Seeing stars: fractionalized (alter-)magnets and superconductors

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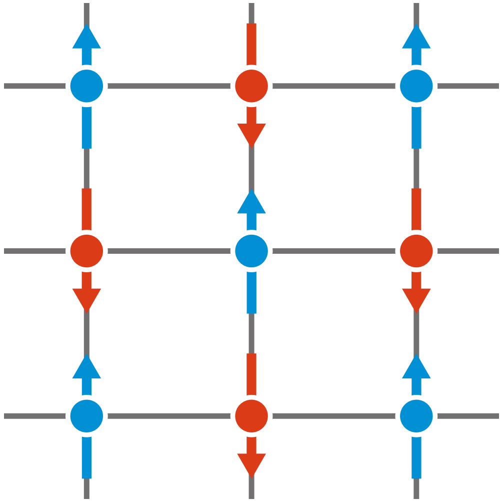
Emmy Noether-Programm

DFG Deutsche Forschungsgemeinschaft

Symmetry-breaking order and magnetism

Consider collinear states of local moments *without spin-orbit coupling*

Antiferromagnet



- Staggered magnetization: $\langle N^z \rangle \neq 0$
- Combination of spin-rotation and translation $R_y(\pi)\, T_x$ is a symmetry $\Rightarrow$ Magnetization $\langle M^z \rangle = 0$

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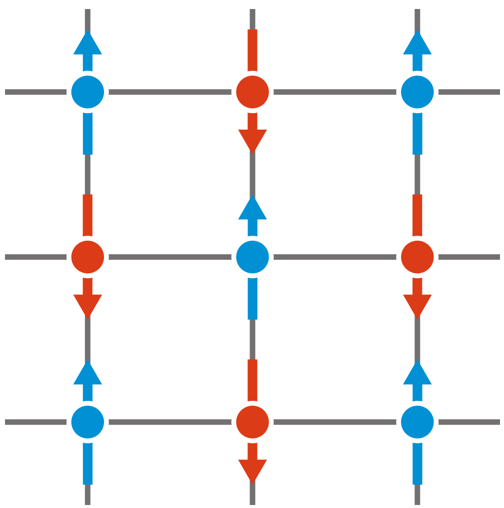

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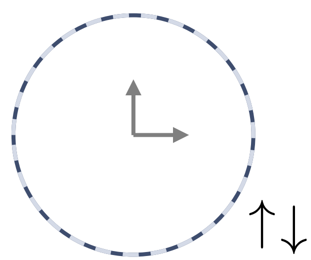
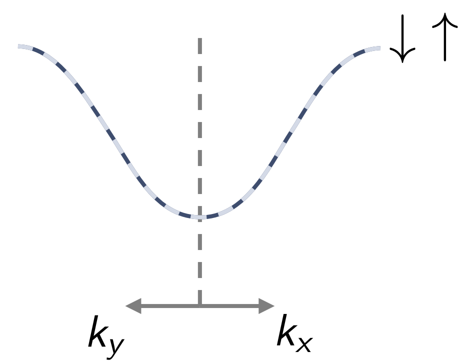
- $R_y(\pi)\,T_x$ symmetry (with $R_y(\pi) : \varepsilon_\sigma(\mathbf{k}) \to \varepsilon_{-\sigma}(\mathbf{k})$ and $T_x : \varepsilon_\sigma(\mathbf{k}) \to \varepsilon_\sigma(\mathbf{k})$) implies spin-degenerate bands



$\varepsilon_\sigma(\mathbf{k}) = \varepsilon_{-\sigma}(\mathbf{k})$



Symmetry-breaking order and magnetism

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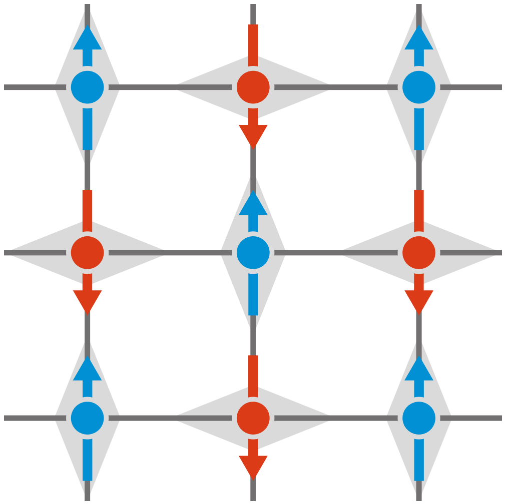

Šmejkal, Sinova, Jungwirth, PRX 2022

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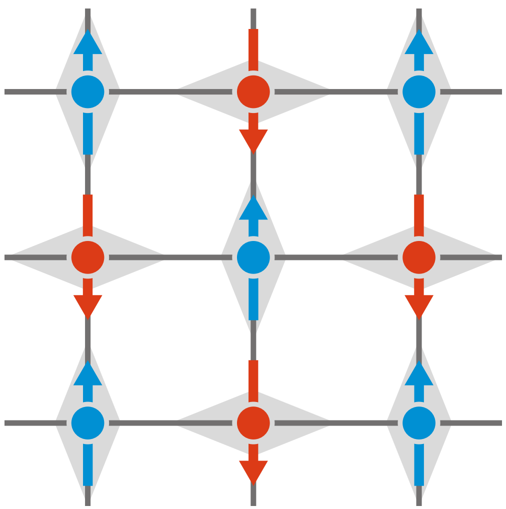

