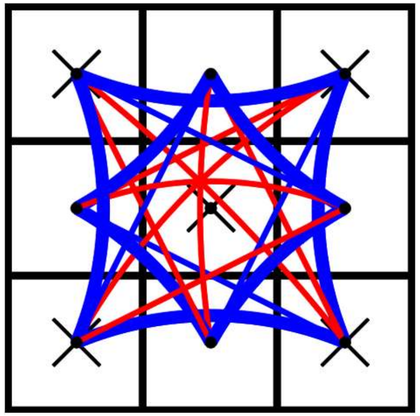
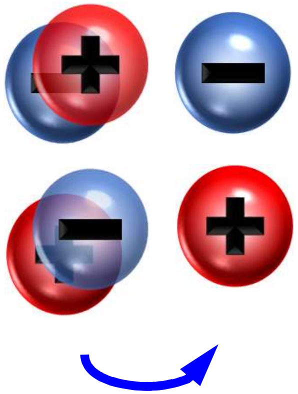
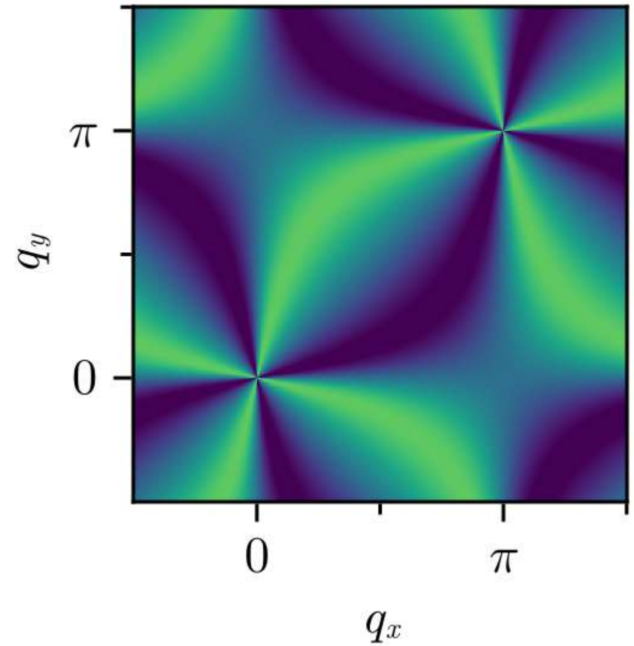






Fracton quantum spin liquid on a square lattice_finalversion_last_v3.pptx

Nils Niggemann, ICTP Trieste
Quantum Spin Liquids, Budapest
October 9th, 2025

Freie Universität Berlin
HZB Helmholtz Zentrum Berlin
ICTP

Collaborators

Johannes Reuther,
Freie Universität Berlin
Helmholtz Zentrum Berlin

Yannik Schaden,
Freie Universität Berlin
Helmholtz Zentrum Berlin

Meghadeepa Adhikary,
Indian Institute of Technology, Madras
SISSA, Trieste

Classical fracton spin liquid and Hilbert space fragmentation in a 2D spin- ½ model

arXiv:**2508.06606 (2025)**

Gapless fracton quantum spin liquid and emergent photons in a 2D spin-1 model

arXiv:**2508.06605 (2025)**

Introduction: Particles in U(1) gauge theories

Rank 1 U(1) gauge theory

Gauss' law:

- Electric field is a vector

$$\nabla \cdot \boldsymbol{E}(\boldsymbol{r}) = \rho(\boldsymbol{r})$$

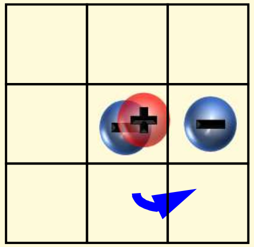
Charge conservation:

- Lowest possible excitation is a **dipole**

$$Q = \int d^3 r \rho(\boldsymbol{r})$$

Charge mobility:

