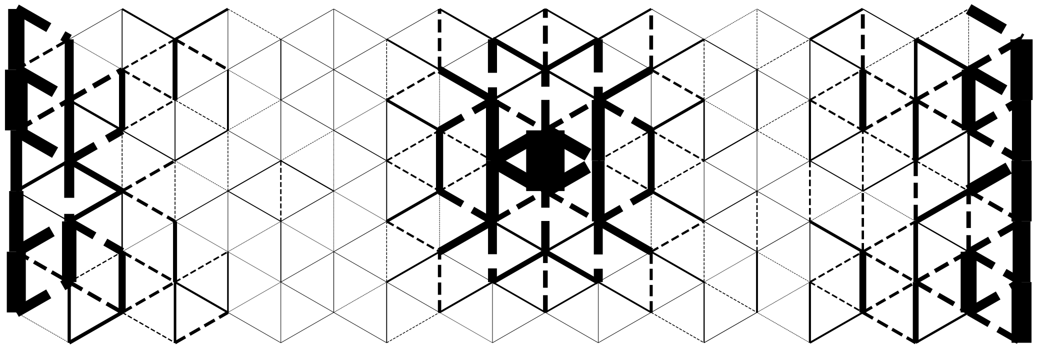
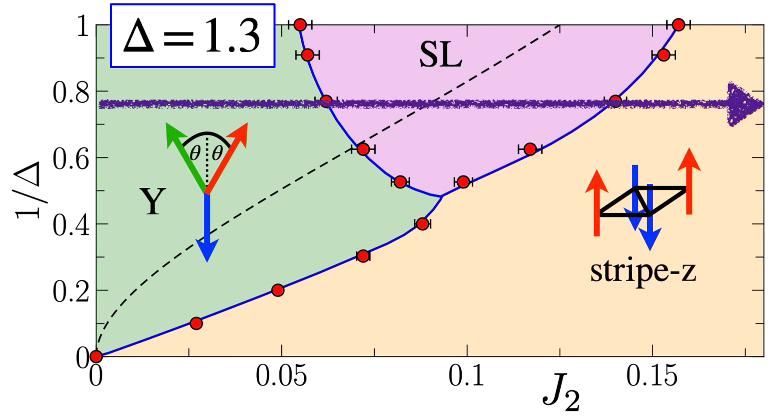
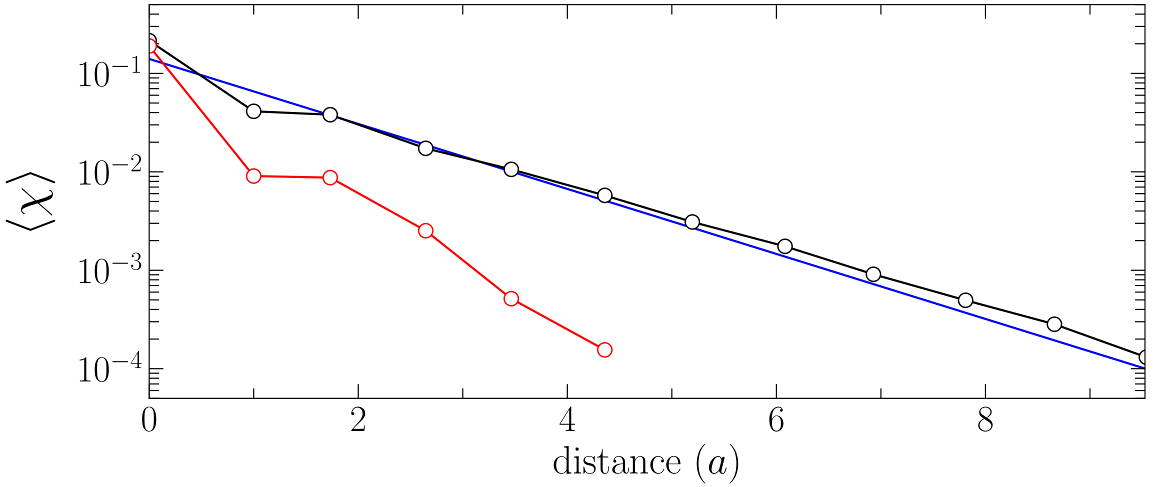
characterizing **SL** …

