

**From spin chains to corner-sharing
triangles in the compound family
 ACuTe_2O_6 A=Sr, Ba, Pb**

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Berlin Technical University, Germany

Outline

ACuTe₂O₆ family of magnets
Isostructural, cubic crystal symmetry
Quantum spin-1/2

Explore the space group and possible magnetic lattices

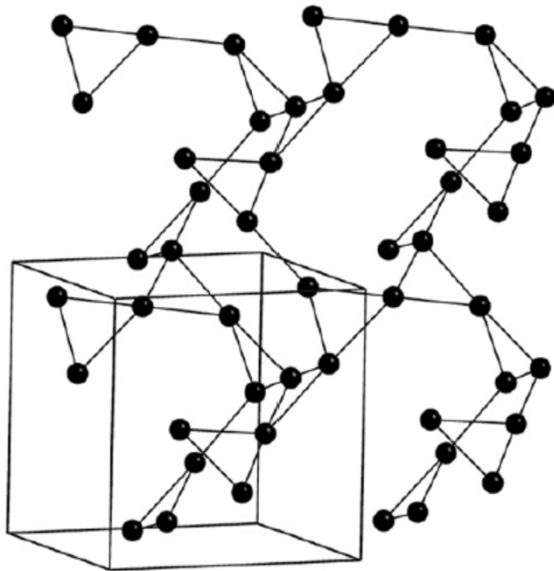
BaCuTe₂O₆ – coupled spin chains

SrCuTe₂O₆ – coupled spin chains

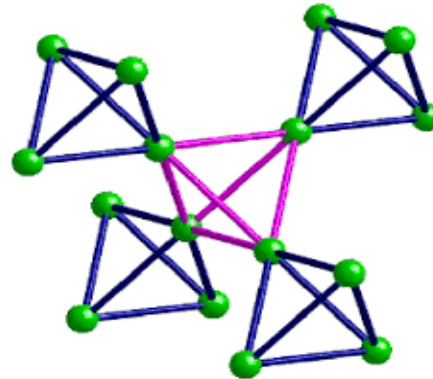
PbCuTe₂O₆ – corner-sharing triangles
– spin liquid candidate

3-dimensional arrangements of corner-sharing triangles

Hyperkagome lattice—
2 triangles at each corner
10 spins loops

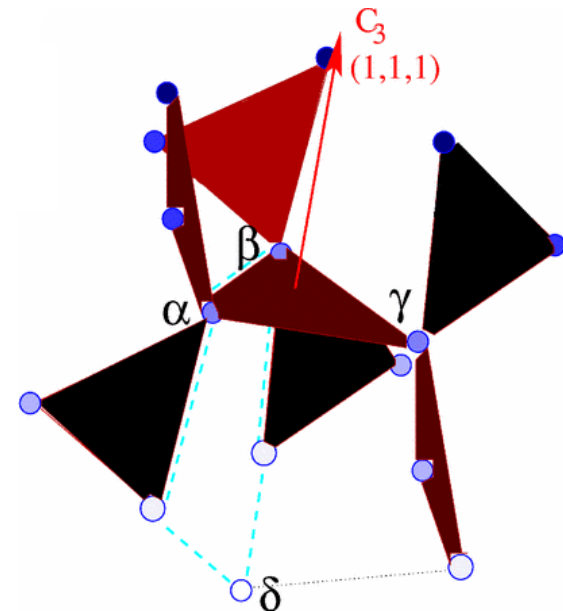


*M. V. Talanov, V. B. Shirokov, V. M. Talanov,
Nanosystems: Physics, Chemistry, Mathematics,
6, 442 (2015)*



Pyrochlore

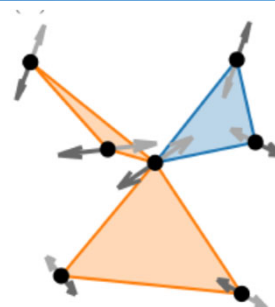
Trillium lattice—
3 triangles at each corner
5 spin loops



*J. M. Hopkinson, H.-Y. Kee
Phys. Rev. B 74, 224441 (2006)*

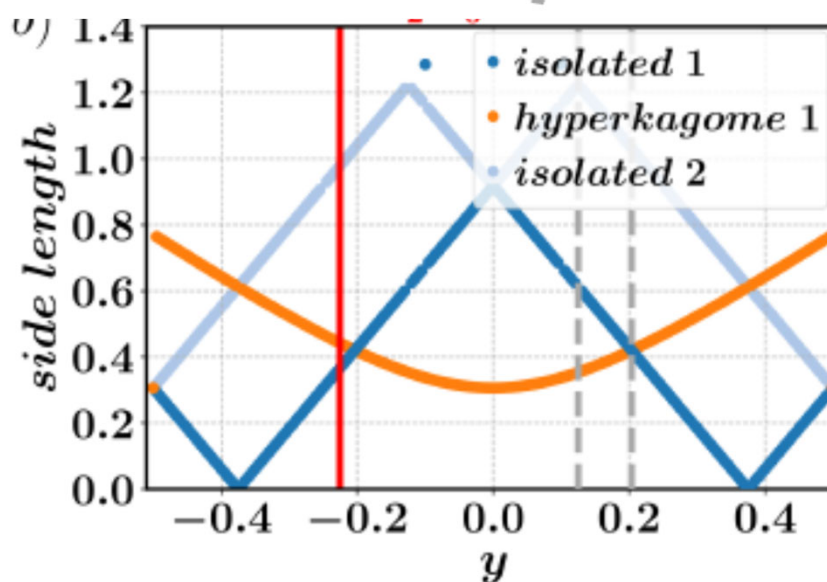
Distorted Windmill Lattice

- Cubic space group $P4_132$
- Magnetic ions at Wyckoff 12d
- 12 equivalent sites per cell
- controlled by parameter y



- For general y , each magnetic ion in
- 2 hyperkagome triangles
 - 1 isolated triangle

Sublattice	Position
1	$(3/4 + y, 3/8, 1 - y)$
2	$(1/2 + y, 1/4 - y, 7/8)$
3	$(5/8, 1/2 - y, 3/4 - y)$
4	$(1/2 - y, 3/4 - y, 5/8)$
5	$(3/4 - y, 5/8, 1/2 - y)$
6	$(7/8, 1/2 + y, 1/4 - y)$
7	$(1 - y, 3/4 + y, 3/8)$
8	$(1/4 - y, 7/8, 1/2 + y)$
9	$(3/8, 1 - y, 3/4 + y)$
10	$(1/4 + y, 1/8, y)$
11	$(1/8, y, 1/4 + y)$
12	$(y, 1/4 + y, 1/8)$



$$y^* = \frac{9 - \sqrt{33}}{16} \approx 0.2035$$

For $-\frac{1}{2} < y < -y^*$ & $y^* < y < \frac{1}{2}$: isolated triangle

For $-y^* < y < y^*$ & $y = \pm \frac{1}{2}$: hyperkagome

For $y = \pm y^*$: hyperkagome & isolated triangles

For $y = \pm \frac{3}{8} = 0.375$: 4 sites, trillium lattice

A. Fancelli, J. Reuther, B. Lake
Phys. Rev. B 108, 184415 (2023)

B. Lake; Budapest, Oct 2025

From hyperkagome to trillium

ACuTe₂O₆ A=Sr, Ba, Pb - spin-1/2 windmill lattices

P4₁32, Cu²⁺ S=1/2 @ 12d Wyckoff site

Neutron, powder by solid state reaction, 15 K

Atom	Mult.	x	y	z	B _{iso}
Te	24e	0.3371(3)	0.9180(3)	0.0640(3)	0.98(9)
Ba1	8c	0.0563(3)	0.0563(3)	0.0563(3)	0.36(19)
Ba2	4b	0.3750	0.6250	0.1250	0.96(24)
Cu	12d	0.4738(2)	0.8750	0.2761(2)	0.80(10)
O1	24e	0.5763(3)	0.9242(3)	0.3737(3)	1.02(10)
O2	24e	0.2662(2)	0.8115(5)	0.9900(3)	1.42(10)
O3	24e	0.2267(3)	0.9739(3)	0.1372(3)	1.39(9)