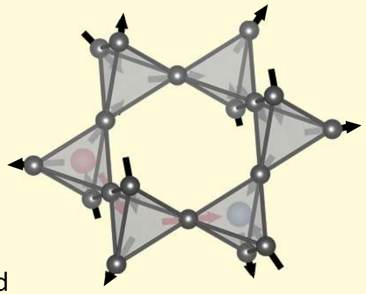
- Moving charges ↔ adding dipole



Realization:

- Spin ice materials

$$H = \frac{J}{2} \sum_{\text{tet}} \left(\sum_{i \in \text{tet}} S_i^z \right)^2$$



D. A. Huse, W. Krauth, R. Moessner, S. L. Sondhi, PRL **91**, 167004 (2003)
Harris M J, Bramwell S T, McMorrow D F, Zeiske T and Godfrey K W 1997 Phys. Rev. Lett. **79** 2554

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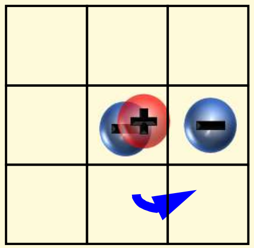

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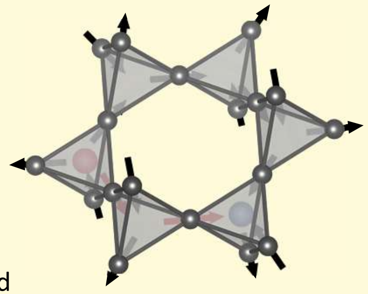

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Harris M J, Bramwell S T, McMorrow D F, Zeiske T and Godfrey K W 1997 Phys. Rev. Lett. **79** 2554

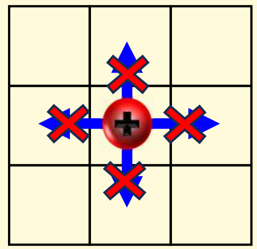
Rank 2 U(1) gauge theory

Gauss' law: $\partial_\mu \partial_\nu E^{\mu\nu} = \rho$

- Electric field is a rank 2 tensor

Charge **and dipole** conservation: $\boldsymbol{d} = \int d^3 r \boldsymbol{r} \rho(\boldsymbol{r})$

- Lowest possible excitation is a **quadrupole**

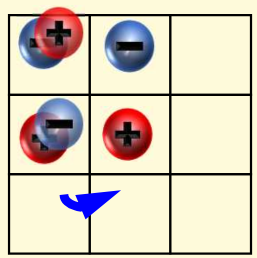


Charge mobility:

- Charges are **immobile → "Fractons"**
- Dipoles **move along line → "Lineons"**

Realization:

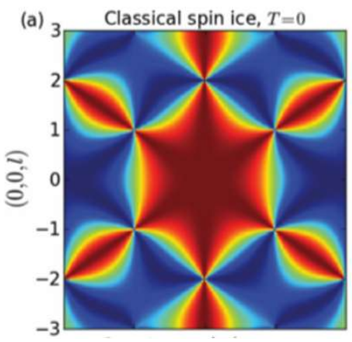
- Classically: designer models (*)
- Quantum: **this talk**



M. Pretko Physical Review B **95**, 115139 (2017)
M. Pretko et. al. International Journal of Modern Physics (2020)
(*) O. Benton and R. Moessner, PRL **127**, 107202 (2021)
(*) Davier, Naïmo, and L. D. C. Jaubert, arXiv:2509.18845 (2025)

Spin correlator: Quantum fluctuations in field theory

Rank 1
(also observed in microscopic models!)



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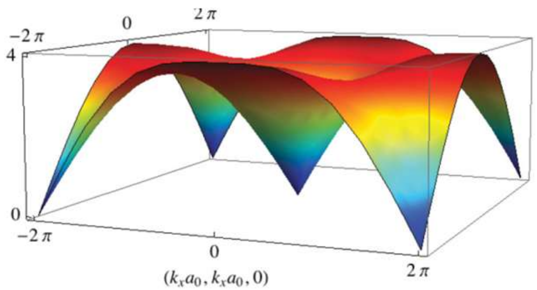