- strongly pinned singlet bond in the center
- bonds: $\langle \mathbf{S}_i \mathbf{S}_j \rangle - \langle \mathbf{S}_i \mathbf{S}_j \rangle_{average}$ $\Rrightarrow$ no VBS response

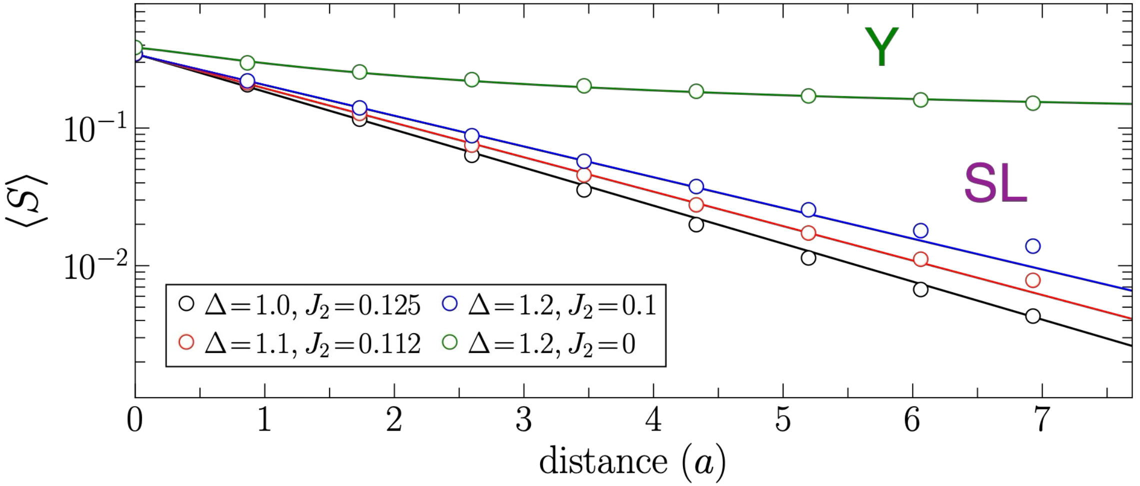




- chirality induced, edge and center $\Rrightarrow$ no response



- spin pinned at the edge
  $\Rrightarrow$ correlation length in **SL** $(\xi \approx 1.6a)$



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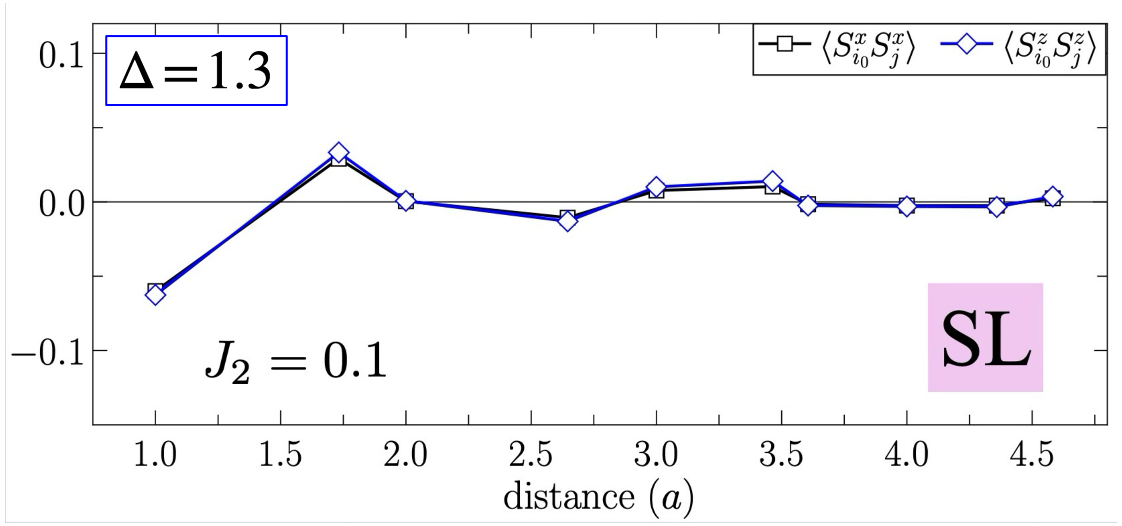
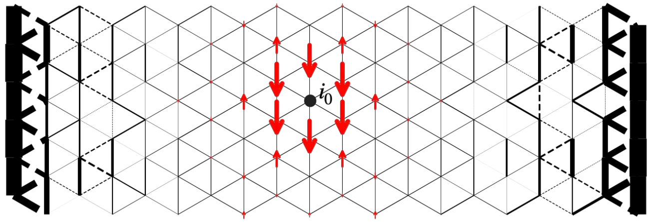
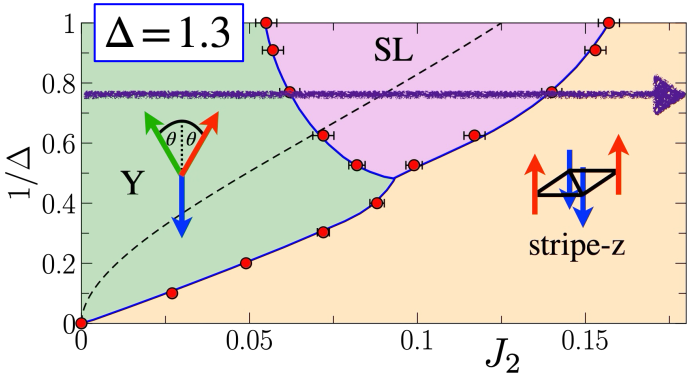
SL seems fully isotropic!