Šmejkal, Sinova, Jungwirth, PRX 2022

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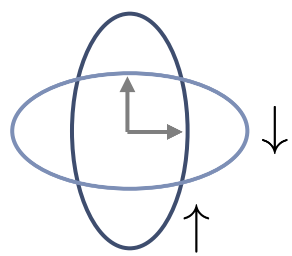
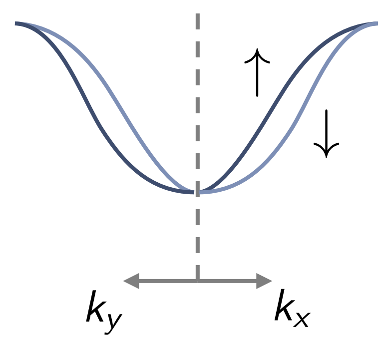
- Crystal environment or orbital ordering (or ...) break translation symmetry: $R_y(\pi) T_x$ not a symmetry!
- Combination of spin and lattice rotation $R_y(\pi) C_4^{\mathrm{p}}$ is a symmetry: enforces $\langle M^z \rangle = 0$ and **d-wave splitting**

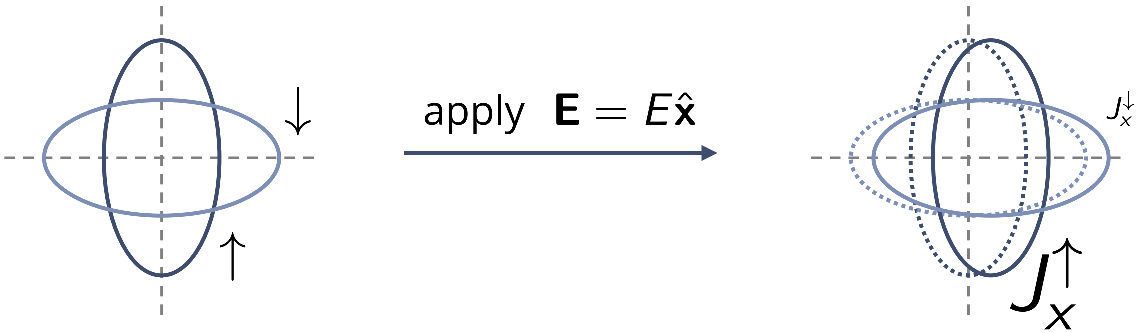


$$\varepsilon_\uparrow(k_x, k_y) = \varepsilon_\downarrow(-k_y, k_x)$$



Altermagnetism — Applications and Experiments

- Interest from spintronics community: spin splittings without ferromagnetic stray fields Šmejkal, Sinova, Jungwirth, PRX 2022



$\Rightarrow$ spin-polarized current
$J_x^{\mathrm{s}} = J_x^{\uparrow} - J_x^{\downarrow} > 0$

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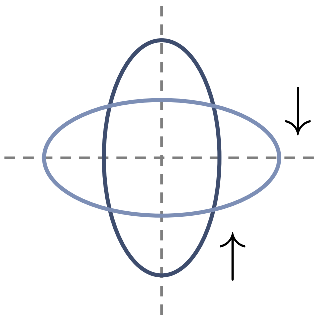
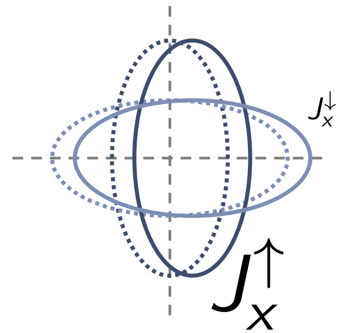