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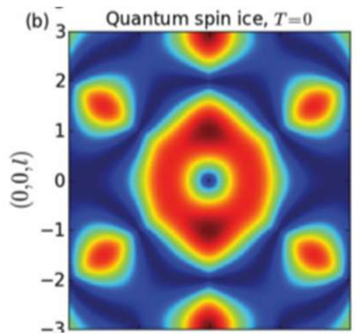
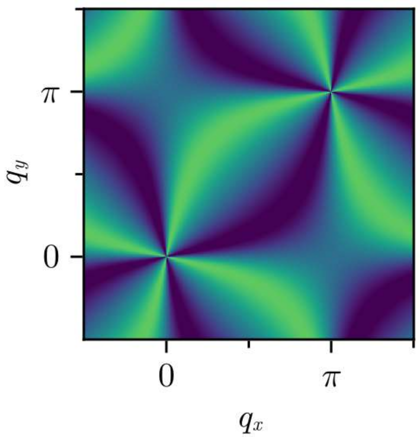


Pinch points:
Gauss' law (Classical)

Photon dispersion

Suppressed pinch points

Rank 2



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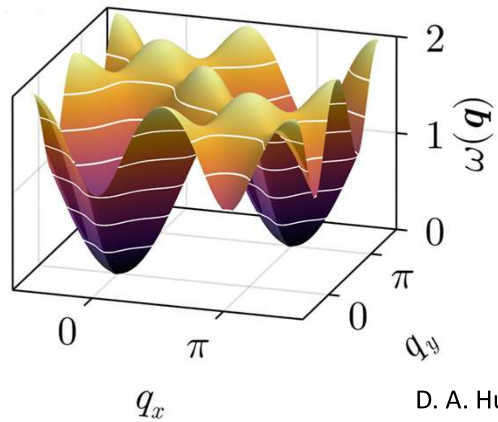

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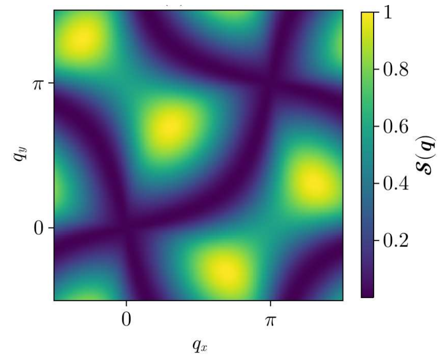


D. A. Huse, W. Krauth, R. Moessner, S. L. Sondhi, PRL **91**, 167004 (2003)
M J P Gingras and P A McClarty *Rep. Prog. Phys.* **77** 056501 (2014)
O. Benton, O. Sikora, and N. Shannon Phys. Rev. B **86**, 075154 (2012)

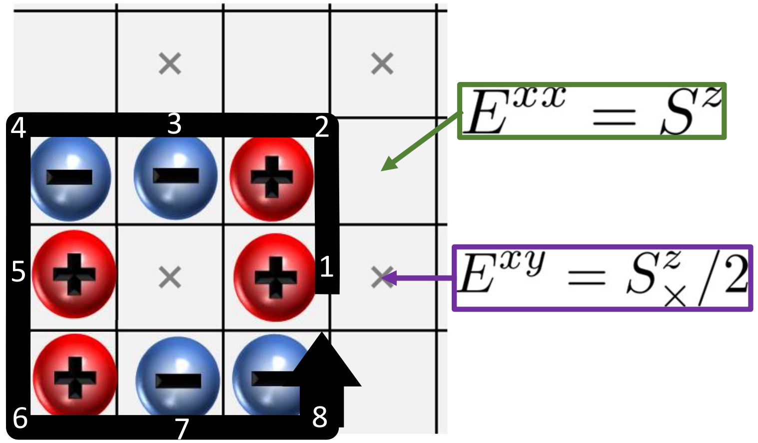
The spiderweb model

Discrete form of rank-2 **gauge theory** $\partial_\mu \partial_\nu E^{\mu\nu} = 0$