$$(1/4 - y, 7/8, 1/2 + y)$$

PbCuTe₂O₆, a=12.497 Å, y = -0.2258

BaCuTe₂O₆, a=12.829 Å, y = -0.2239

SrCuTe₂O₆, a=12.437 Å, y = -0.2255

For all three compounds

$$|y| > |y^*| = 0.2035$$

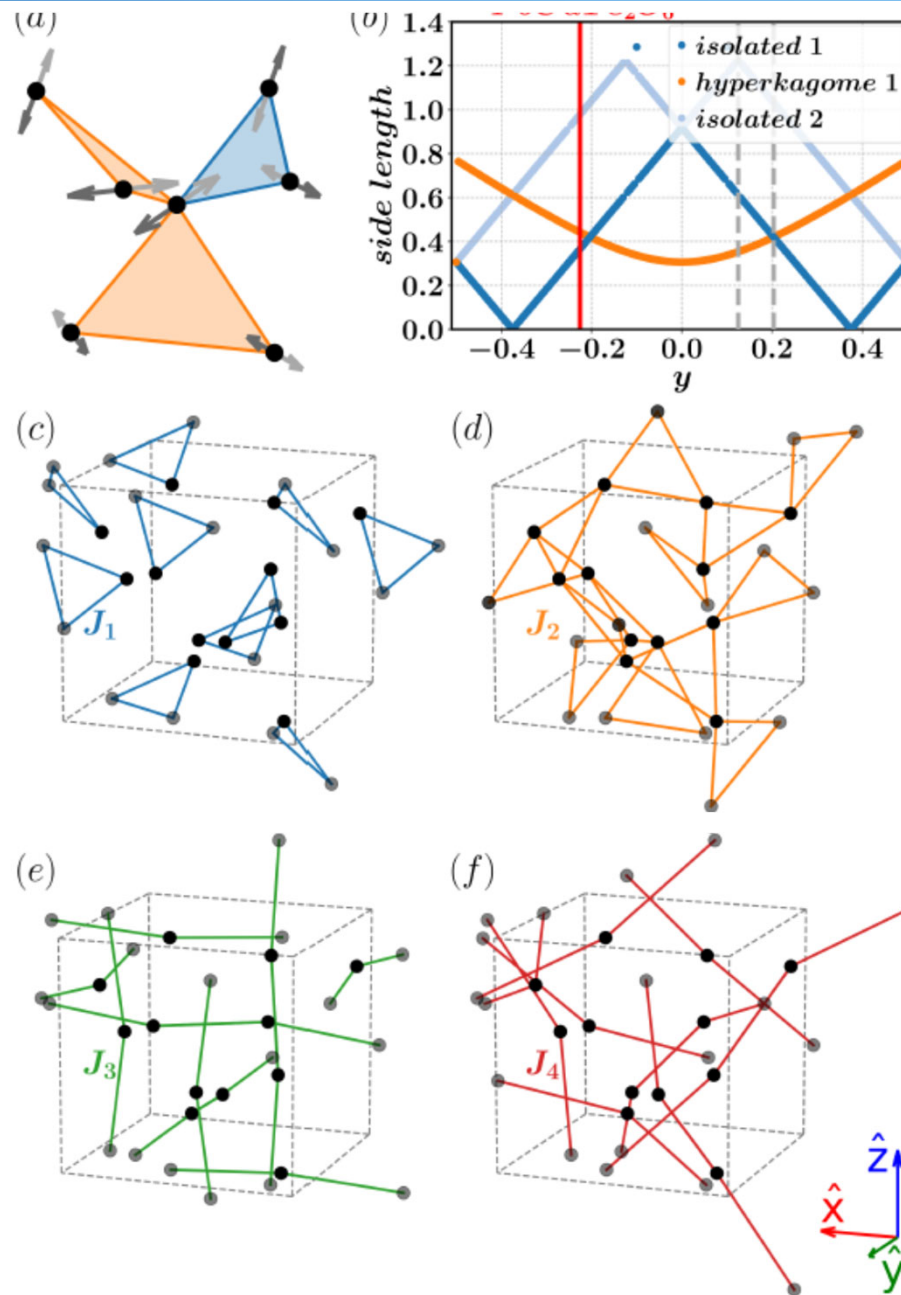
1st neighbor J_1 is the isolated triangle

2nd neighbor J_2 is the hyperkagome

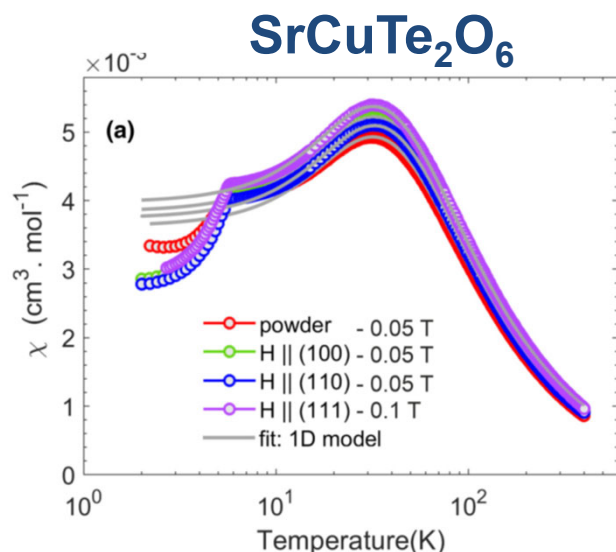
But bond distances do not translate simply into interaction strengths

J_3 gives chains parallel to a, b, c

J_4 gives chains parallel to [1,1,0] etc

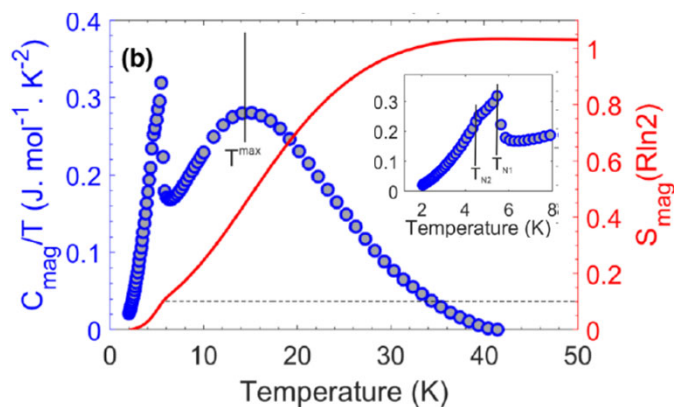


ACuTe₂O₆ A=Sr, Ba, Pb - DC susceptibility, heat capacity



$$\theta_{CW} = -28 \pm 0.5 \text{ K} \Rightarrow \text{AFM}$$

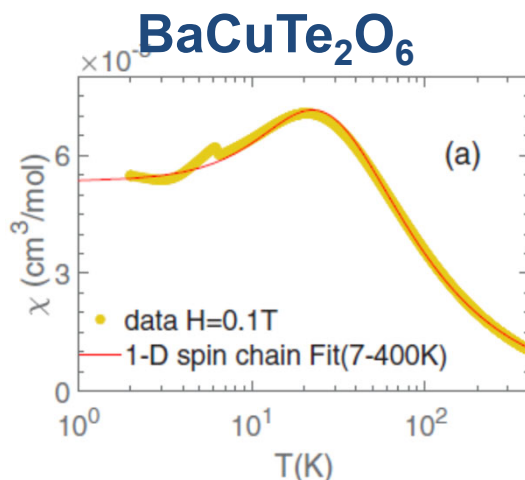
$$\mu_{eff} = 1.85 \mu_B \Rightarrow S=1/2$$



$$T_{N1} \approx 5.5 \text{ K} \text{ \& } T_{N2} \approx 4.5 \text{ K}$$

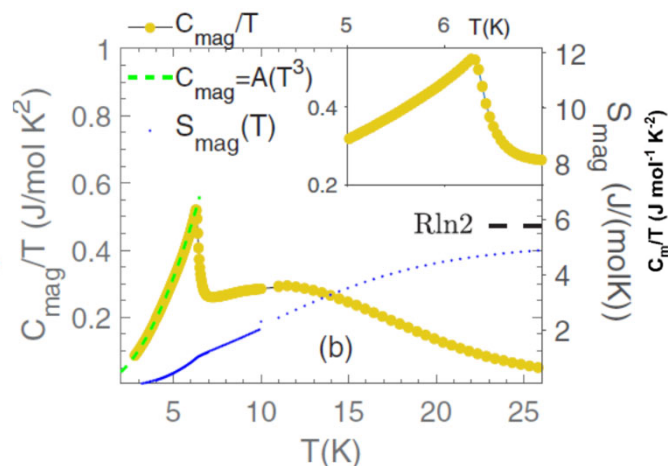
$$\Rightarrow \text{AFM order}$$

S. Chillal et al
PRB 102, 224424 (2020)



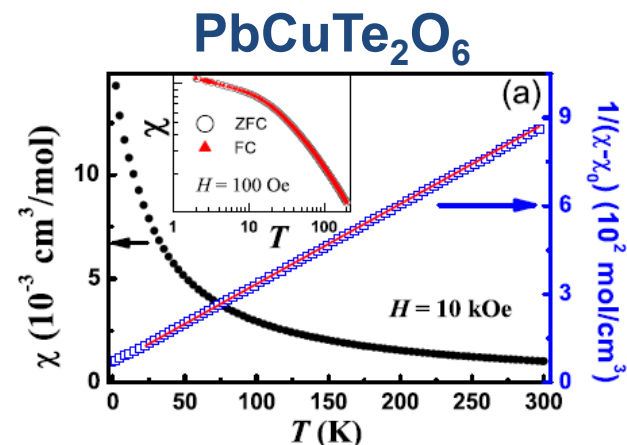
$$\theta_{CW} = -33 \pm 0.5 \text{ K} \Rightarrow \text{AFM}$$

$$\mu_{eff} = 1.96 \mu_B \Rightarrow S=1/2$$



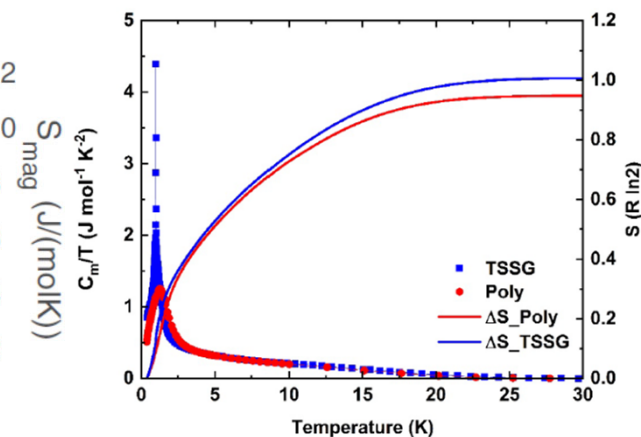
$$T_N = 6.31 \text{ K} \Rightarrow \text{AFM order}$$