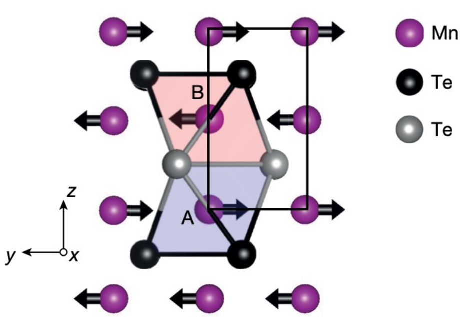
apply $\mathbf{E} = E\hat{\mathbf{x}}$

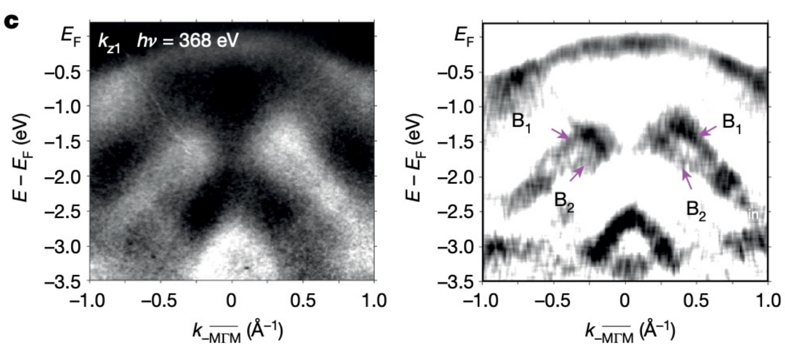


⇒ spin-polarized current

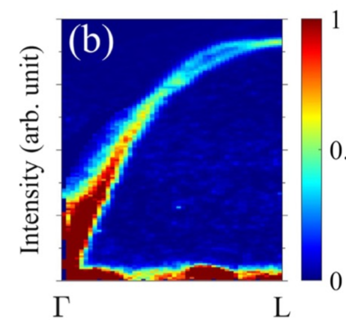
$$J_x^s = J_x^\uparrow - J_x^\downarrow > 0$$

- Most clear evidence in MnTe (so far) Krempasky *et al.*, Nature 2024; Liu *et al.*, PRL 2024





ARPES spectra: band splittings



Neutron scattering (chiral magnons)

Unconventional magnetism: Landau theory and beyond

⇒ Altermagnets fit into classical Landau theory: $F_{\mathrm{AM}} \sim \vec{N} \cdot \vec{O}_{\mu\nu}$

magnetic octupole

PHYSICAL REVIEW X **14,** 011019 (2024)

Ferroically Ordered Magnetic Octupoles in *d*-Wave Altermagnets

Sayantika Bhowal and Nicola A. Spaldin
Materials Theory, ETH Zurich, Wolfgang-Pauli-Strasse 27, 8093 Zurich, Switzerland

PHYSICAL REVIEW LETTERS **132,** 176702 (2024)

Editors' Suggestion

Landau Theory of Altermagnetism

Paul A. McClarty[1,2,*] and Jeffrey G. Rau[3,†]
[1]*Laboratoire Léon Brillouin, UMR12 CEA-CNRS, Université Paris-Saclay, CEA Saclay, 91191 Gif-sur-Yvette, France*
[2]*Max Planck Institute for the Physics of Complex Systems, Nöthnitzer Strasse 38, 01187 Dresden, Germany*
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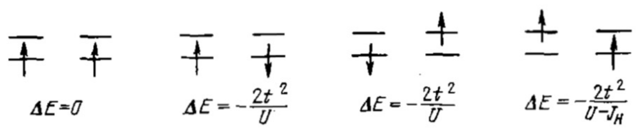
Are there *quantum spin liquids* with altermagnetic features?

Outline

1. Introduction
2. A solvable model for a quantum spin (-orbital) liquid
3. Fractionalized altermagnetism (AM*)
4. Fractionalized superconductivity (SC*)
5. Conclusion

A solvable model for quantum spin(-orbital) liquid

Consider model for $J_{\text{eff}} = 3/2 \simeq$ Spin-1/2 $\otimes$ Pseudospin-1/2 (*e.g.*: twofold-degenerate orbital)



$$\Rightarrow \quad H = \sum_{\langle i,\, j\rangle} (J_1 \mathbf{S}_i \mathbf{S}_j + J_2 \boldsymbol{\tau}_i \boldsymbol{\tau}_j + 4 J_3 \mathbf{S}_i \mathbf{S}_j \boldsymbol{\tau}_i \boldsymbol{\tau}_j),$$

Kugel, Khomskii, Sov. Phys. Usp. 1983
cf. *Altermagnetism from orbital ordering*, Leeb, Mook, Šmejkal, Knolle, PRL 2024

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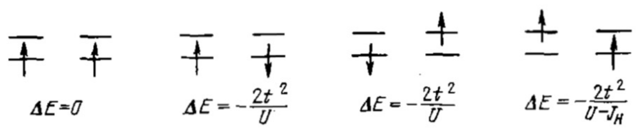

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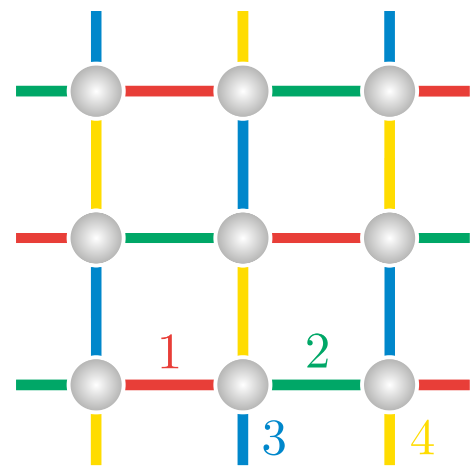
Here: bond-anisotropic XY interactions

$$H = J \sum_{\langle ij\rangle_\mu} \left(s_i^x s_j^x + s_i^y s_j^y \right) t_i^\mu t_j^\mu \qquad t^\mu = \begin{pmatrix} t^x \\ t^y \\ t^z \\ \mathbb{1} \end{pmatrix}$$



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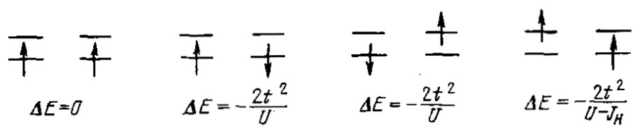