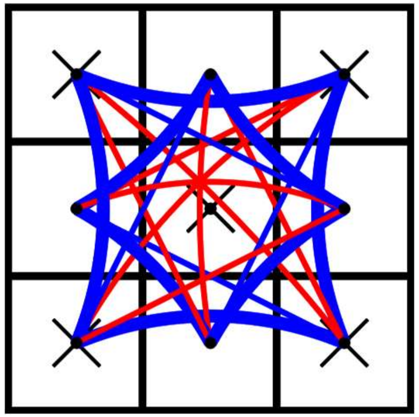
Ground state constraint:

$$S_1^z + S_2^z - S_3^z - S_4^z + S_5^z + S_6^z - S_7^z - S_8^z = 0 \quad \forall$$

4	3	2
5	$\times$	1
6	7	8



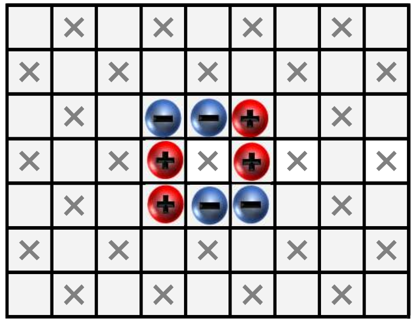
J_{ij}



$$\mathcal{H}_1 = \frac{J}{2} \sum_{\boxtimes} (S^z_{\boxtimes_1} + S^z_{\boxtimes_2} - S^z_{\boxtimes_3} - S^z_{\boxtimes_4} + S^z_{\boxtimes_5} + S^z_{\boxtimes_6} - S^z_{\boxtimes_7} - S^z_{\boxtimes_8})^2$$

Classical fractons and lineons

- A single spin flip on a ground state creates four fractons



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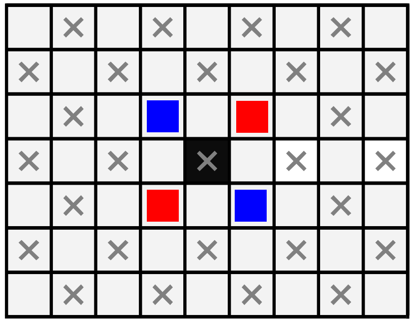
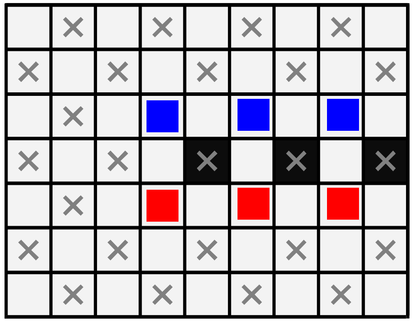
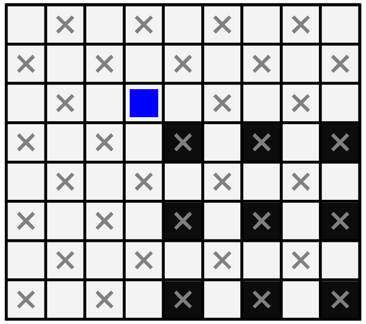


- Lineons are pairs of fractons
  can move in one dimension



- Isolated fractons can be found at corners of phase boundaries between two phases
  - Cannot be moved without re-arranging the entire boundary