A. Samartzis et al
PRB 103, 094417 (2021)



$$\theta_{CW} = -22 \pm 0.5 \text{ K} \Rightarrow \text{AFM}$$

$$\mu_{eff} = 1.75 \mu_B \Rightarrow S=1/2$$



$$T_C < 1.0 \text{ K} \Rightarrow \text{NO order}$$

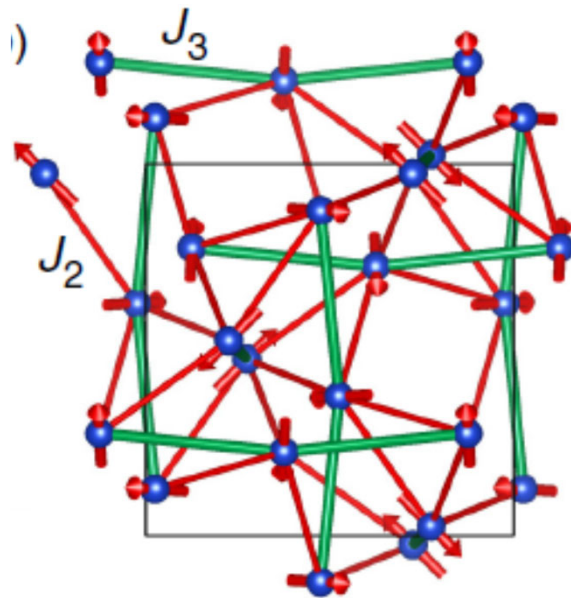
B. Koteswararao et al
PRB 90 035141 (2014)
A. Hanna et al. PRM 5 113401 (2021)

All compounds show suppression of magnetic order



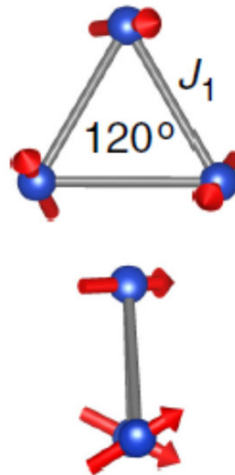
Magnetic structure of BaCuTe₂O₆

$q=(0,0,1)$, $m=0.46 \mu_B/\text{Cu}^{2+}$ suppressed compared to $1 \mu_B/\text{Cu}^{2+}$
 $\Rightarrow$ low-dimensional / frustrated



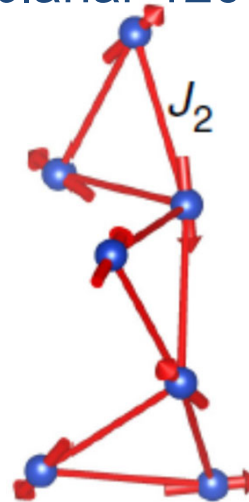
180° order along J_3
 chains $\Rightarrow J_3$ is AFM

J_1 triangles
 canted 120°



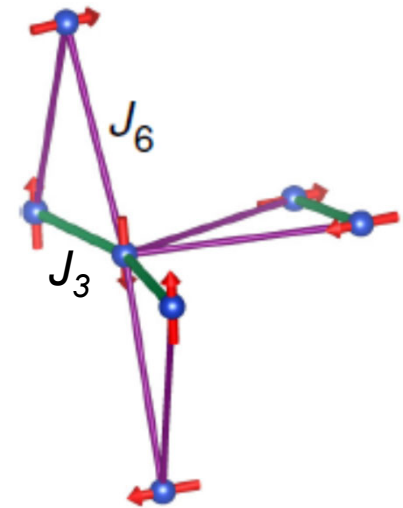
isolated triangles
 J_1 is very weak

J_2 triangles
 planar 120°



hyperkagome
 triangles
 $\Rightarrow J_2$ is AFM

J_3/J_6 perpendicular
 order between chains

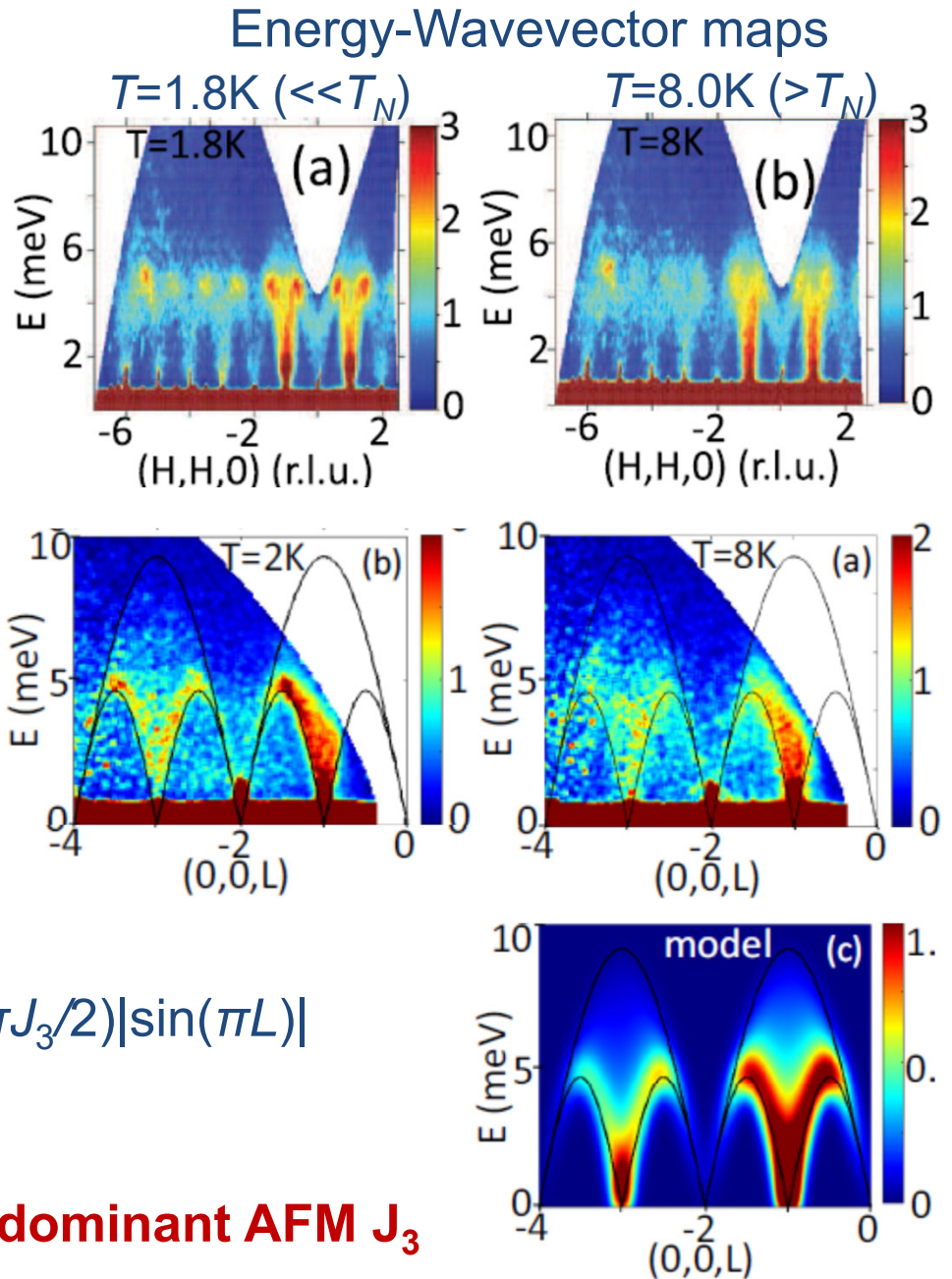
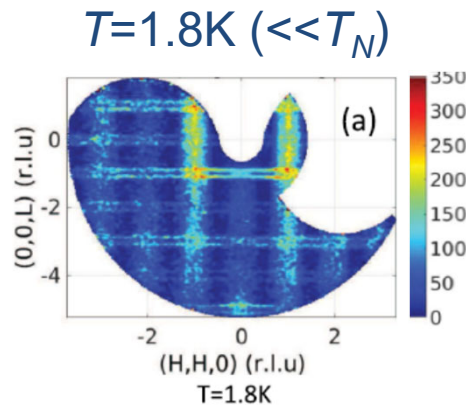


Neighboring parallel
 chains have 90° spins
 $\Rightarrow$ FM/AFM J_6 possible

**Spin-1/2 Heisenberg AFM chains // a, b, c axes coupled
 by frustrated interchain interactions into a non-coplanar order**

High energy magnetic excitations of BaCuTe₂O₆

INS, [H,H,L] plane, $1.8 < E < 2.2$ meV



- Streaks indicate dispersionless scattering $\Rightarrow$ 1D magnetism from J_3
- multi-spinon continuum typical of AFM S-1/2 chain
- Minor changes below T_N
- Comparison to the Bethe Ansatz
- des Cloiseaux-Pearson dispersion $(\pi J_3/2)|\sin(\pi L)|$ gives $J_3 = 2.90(6)$ meV or $33.7(7)$ K