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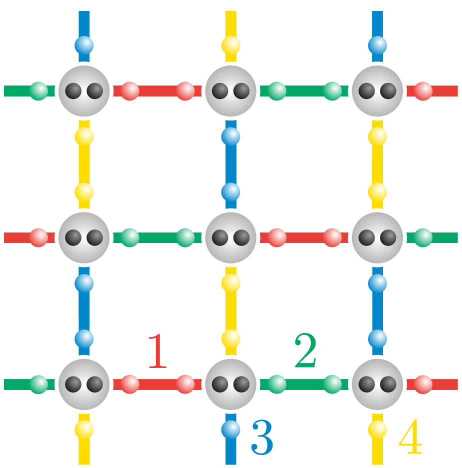

$\Rightarrow$ Exactly solvable (cf. Kitaev honecyomb model):

$$s^z = -i c^x c^y \qquad s^x = -i c^y b^4 \qquad t^\alpha = -i \epsilon^{\alpha\beta\gamma} b^\beta b^\gamma / 2$$

Majorana fermions

with constraint $\quad D_i = -i b_i^1 b_i^2 b_i^3 b_i^4 c_i^x c_i^y \stackrel{!}{=} +1$

Kitaev, AoP 2006
Yao, Lee, PRL 2011

Exact solution — Z_2 gauge theory with fermionic matter

After parton construction: $$H = 2J \sum_{\langle ij \rangle} u_{ij} \left(f_i^\dagger f_j + \text{h.c.} \right)$$

complex (spinless) fermion

Z_2 gauge field (on links)

Conserved plaquette operators $W_\square = \prod_{\langle ij \rangle \in \square} u_{ij} = \pm 1 \;\Rightarrow$ static gauge field

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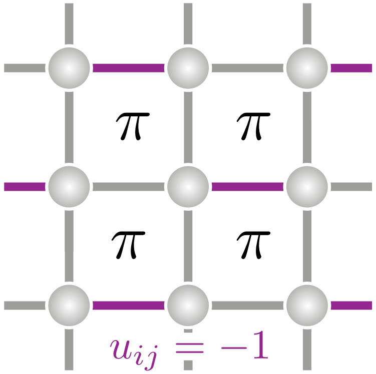

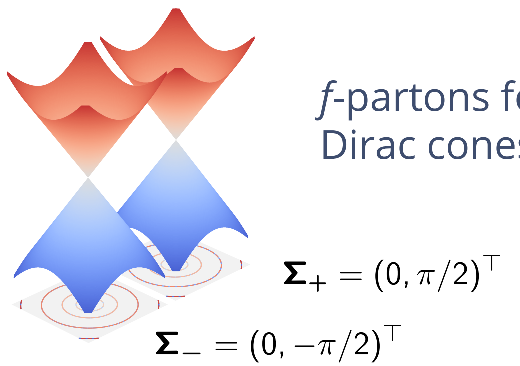

f-partons form
Dirac cones

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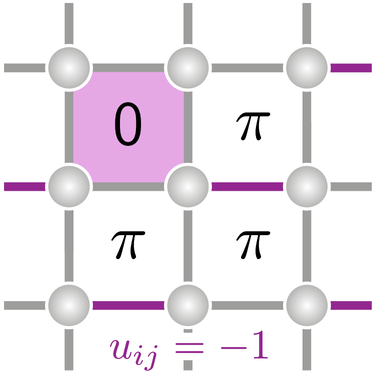
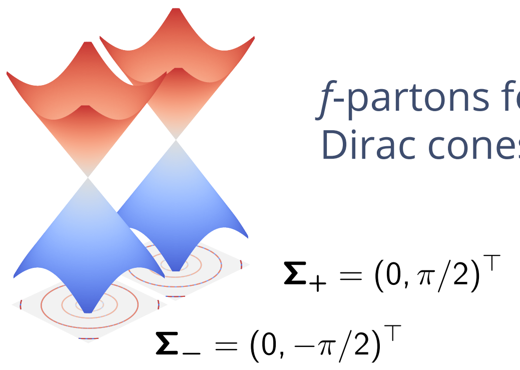
gapped vison



f-partons form Dirac cones



⇒ spins/orbitals fractionalize into visons and complex spinless fermions

Adding Ising interactions

UFPS, Dong, Chulliparambil, Vojta, Tu, Janssen, PRL 2020

$$H = J \sum_{\langle ij \rangle_\mu} \left(\mathrm{s}_i^x \mathrm{s}_j^x + \mathrm{s}_i^y \mathrm{s}_j^y \right) \mathrm{t}_i^\mu \mathrm{t}_j^\mu \longrightarrow H = 2J \sum_{\langle ij \rangle} u_{ij} \left(f_i^\dagger f_j + \mathrm{h.c.} \right)$$

Parton construction

$$+ g \sum_{\langle ij \rangle} \mathrm{s}_i^z \mathrm{s}_j^z \longrightarrow + 4g \sum_{\langle ij \rangle} \left(f_i^\dagger f_i - \frac{1}{2} \right) \left(f_j^\dagger f_j - \frac{1}{2} \right)$$

Gauge field remains static!