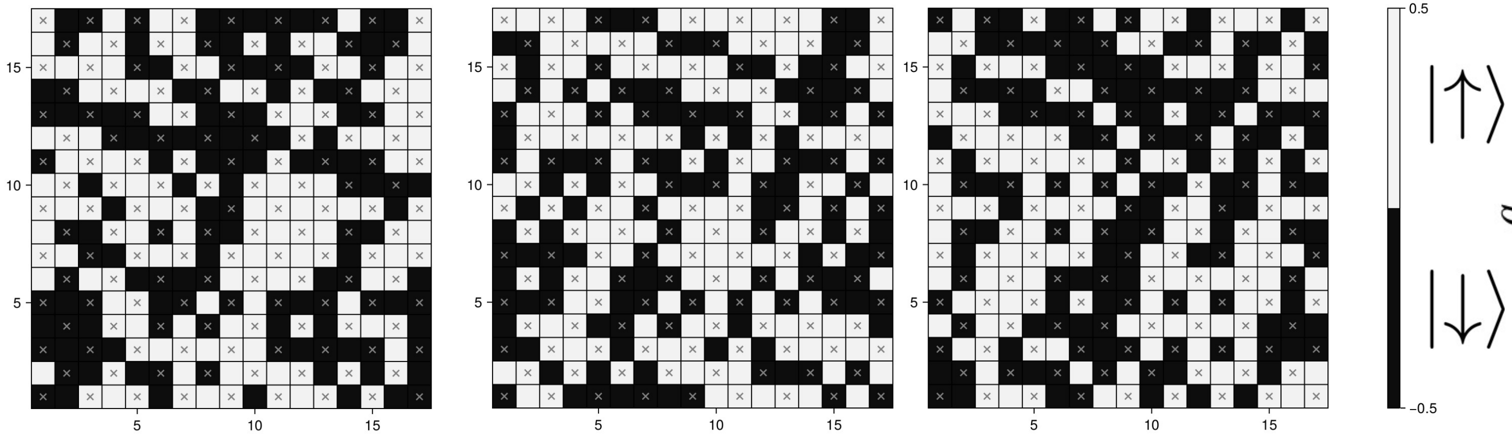


The Spiderweb Model – Classical Ising case

Extensively many ground states! $S_1^z + S_2^z - S_3^z - S_4^z + S_5^z + S_6^z - S_7^z - S_8^z = 0 \quad \forall \begin{smallmatrix} 4 & 3 & 2 \\ 5 & \times & 1 \\ 6 & 7 & 8 \end{smallmatrix}$

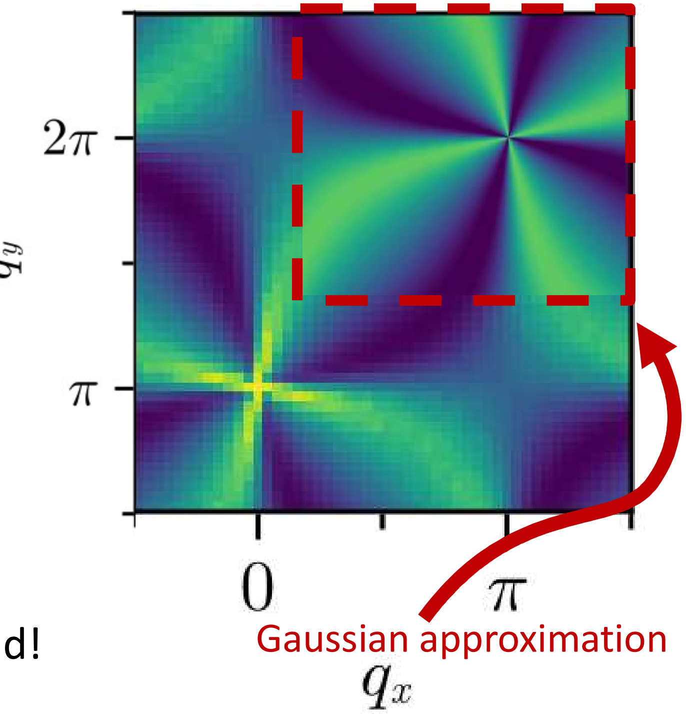
Challenging for Monte Carlo: Fractons are immobile → Glassy dynamics [B. Placke et. al. arXiv:2306.13151 (2023)]

Stochastic sampling of ground states:



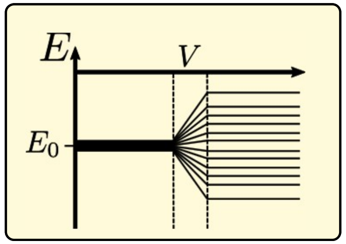
Spin structure factor

$\mathcal{S}(\boldsymbol{q}) = \langle S^z(-\boldsymbol{q}) S^z(\boldsymbol{q}) \rangle$



4-fold pinch point visible → classical rank-2 U(1) spin liquid!