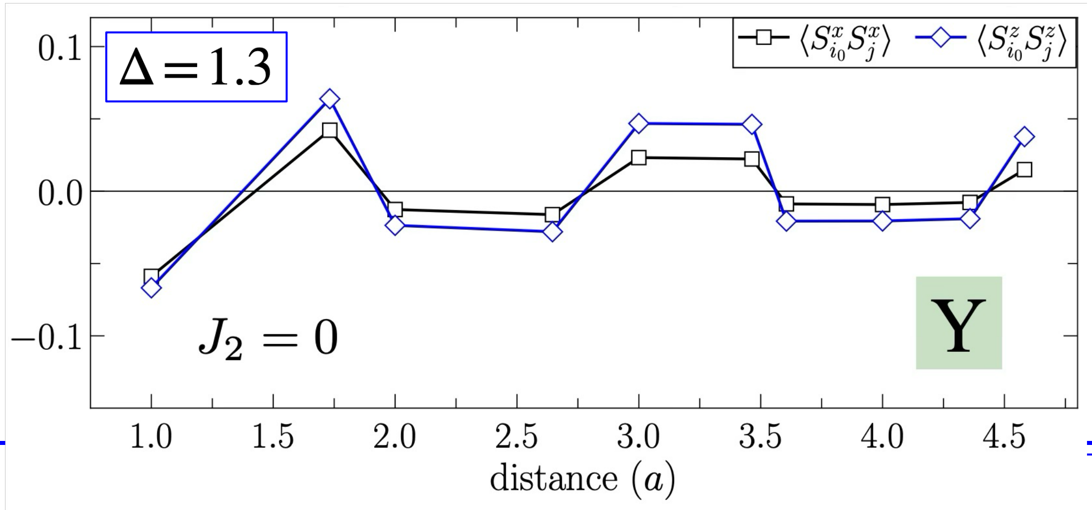
- real-space correlation function

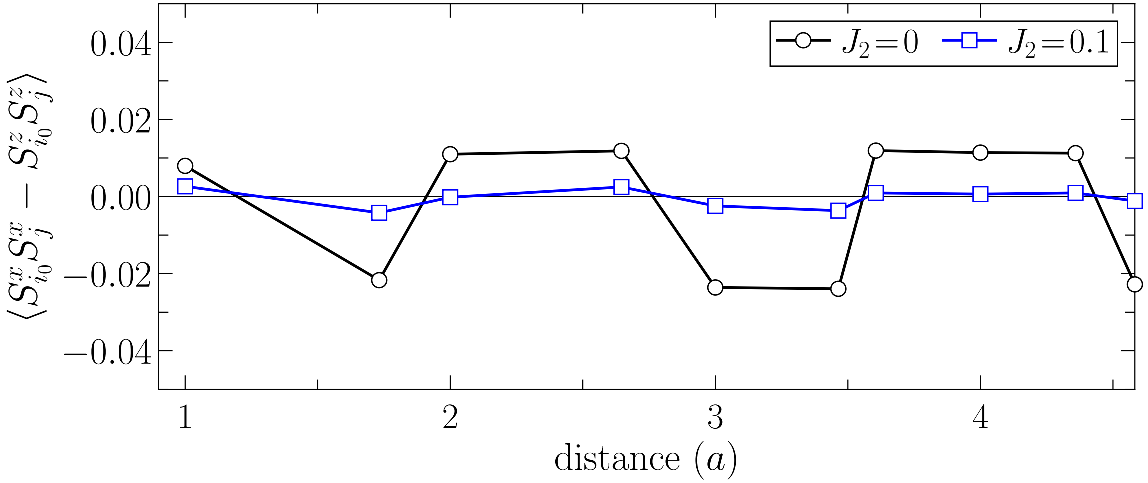






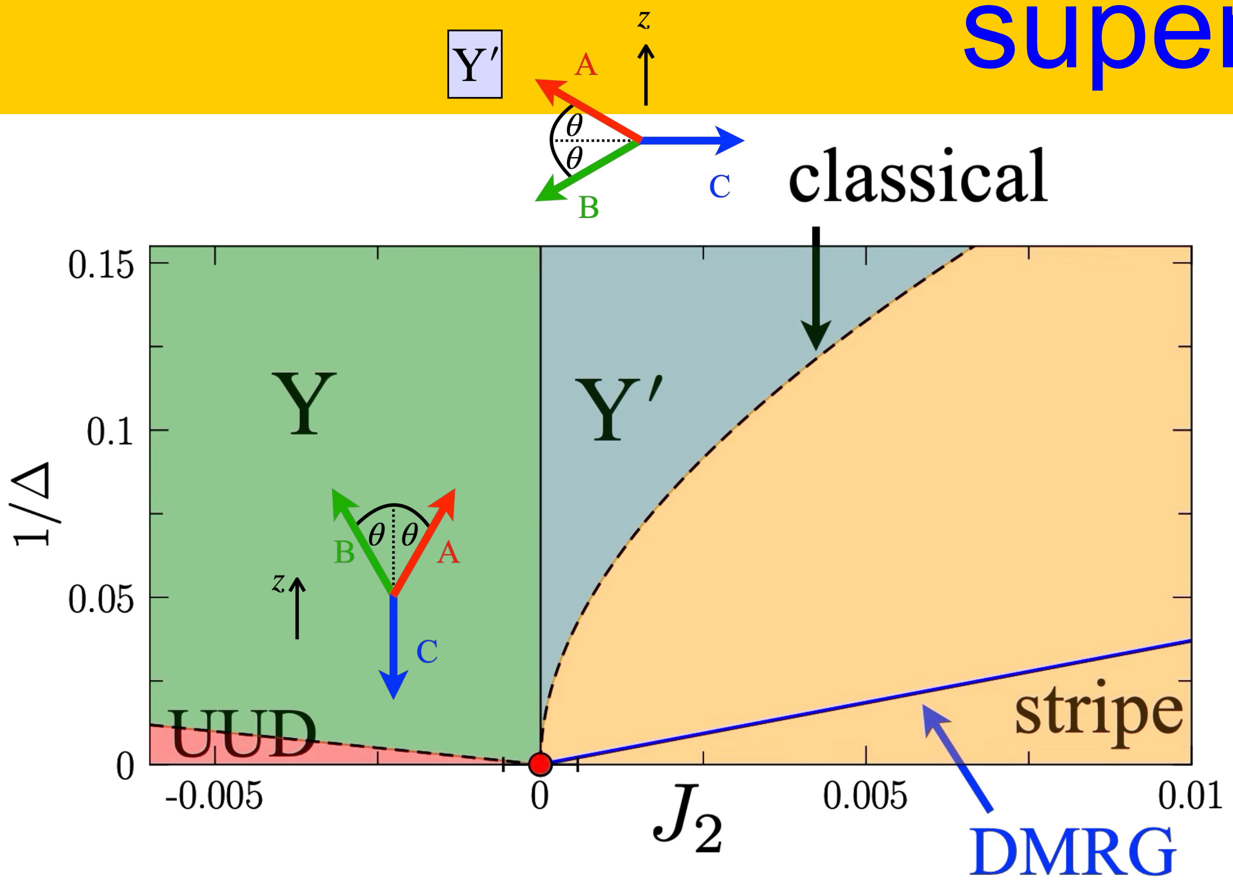
- far away from Heisenberg, yet in-plane and out-of-plane correlations are nearly equal
- no quantum numbers kept in DMRG
- ⇛ **symmetry enrichment?**