B. Lake; Budapest, **Spin chain physics – dominant AFM J_3**

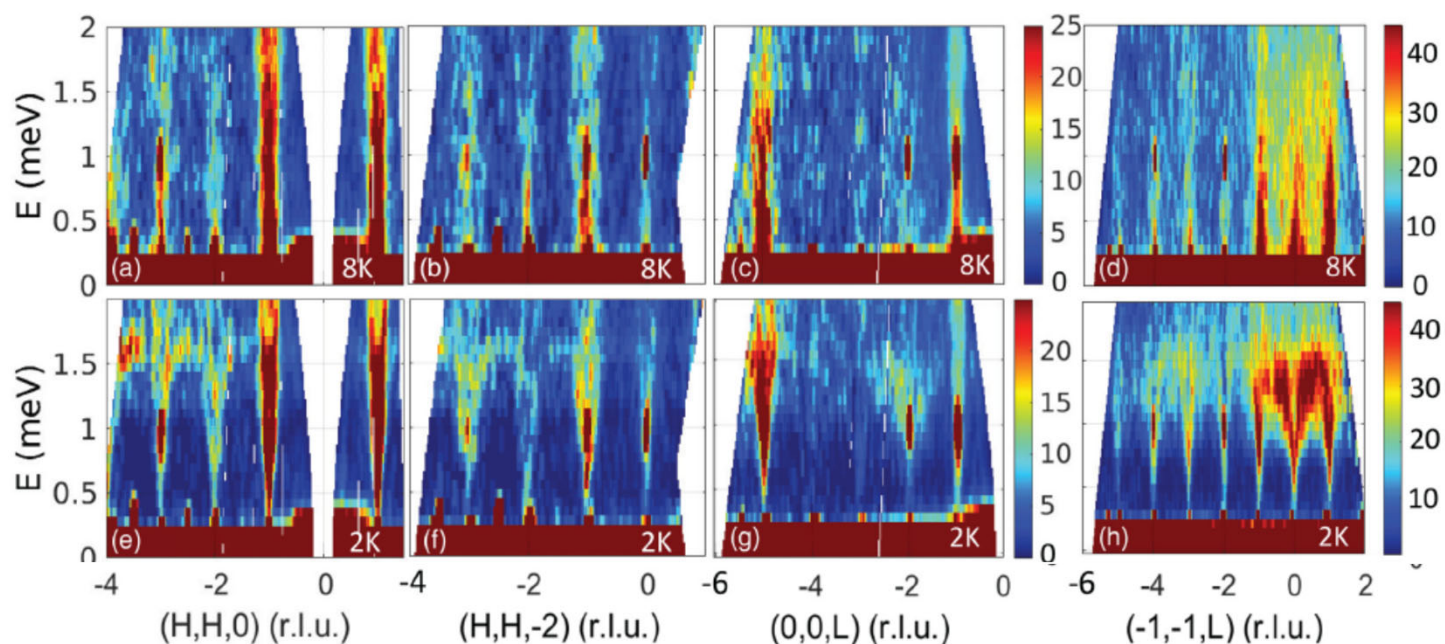
Low Energy magnetic excitations of BaCuTe₂O₆

$T = 8.0\text{K} (>T_N)$

odd integer H, L –
spinon continuum

$T = 1.8\text{K} (<<T_N)$

Band at ≈ 1.6 meV,
Spin-wave $E < 1$ meV



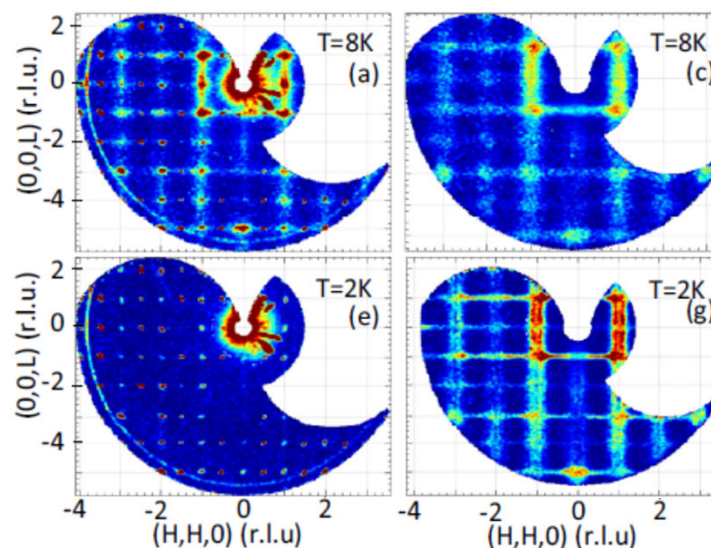
$0.2 < \Delta E < 0.6$ meV $1.2 < \Delta E < 1.6$ meV

$T = 8.0\text{K} (>T_N)$

Scattering streaks -
uncoupled chains

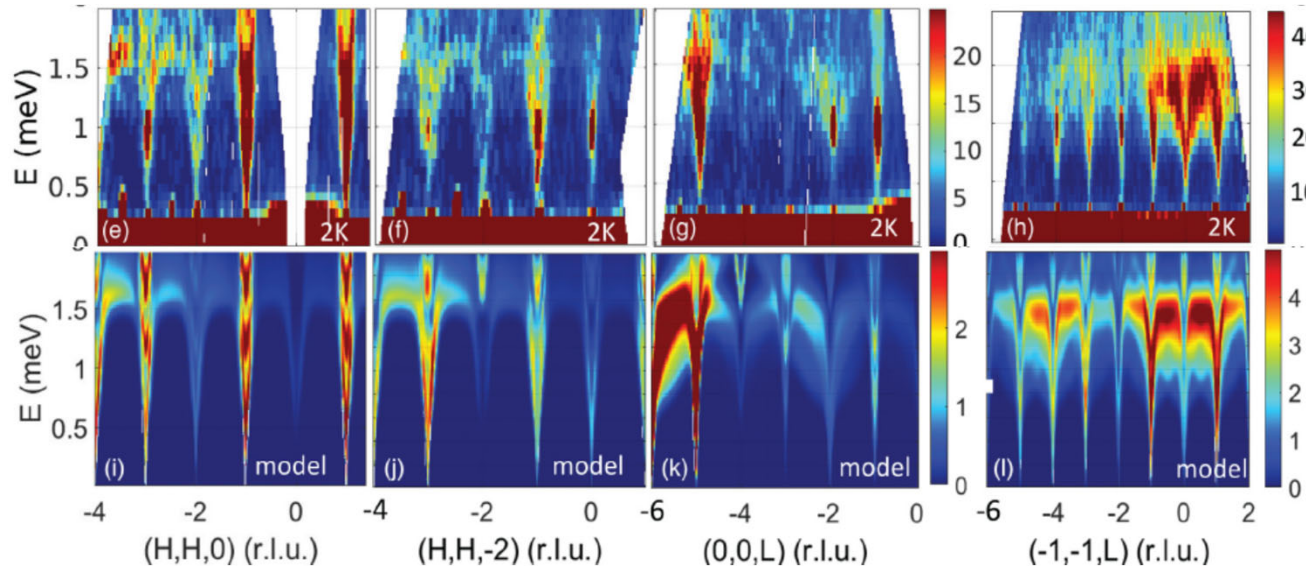
$T = 1.8\text{K} (<<T_N)$

Scattering dots -
Spin-wave
dispersion



**Confinement of
spinon-pairs into
spin-waves at
low E and T**

Hamiltonian of BaCuTe₂O₆



Data, $T=1.8\text{K}$

Spin-waves

$$\begin{aligned} J_3 &= 2.90 \text{ meV}, J_3' = 4.56 \text{ meV} \\ J_2 &= 0.30 \text{ meV}, \\ J_1 &= 0.05 \text{ meV}, \\ J_6 &= 0.03 \text{ meV} \end{aligned}$$

Spin-wave theory, $T=2\text{K}$, $E < 2 \text{ meV}$

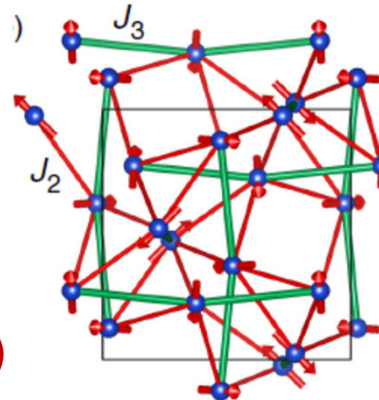
Fixed intrachain $J_3' = \pi J_3 / 2 = 4.56 \text{ meV}$

$J_3 = 2.90 \text{ meV}$ **AFM dominant**

$0.17 < J_2 < 0.44 \text{ meV}$ **AFM hyperkagome subleading**

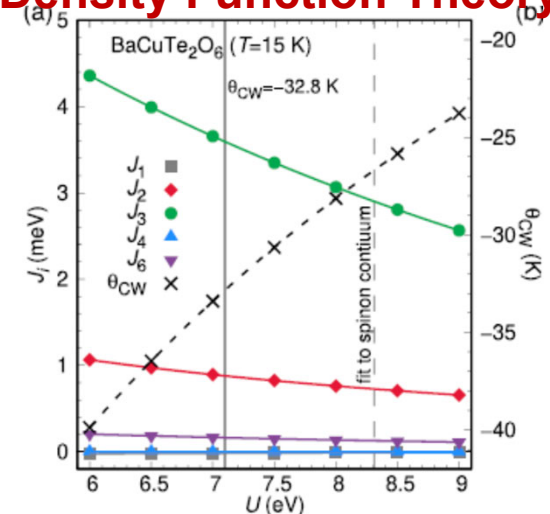
$J_1 = 2J_2 - 0.54 \text{ meV}$, **isolated triangle weak**

J_6 **undetermined**



AFM spin chain // a, b, & c axes (J_3)
Weakly coupled by hyperkagome (J_2)

Density Function Theory



$J_3 = 2.90 \text{ meV}$

$0.44 < J_2 < 0.73 \text{ meV}$

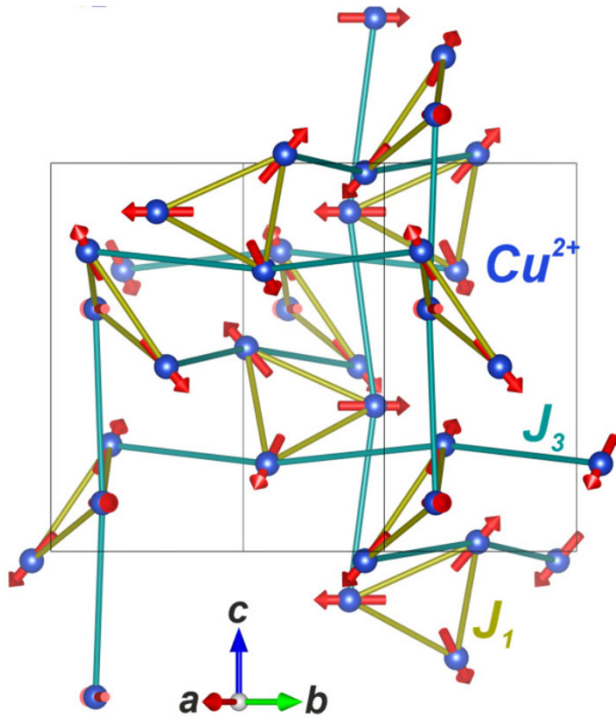
$J_1 = -0.02 \text{ meV}$

$J_6 = 0.12 \text{ meV}$



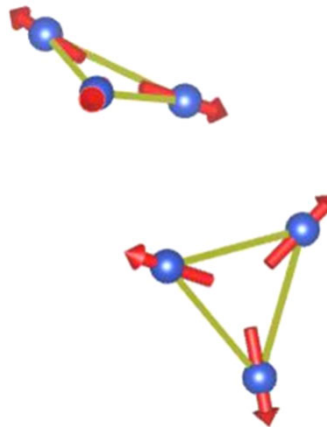
Magnetic structure of $\text{SrCuTe}_2\text{O}_6$