Quantum fluctuations



Degenerate perturbation theory

- Tunneling between fracton-free configurations

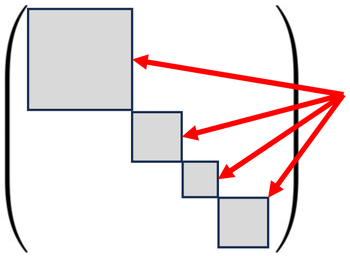
Lowest order tunnelling process (~ring exchange)

$$\mathcal{F}_\square = S^+_{\square_1} S^-_{\square_2} S^-_{\square_3} S^+_{\square_4} S^+_{\square_5} S^-_{\square_6} S^-_{\square_7} S^+_{\square_8}$$

$\mathcal{F}_\square =$ 

$$\mathcal{H} = -\sum_\square \left(\mathcal{F}_\square + \mathcal{F}_\square^\dagger\right) + \mu \sum_\square \left(\mathcal{F}_\square^\dagger \mathcal{F}_\square + \mathcal{F}_\square \mathcal{F}_\square^\dagger\right)$$

At μ = 1 $\quad |\psi\rangle = \sum_x |x\rangle$

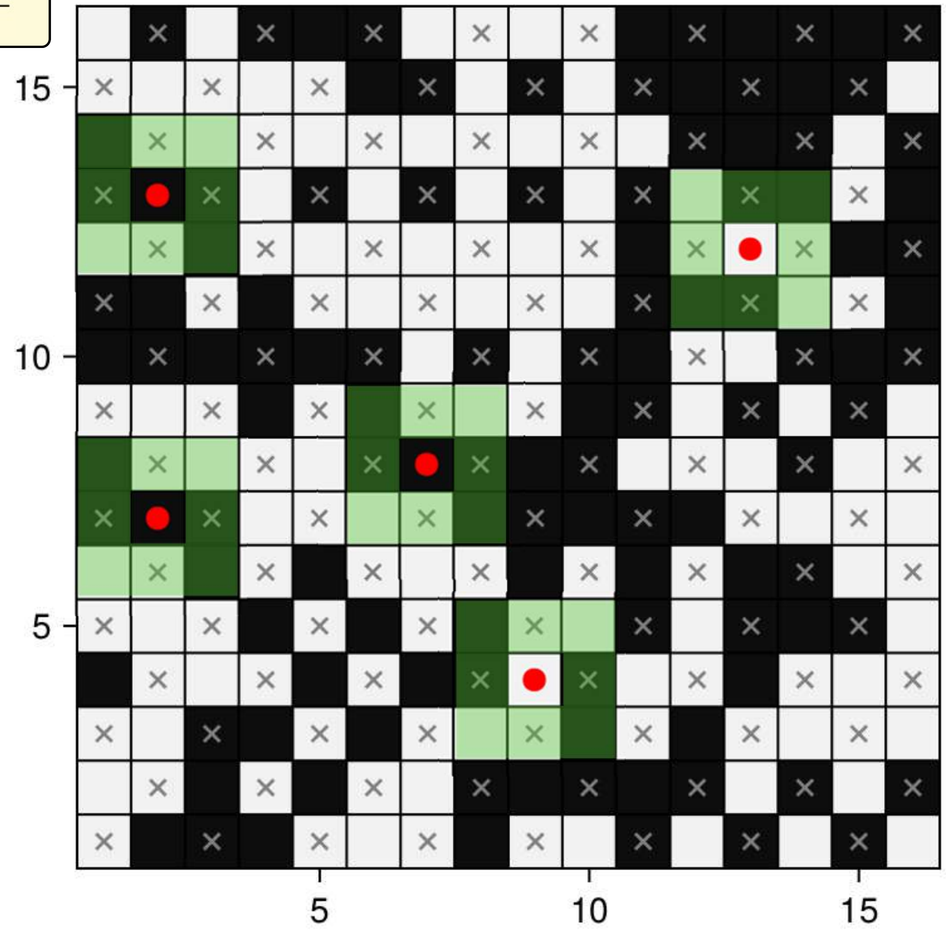
$\mathcal{H} =$ 

No quantum spin liquid!

Generic case:

Loop moves are rare: ● = flippable plaquette

(*)Hilbert space fragmentation!