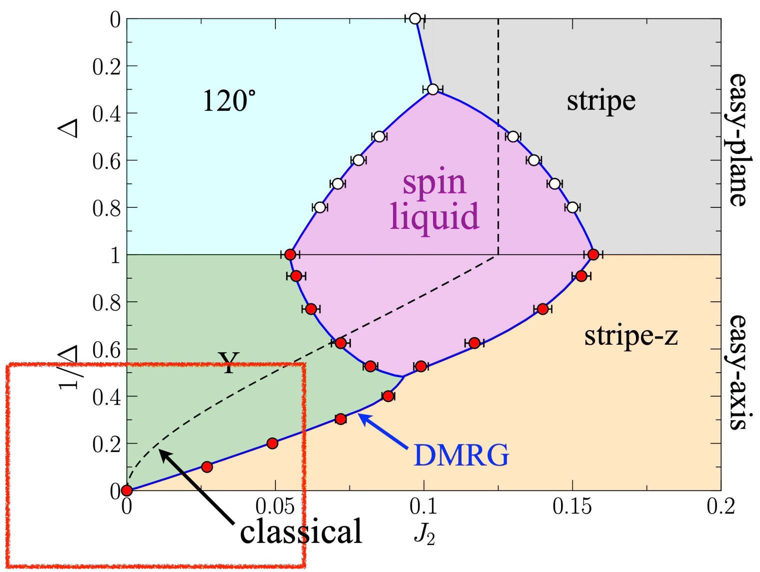




R. White, and SC, PRL 134, 196702 (2025).

supersolid, I





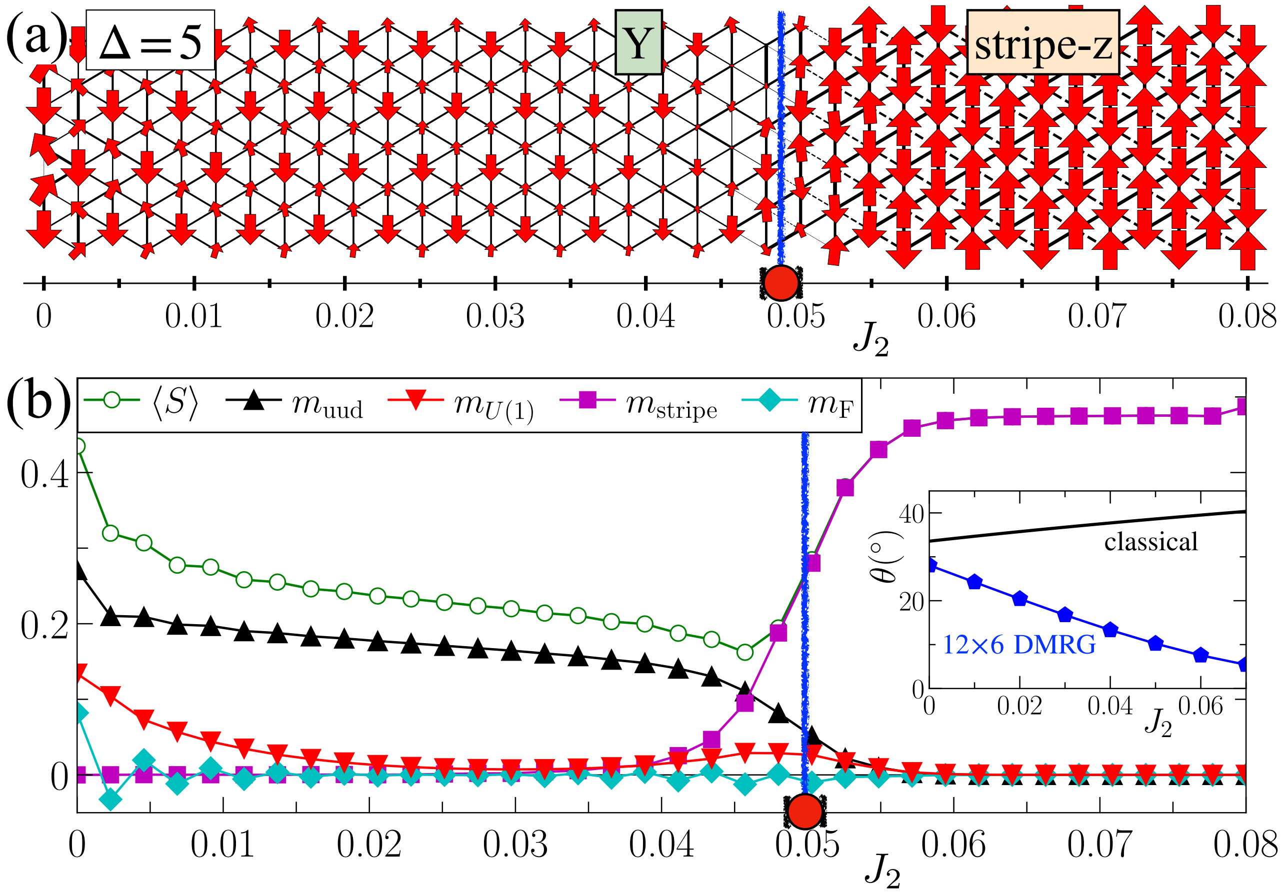
- classically, Y′ is the GS for $J_2>0$, and Y′-to-stripe boundary is anomalous: $1/\Delta \propto \sqrt{J_2}$