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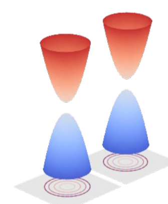
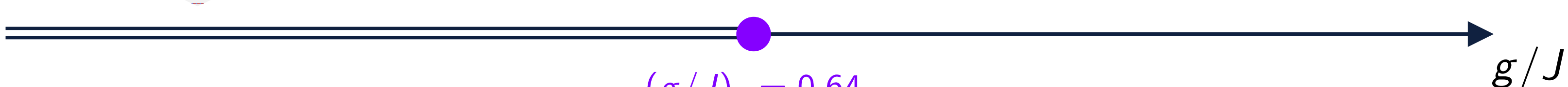
Repulsive fermions on π-flux square lattice: Quantum Monte Carlo

Wang, Corboz, Troyer, NJP 2014

Dirac semimetal
of f-fermions



Charge Density Wave
with $\langle f_i^\dagger f_i \rangle \propto (-1)^i$



$(g/J)_c = 0.64$

(Z_2 Gross-Neveu universality)

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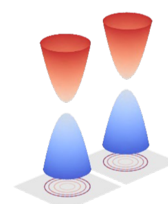

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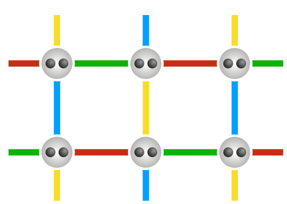
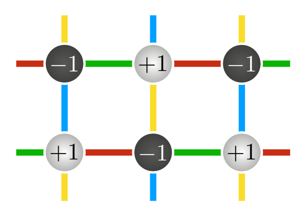




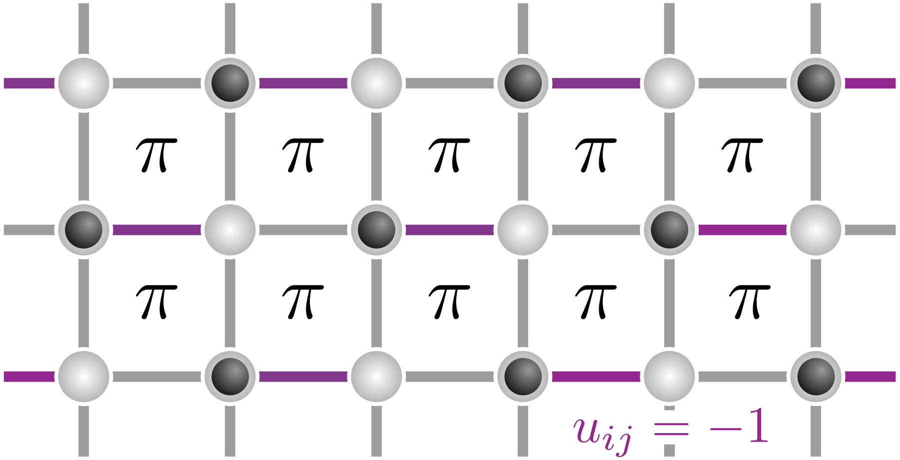
Gapless quantum spin liquid

Ising antiferromagnet
with Z_2 topological order (AF*)

Topological order in AF*

AF*: both antiferromagnetic order *and* Z_2 topological order

(PBC along *x*,*y*)



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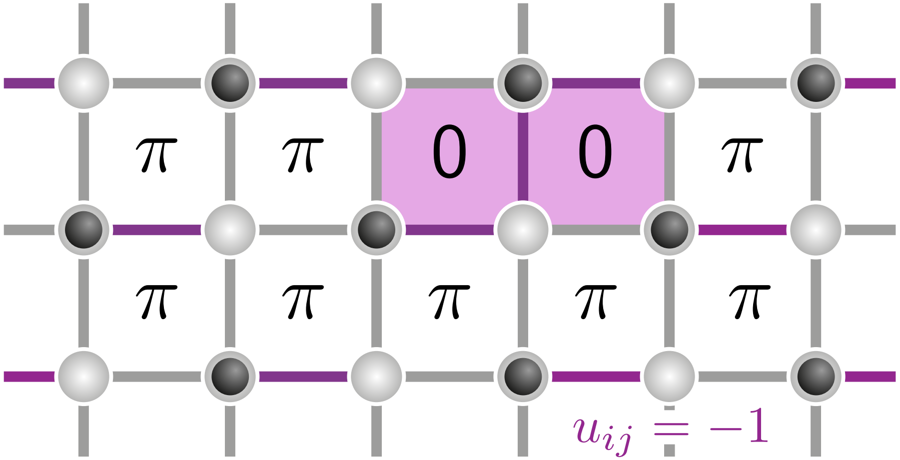