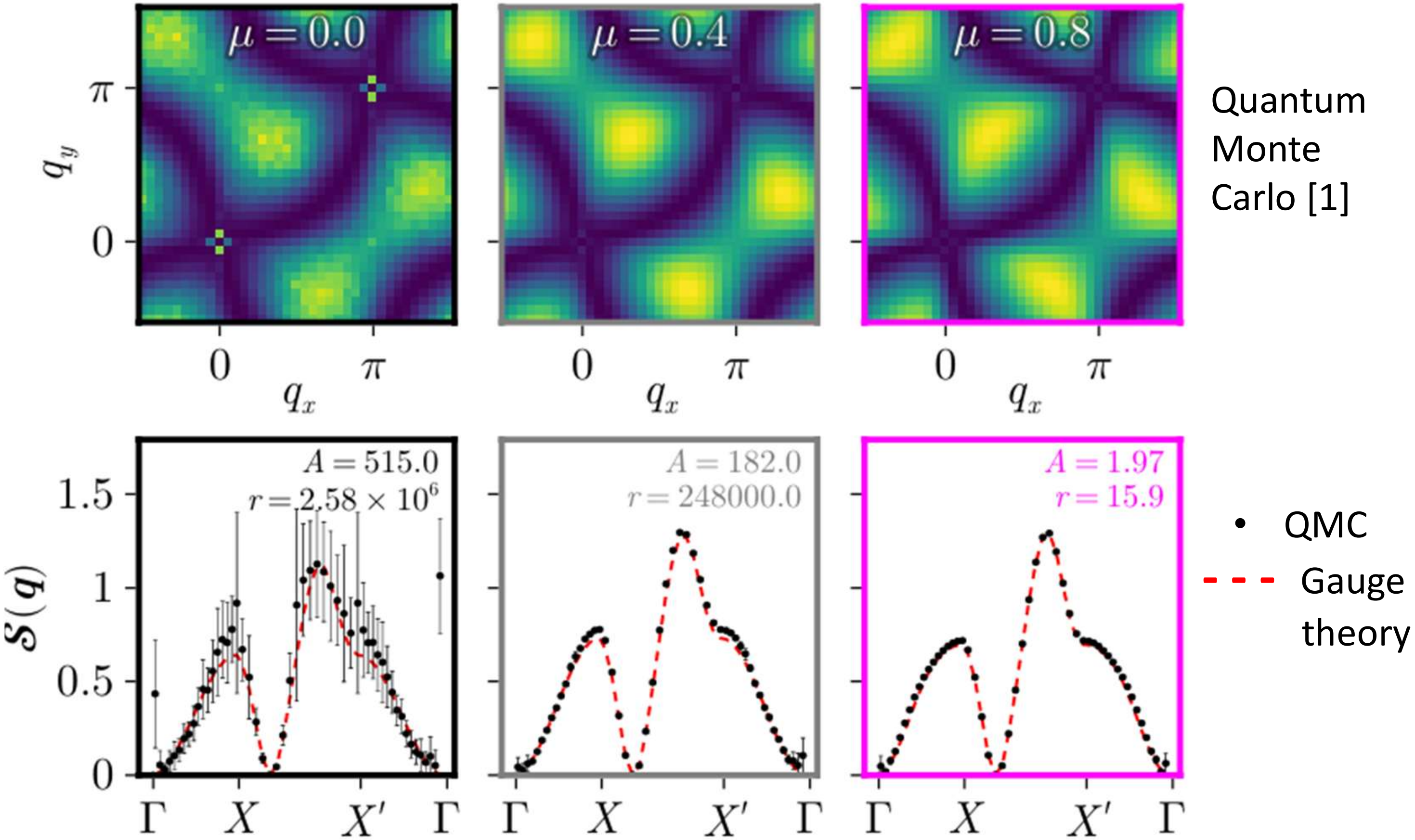
(*)