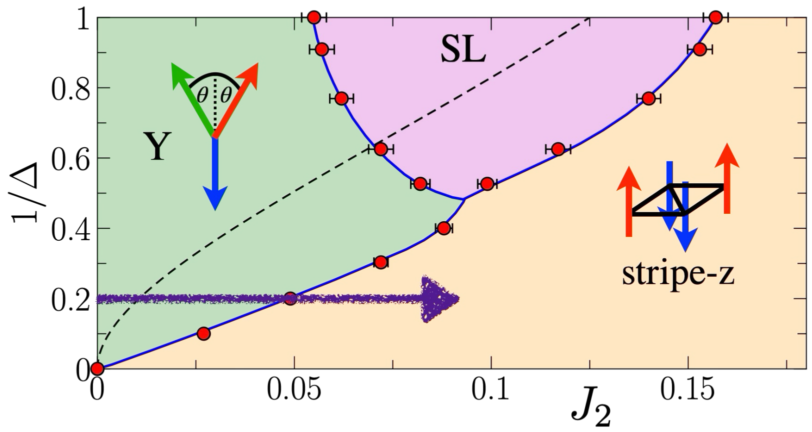
  this is related to the accidental degeneracy along the $J_2=0$ line and $E_{\rm cl} \propto -1-1/\Delta^2$

- quantum effects select Y for the GS and make the Y-stripe boundary "normal": $1/\Delta \propto J_2$

C. A. Gallegos, S. Jiang, S. R. White, and **SC**, PRL **134**, 196702 (2025).

supersolid, II, more scans





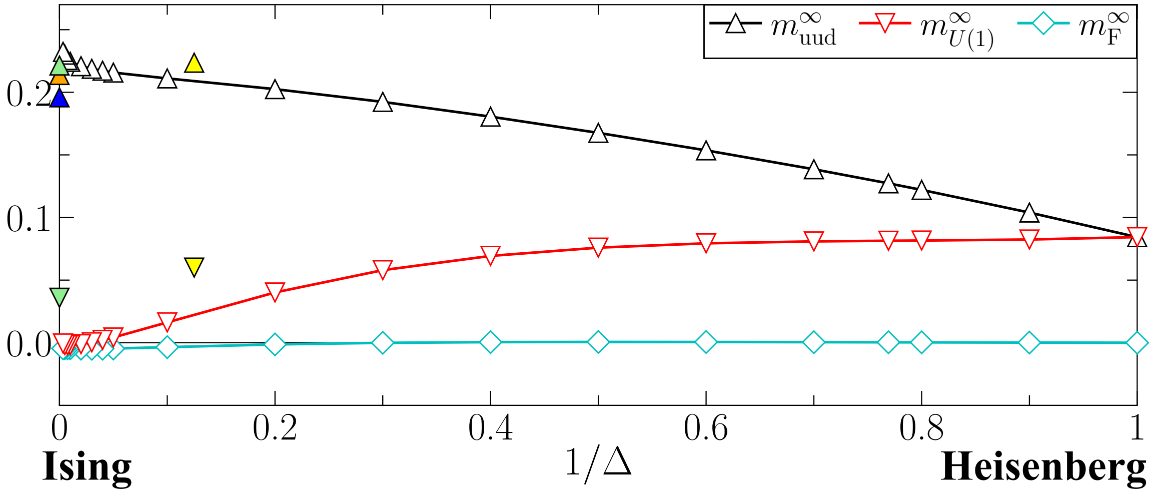
- direct Y-to-stripe transition, small U(1) component
- total moment, m_F, is close to zero, as if canceling for <u>every unit cell</u>: **down + 1/2 up + 1/2 up**