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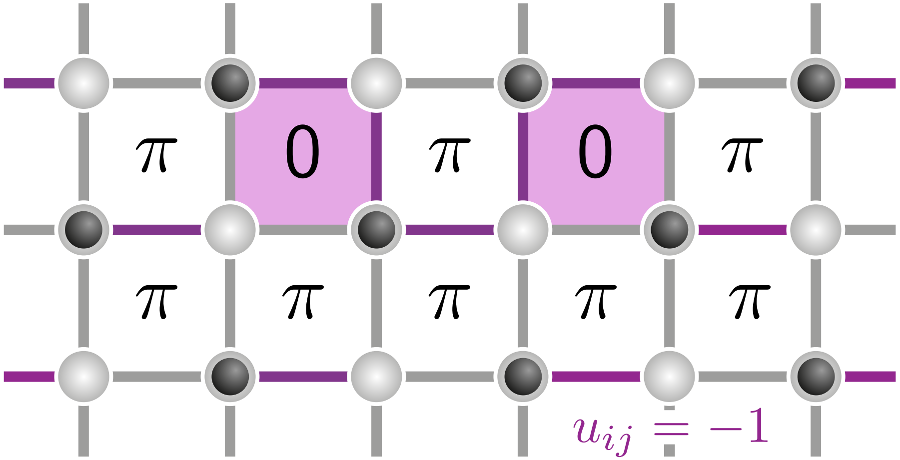

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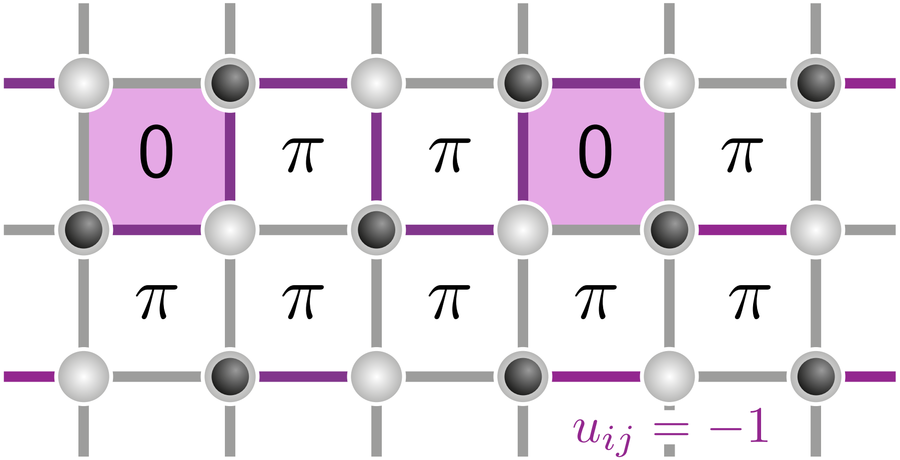

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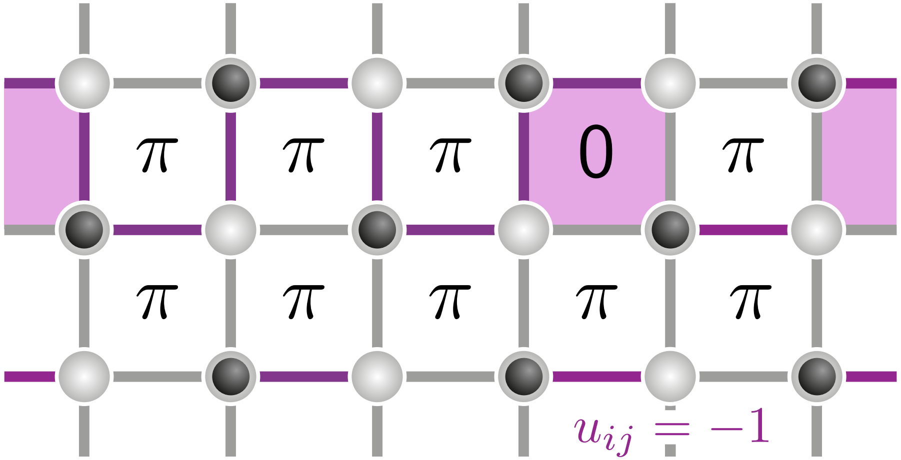

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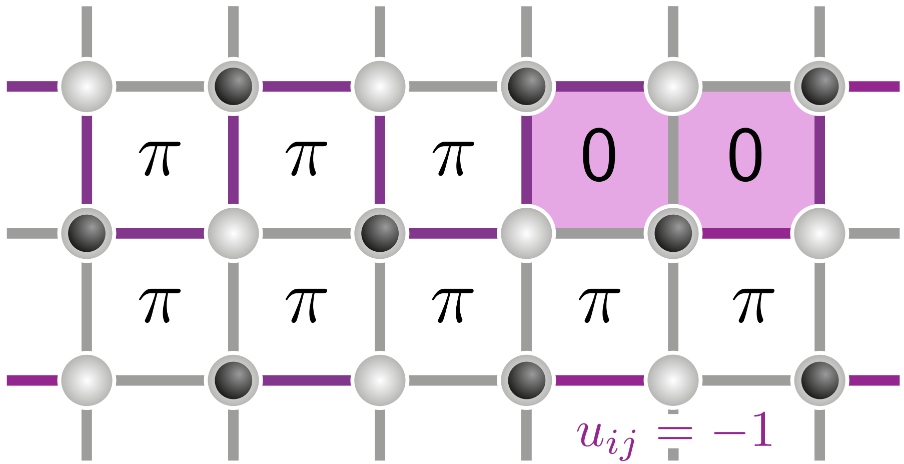