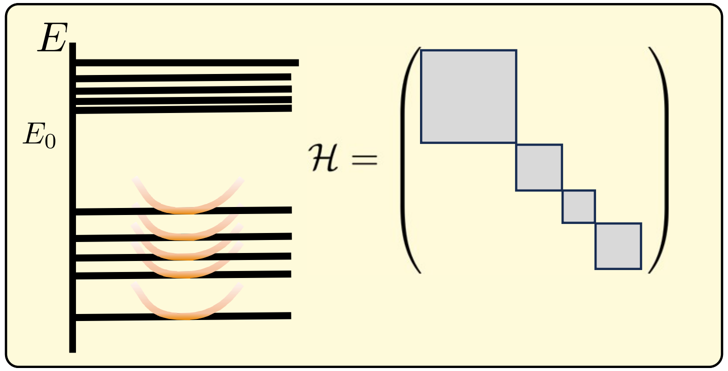
P. Sala, T. Rakovszky, R. Verresen, M. Knap, and F. Pollmann, Phys.Rev. X **10**, 011047 (2020).
A. Khudorozhkov, A. Tiwari, C. Chamon, and T. Neupert, SciPost Physics **13**,098 (2022)

$$\mathcal{H} = -\sum_{\square}\left(\mathcal{F}_{\square} + \mathcal{F}_{\square}^{\dagger}\right) + \mu \sum_{\square}\left(\mathcal{F}_{\square}^{\dagger}\mathcal{F}_{\square} + \mathcal{F}_{\square}\mathcal{F}_{\square}^{\dagger}\right)$$

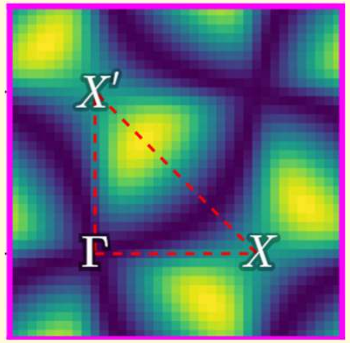
Solution: spin-1

Quantum fracton spin liquid in in low-energetic sectors!





U(1) Lattice gauge theory

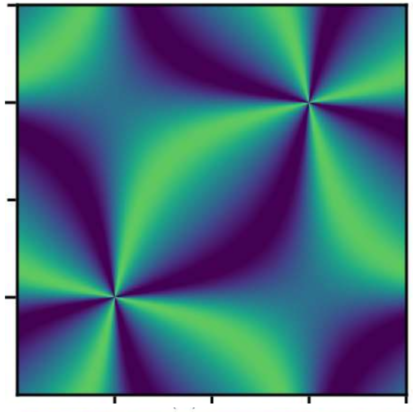


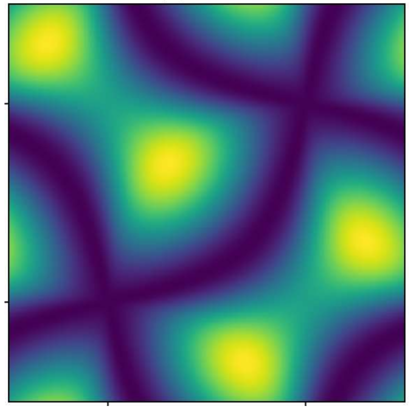
[1] Matteo Calandra Buonaura and Sandro Sorella Phys. Rev. B **57**, 11446 (1998)

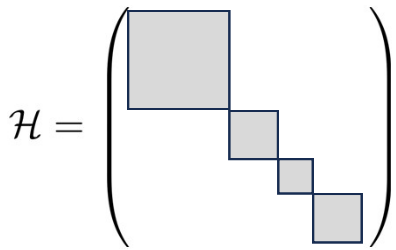
[2] Becca F, Sorella S. Quantum Monte Carlo Approaches for Correlated Systems. Cambridge University Press (2017)

Conclusions

- Fourfold pinch points:
  hallmarks of emergent classical **fractons**
- Gapless photons:
  suppression of $S(\boldsymbol{q} \approx \mathbf{0})$
- Hilbert space fragmentation can prohibit formation of a quantum spin liquid (spin - $\frac{1}{2}$)
  Likely generic property for constrainer fracton models
- Quantum fracton spin liquid for spin 1 with emergent gapless photon







Thanks for your attention!

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