$q=(0,0,1)$, $m= 0.46 \mu_B/\text{Cu}^{2+}$ compared to $1 \mu_B/\text{Cu}^{2+}$
 $\Rightarrow$ low-dimensional / frustrated



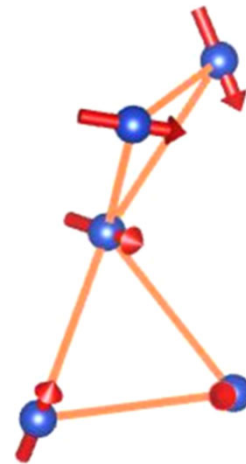
180° order along J_3 chains $\Rightarrow J_3$ is AFM

J_1 triangles planar 120°



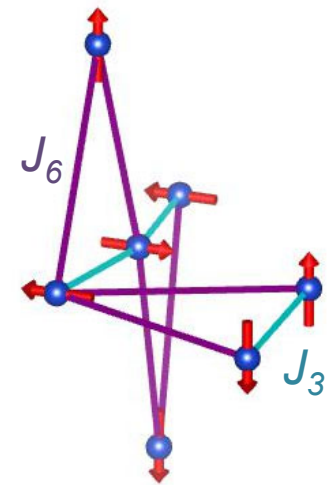
isolated triangles $\Rightarrow J_1$ is AFM

J_2 triangles canted 120°



hyperkagome triangles $\Rightarrow J_2$ is very weak

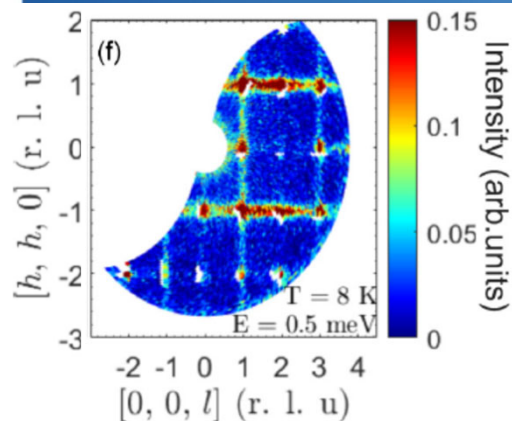
J_3/J_6 perpendicular order between chains



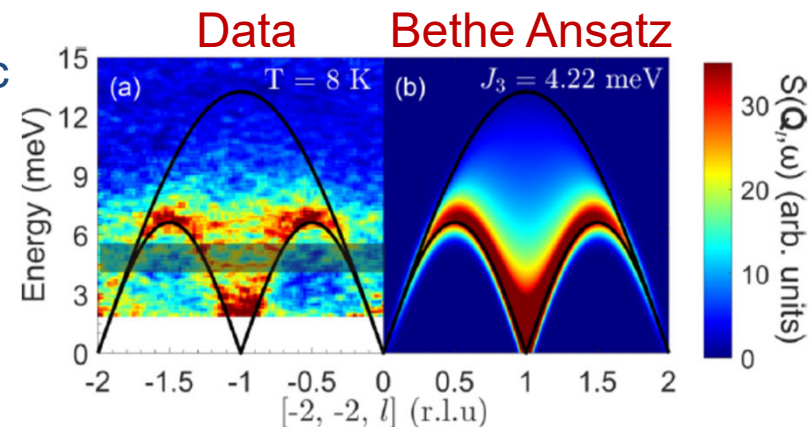
Neighboring parallel chains have 90° spins $\Rightarrow$ FM/AFM J_6 possible

Spin-1/2 Heisenberg AFM chains // a , b , c axes coupled by frustrated interchain interactions into a non-coplanar order

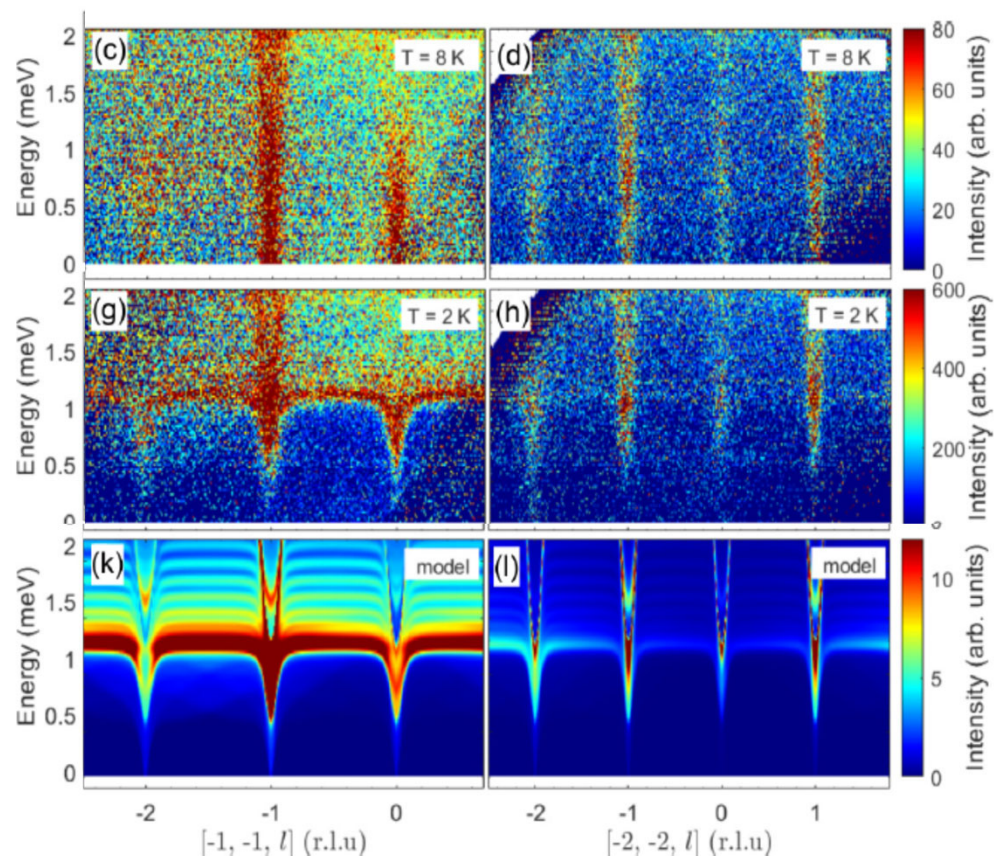
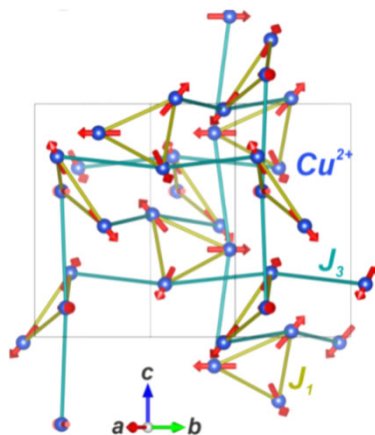
Hamiltonian of SrCuTe₂O₆



- Spin chains // a, b, c
- Spinon continuum
- $(\pi J_3/2) |\sin(\pi H)|$
- $\Rightarrow J_3 = 4.22 \text{ meV}$



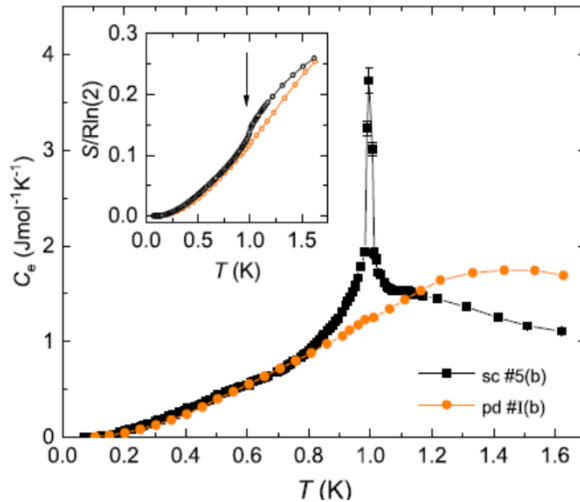
Spin-wave theory: $T=2\text{K}$, $E < 2 \text{ meV}$
 Fixed intrachain $J'_3 = \pi J_3/2 = 6.62 \text{ meV}$
 $J_3 = 4.22 \text{ meV}$ AFM dominant
 $J_2 = -0.037 \pm 0.012 \text{ meV}$ very weak
 $J_1 = 0.17 \pm 0.025 \text{ meV}$, AFM subleading
 J_6 undetermined



AFM spin chain // a, b, & c axes (J_3)
Weakly coupled by triangular (J_1)