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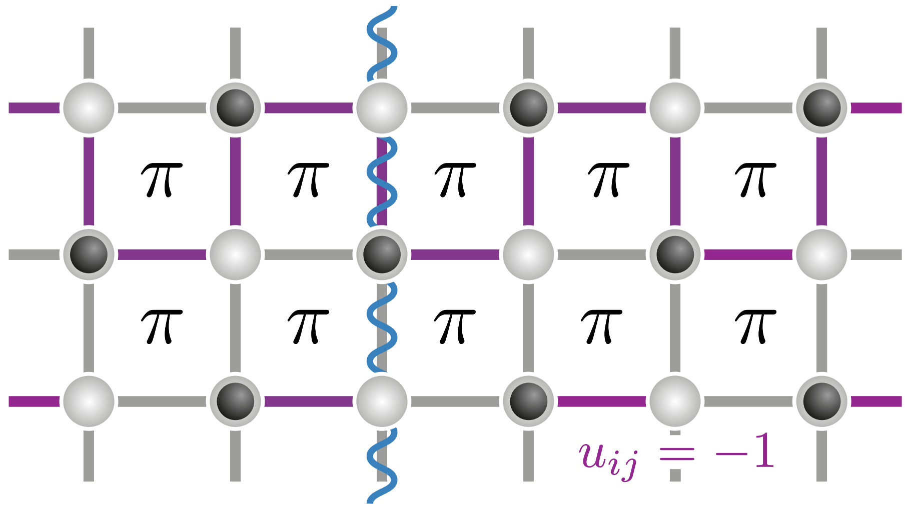

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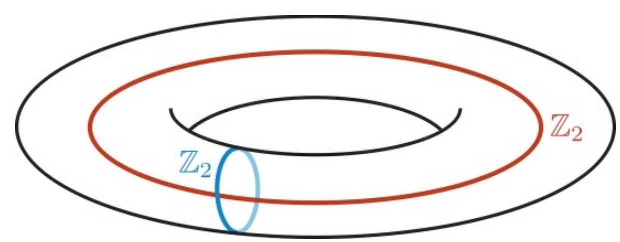

Topological ground state degeneracy on torus: labelled by “large” Wilson loops

Projective symmetry groups

Altermagnetic analogue to AF*? ⇒ Symmetry analysis of parton bands

Complications:

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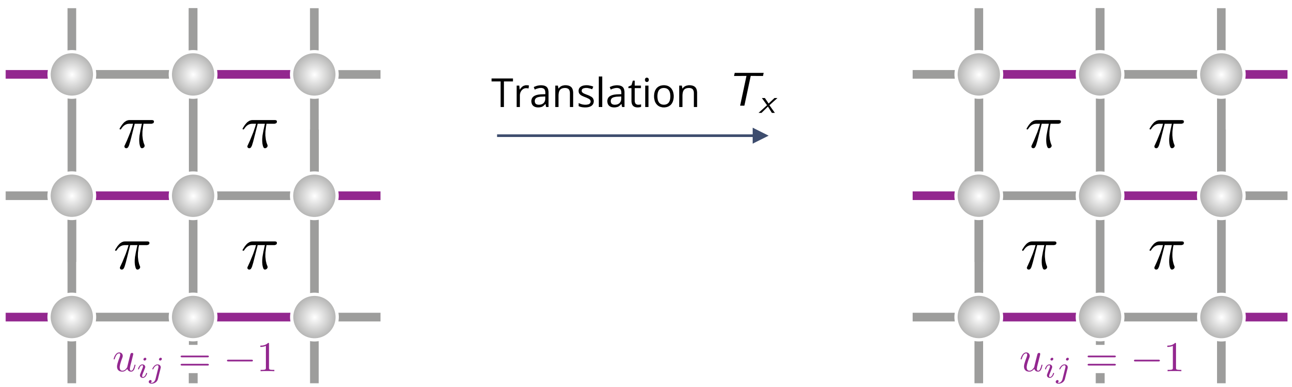

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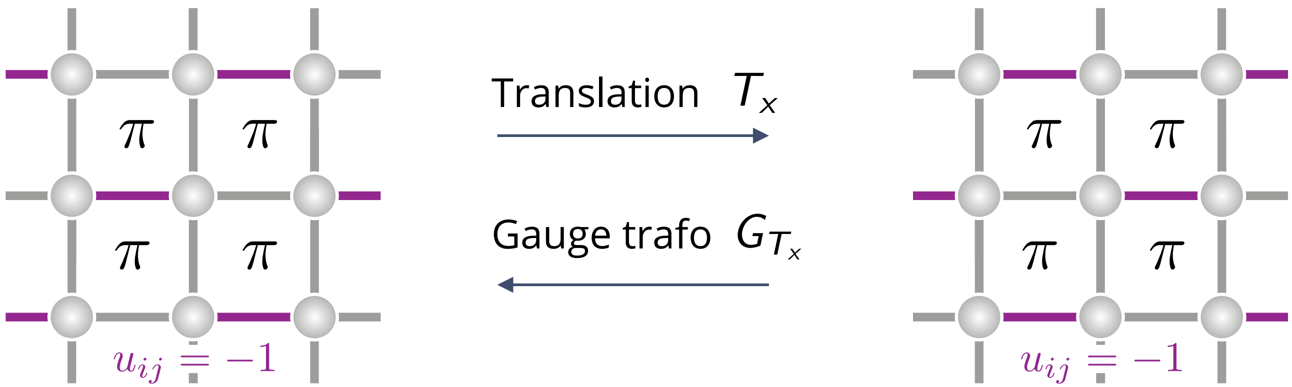