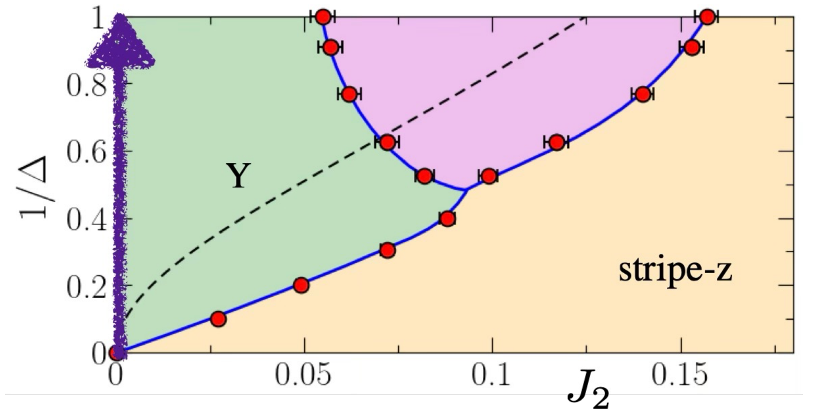
C. A. Gallegos, S. Jiang, S. R. White, and **SC**, PRL **134**, 196702 (2025).

supersolid, III, J_1–only

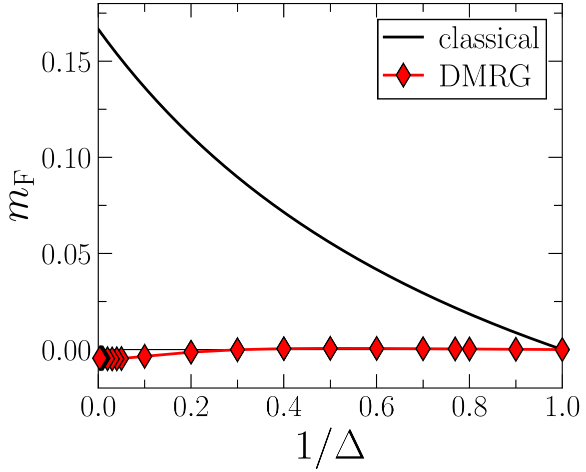
$$\hat{\mathcal{H}} = J_1 \sum_{\langle ij \rangle_1} \left(S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z \right)$$

[full symbols = MC data from earlier works]





- classically, m_F → 1/6 in the Ising limit, DMRG ⇛ zero
- resolution ⇛ rewrite Hamiltonian (*)



$$\boxed{\hat{\mathcal{H}}_1 = \frac{1}{2} \sum_{\triangle} \left(\left(S_{\triangle}^{\perp}\right)^2 + \Delta \left(S_{\triangle}^{z}\right)^2 \right) - \frac{3}{2}(\Delta - 1) \cancel{\sum_i \left(S_i^z\right)^2}}$$

- **S=1/2 is really special! last term = const**
- the rest implies GS with $S_{\triangle}^{\alpha} = 0$

C. A. Gallegos *etal.*, PRL **134**, 196702 (2025).

(*) as in the Heisenberg limit: $\hat{\mathcal{H}}_1 = \frac{1}{2} \sum_{\triangle} \mathbf{S}_{\triangle}^2 + \mathrm{const}$

conclusions

☑ everyone makes mistakes, it takes courage to correct them

☑ **anisotropic directions ⇛ rich physics**

☑ large SL region in the XXZ phase diagram

☑ SL ⇛ isotropic (?); (why? what kind?)

☑ zero m_F in the S=1/2 case

☑ more puzzles near Ising (dynamics, etc.)