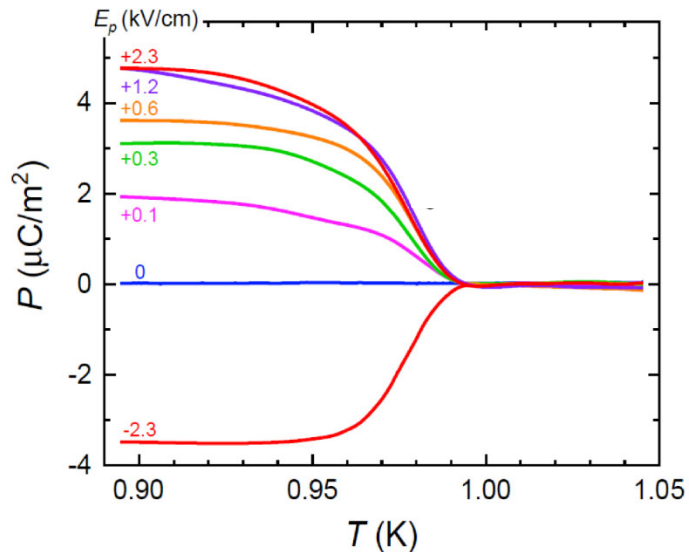
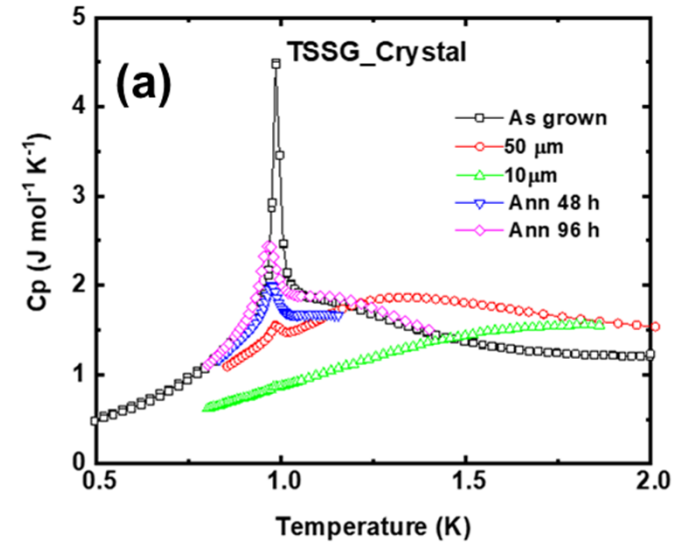
PbCuTe₂O₆ – Ferroelectric transition



Transition $T_c = 1.1\text{K}$

Present in crystal
but absent in powder

A.R. N. Hanna, A. T. M. N. Islam, R. Feyerherm, K. Siemensmeyer,
K. Karmakar, S. Chillal, B. Lake *Phys. Rev. Mat.* 5 113401 (2021)



- Electric field applied on cooling - monodomain sample
- Electric field switched off Polarization measured on heating
- Clear spontaneous polarization below T_{FE} .

Ch. Thurn, P. Eibisch, A. Ata, M. Winkler, P. Lunkenheimer, I. Kézsmárki, U.
Tutsch, Y. Saito, S. Hartmann, J. Zimmermann, A. R. N. Hanna, A. T. M. N. Islam,
S. Chillal, B. Lake, B. Wolf, M. Lang *npj Quantum Materials* 6 95 (2021)

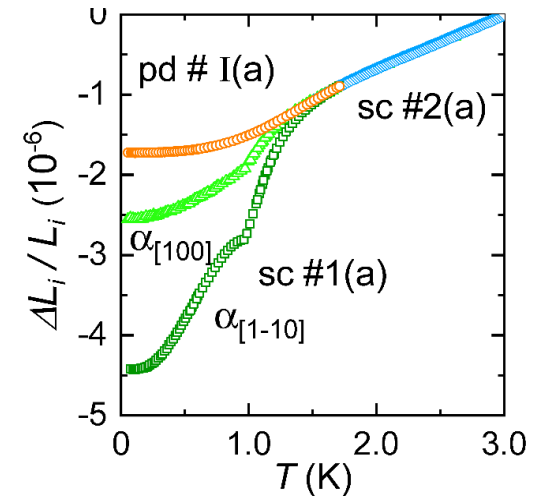
The transition is ferroelectric

PbCuTe₂O₆ – Structural & magnetic changes at T_{FE}

- Thermal expansion reveals structural distortion at T_{FE} to non-cubic phase.
- However no distortion observed by neutron diffraction (HRPT, PSI and WISH, ISIS)

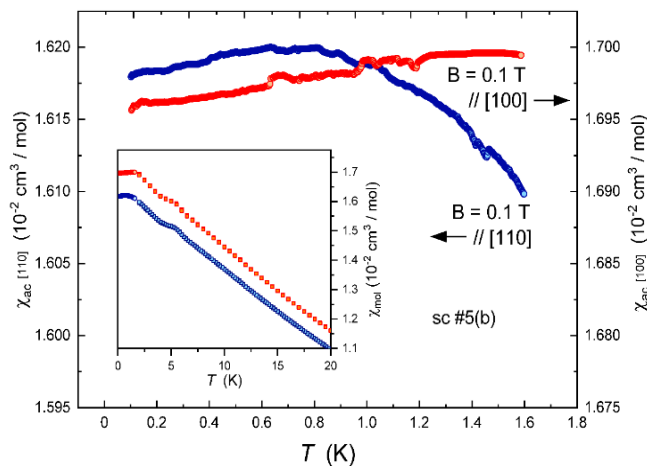
Ferroelectric transition accompanied by weak structural distortion

Symmetry lowering might lift frustration & induce magnetic order



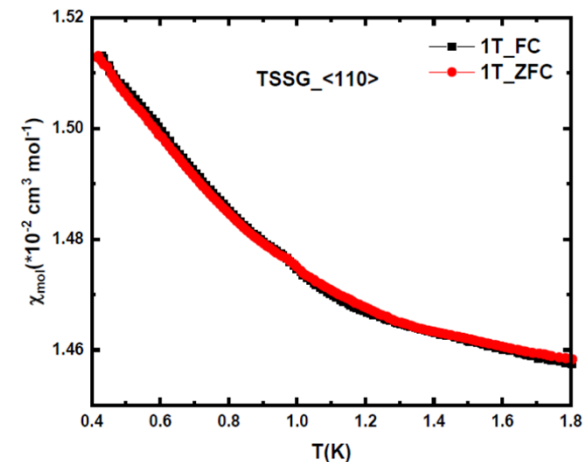
Ch. Thurn, *npj Quantum Materials* 6 95 (2021)

AC susceptibility – no transition



Ch. Thurn, *npj Quantum Materials* 6 95 (2021)

DC susceptibility – very weak signature

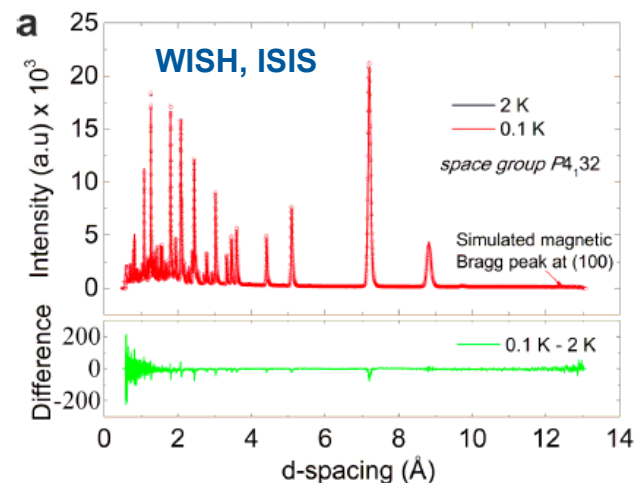


A.R. N. Hanna, *Phys. Rev. Mat.* 5 113401 (2021)

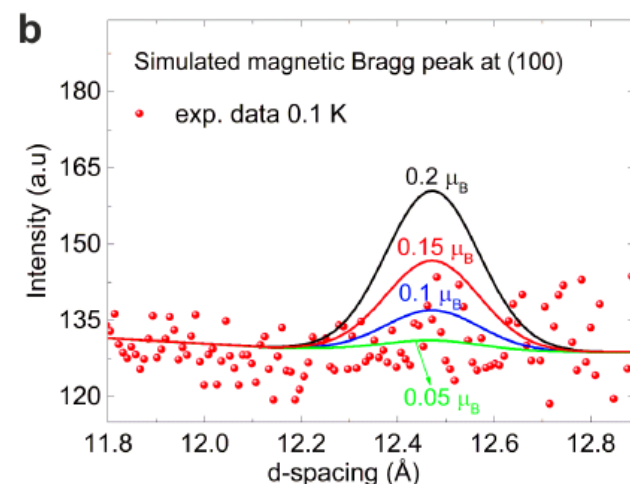
PbCuTe₂O₆ – No observable magnetic order

Neutron diffraction

No new Bragg peaks below $T_{FE}=1$ K



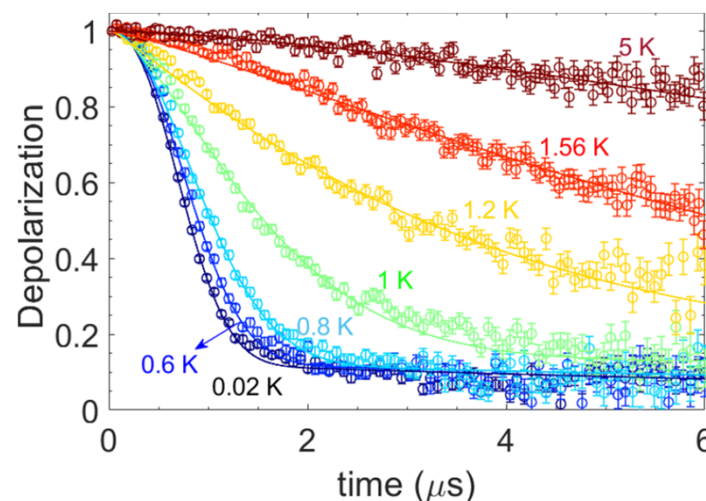
- Different magnetic structures simulated
- $\Rightarrow$ Magnetic order must be $< 0.05 \mu_B$



Muon Spectroscopy

- No oscillations down to 20 mK
- $\Rightarrow$ No long-range magnetic order
- Polarization at long times
- $P(t \rightarrow \infty, T \rightarrow 0) < 1/3$
- $\Rightarrow$ No static magnetism, no spin glass
- Slowing of spin dynamics

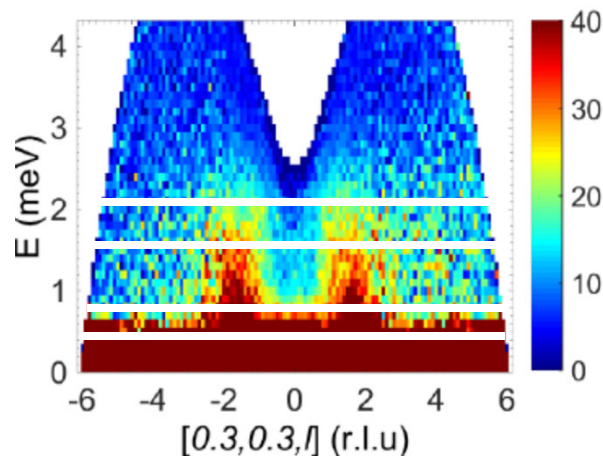
Zero Field LTF, GPS, PSI



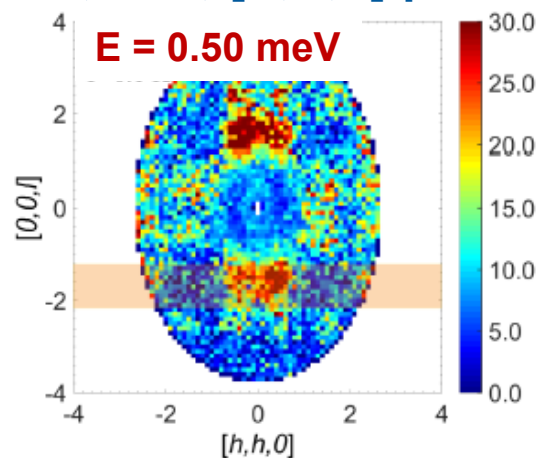
Long-range order and static magnetism is suppressed

PbCuTe₂O₆ – Inelastic Neutron Scattering

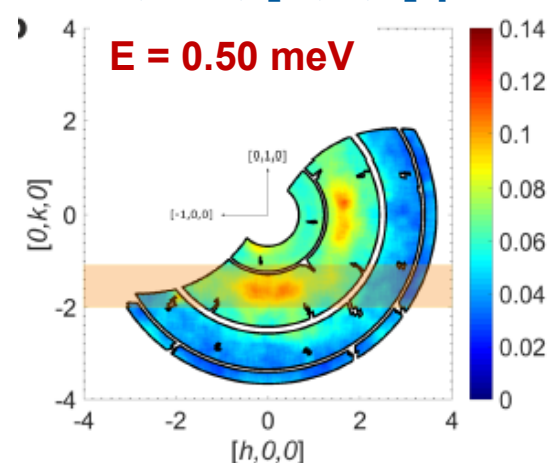
LET, ISIS; T=0.1K



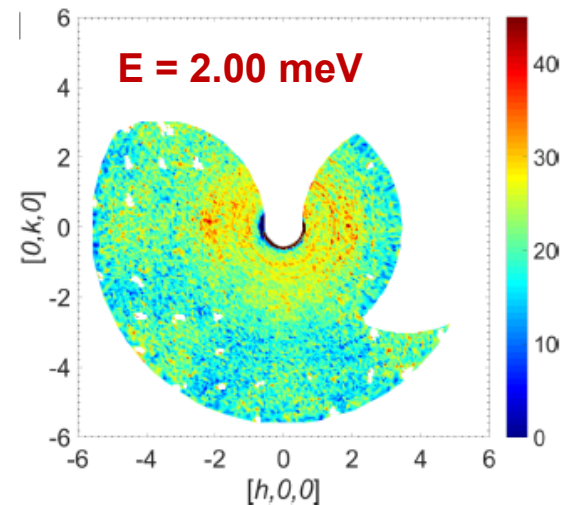
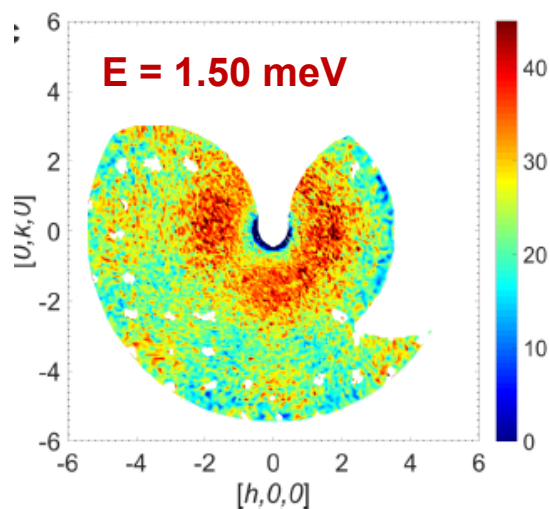
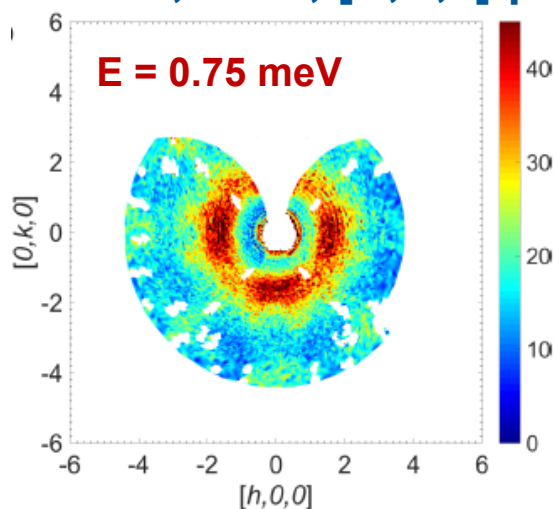
LET, ISIS; [H,H,L]-plane



Thales, ILL; [H,K,0]-plane



MACS, NIST; [H,K,0]-plane T=0.1K



- For all energies diffuse sphere at $Q=0.8\text{\AA}^{-1}$
- No sharp features, no spin-waves

consistent with spinon excitations

S. Chillal, Y. Iqbal, H. O. Jeschke, J.A. Rodriguez-Rivera, R. Bewley, D. Khalyavin, P. Steffens, R. Thomale, A.T.M.N Islam, J. Reuther, B. Lake Nature Communications 11 2348 (2020)

PbCuTe₂O₆ – Exchange interactions

Density Functional Theory (FPLO) (GGA)

Function of onsite potential U (5.5-8.0 eV)

Hund's rule coupling $J_H = 1$ eV

All antiferromagnetic

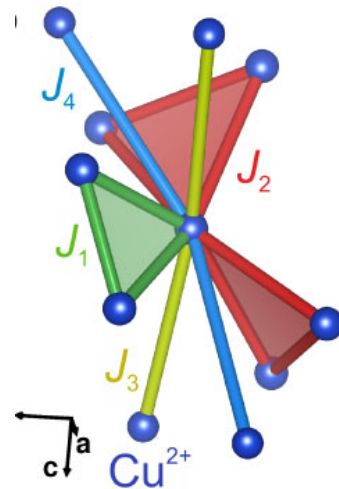
$J_1 = 1.13$ meV

$J_2 = 1.07$ meV

$J_3 = 0.59$ meV

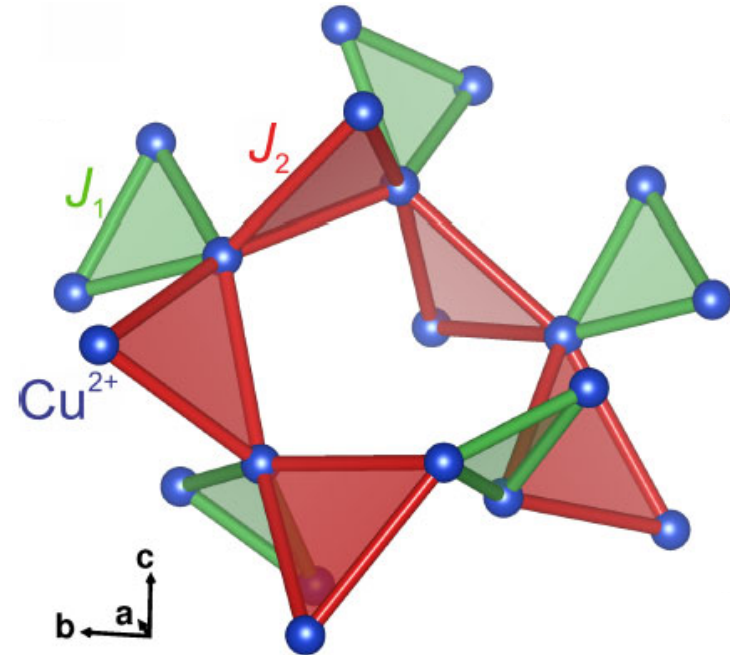
$J_4 = 0.12$ meV

$J_1:J_2:J_3:J_4 = 1:1:0.5:0.1$



Harald Jeschke

- Strong frustrated interactions, $J_1 \approx J_2$
- Chain interaction J_3 is significant



- Distorted windmill lattice with $J_1 \approx J_2$
- 3D network of corner-sharing triangles
- 3 triangles per spin,
 - 2 hyperkagome J_2
 - 1 isolated triangle J_1

S. Chillal, Y. Iqbal, H. O. Jeschke, J.A. Rodriguez-Rivera, R. Bewley, D. Khalyavin, P. Steffens, R. Thomale, A.T.M.N. Islam, J. Reuther, B. Lake Nature Communications 11 2348 (2020)

PbCuTe₂O₆ – Comparison to theory PFFRG

Pseudo-Fermion Functional Renormalisation Group

- Identifies long-range magnetic order
- Calculates spin-spin correlations

Johannes Reuther, Yasir Iqbal

Using the Hamiltonian from DFT

All antiferromagnetic

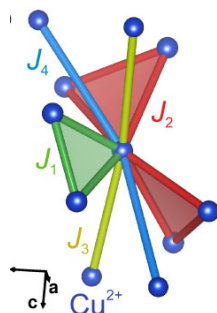
J1 = 1.13 meV

J2 = 1.07 meV

J3 = 0.59 meV

J4 = 0.12 meV

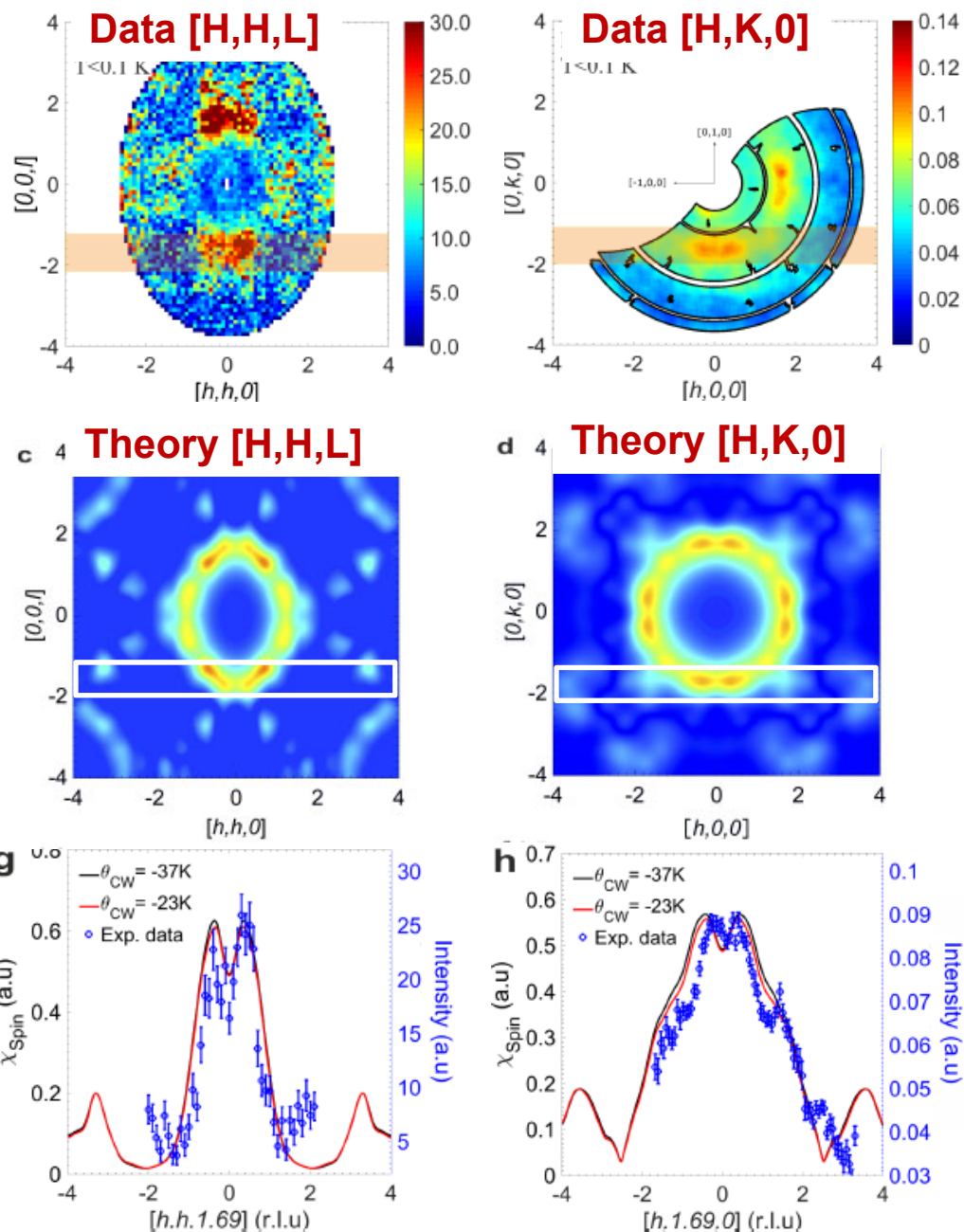
J1:J2:J3:J4=1:1:0.5:0.1



PFFRG confirms

- ⇒ no long-range order
- ⇒ diffuse scattering pattern

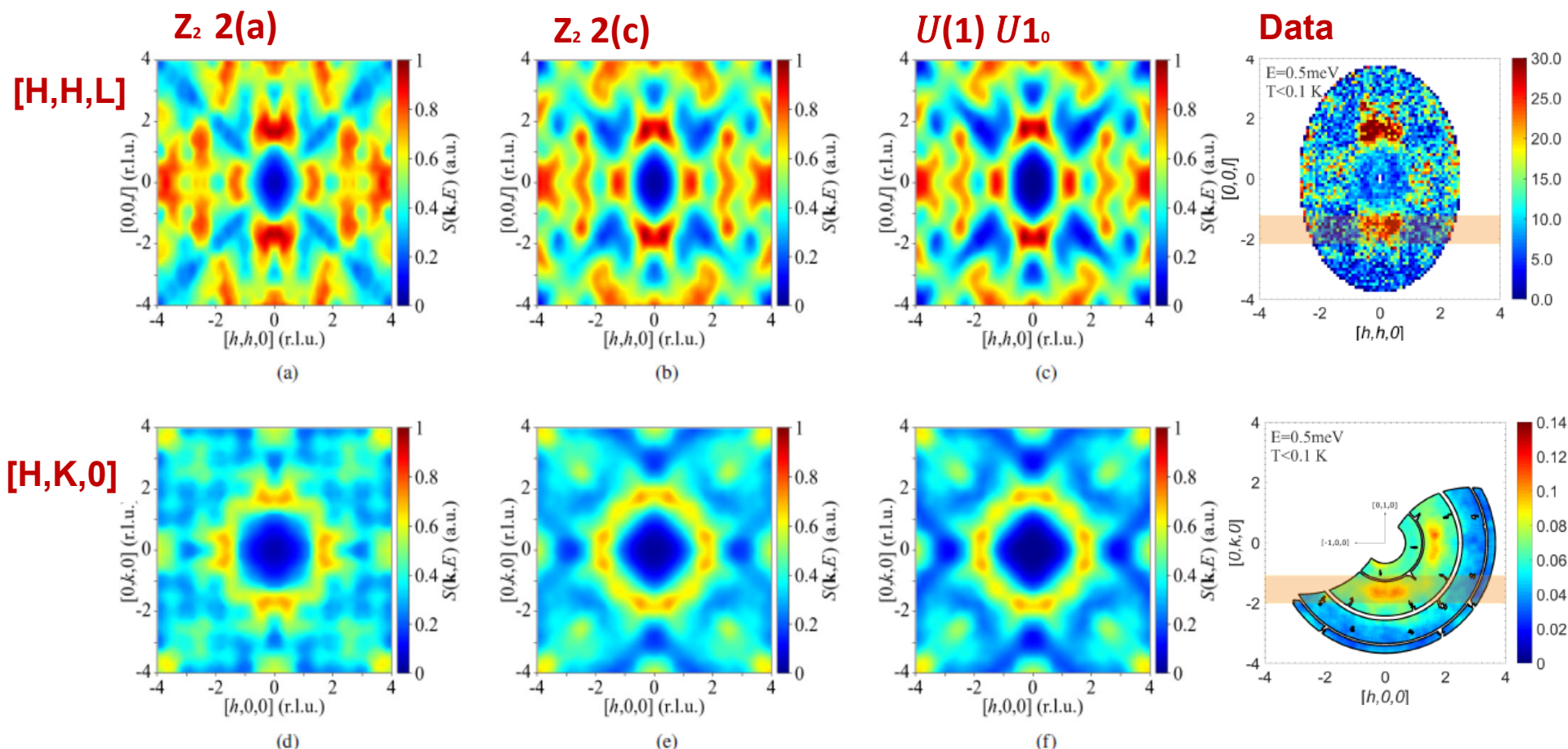
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PbCuTe₂O₆ – Projective symmetry group analysis

- Parton construction and projective symmetry group
- identify 3 spin liquids compatible with lattice of PbCuTe₂O₆ (J_1 - J_2 model)
 - 2 Z_2 spin liquids (gapped and ungapped)
 - 1 $U(1)$ spin liquid

*Li Ern Chern and Yong Baek Kim
Phys. Rev. B 104, 094413 (2021)*



**General agreement with data, but
difficult to distinguish between them**

Summary

**P4₁32 with magnetic site 12d spans a range of geometries
spin chains, distorted windmill, hyperkagome and trillium**

Explore the space group and possible magnetic lattices

BaCuTe₂O₆ – spin chains coupled by hyperkagome interaction

SrCuTe₂O₆ – spin chains coupled isolated triangles

**PbCuTe₂O₆ – corner-sharing triangles 3 triangles at each site
– spin liquid candidate**

The ACuTe₂O₆ Teams



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