Symmetry is restored *with* gauge transformation $\quad G_{T_x} : u_{ij} \to s_i u_{ij} s_j$

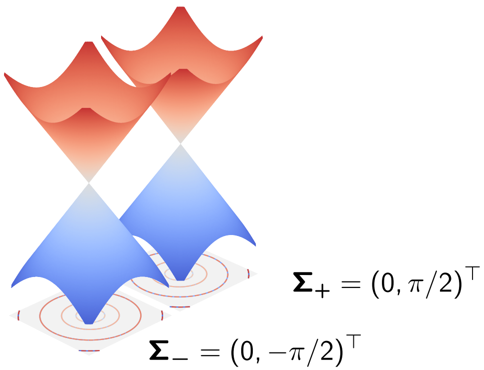
⇒ "Projective Symmetry Group": symmetries act *projectively* in parton theory

$$(G_U U)^{-1} H (G_U U) = H$$

X.-G. Wen, PRB 1990

Projective symmetry operations

How do parton bands transform under projective symmetries?



Translations

swaps valleys

$$\psi(\mathbf{q}) = \begin{pmatrix} \psi_{+,A} \\ \psi_{+,B} \\ \psi_{-,A} \\ \psi_{-,B} \end{pmatrix} \xrightarrow{G_{T_x} T_x} [\tau^y \otimes U_{T_x}(\mathbf{q})]\, \psi(\mathbf{q})$$

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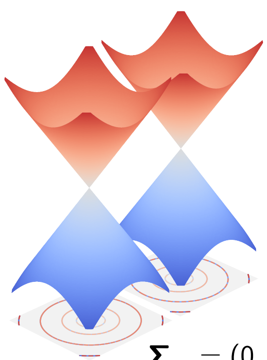

$\mathbf{\Sigma}_+ = (0, \pi/2)^\top$

$\mathbf{\Sigma}_- = (0, -\pi/2)^\top$

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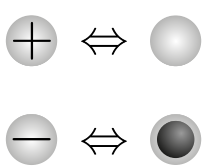
s^z



$$s_i^z = 1 - 2 f_i^\dagger f_i$$

Time-reversal

Projective implementation: particle-hole transformation!

$$\psi(\mathbf{q}) \xrightarrow{G_\mathcal{T} \mathcal{T}} \mathbb{1} \otimes \sigma^z \psi^\dagger(\mathbf{q})$$

$\Rightarrow$ Time-reversal symmetry implies $\quad \varepsilon^+_\tau(\mathbf{q}) = -\varepsilon^-_\tau(\mathbf{q})$

Altermagnetic deformations

What deformations to spin liquid are allowed by “altermagnetic” symmetries?

Assume symmetries of altermagnetic state:

1. Combination of $R_y(\pi) : (s^x, s^y, s^z) \to (-s^x, s^y, -s^z)$ *and* C_4 lattice rotations
2. Lattice reflections $M_{yz} : (x, y) \to (-x, y)$

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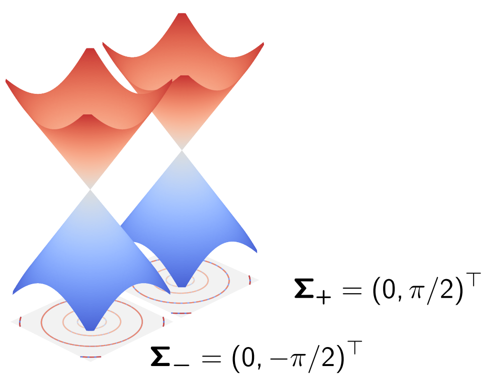
⇒ Weak perturbations (below vison gap): use low-energy parton theory



$$H(q_x, q_y) = \left(a_{\mu\nu} + b^i_{\mu\nu} q_i + c^{ij}_{\mu\nu} q_i q_j\right) \tau^\mu \sigma^\nu$$

constrain via *projectively implemented* symm. trafos

Altermagnetic deformations

Neehus, Rosch, Knolle, UFPS, arXiv:2025.12298

σ^α : sublattice index

τ^α : valley index

$$H^{\text{eff}}(q_x, q_y) = H_0(q_x, q_y) \quad + \quad n\sigma^z \quad + \quad n\kappa(q_x^2 - q_y^2)\,\mathbb{1} \quad + \quad \lambda q_x q_y \tau^z$$

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$\Rightarrow$ $n\kappa \neq 0$ breaks $T_x R_y(\pi)$
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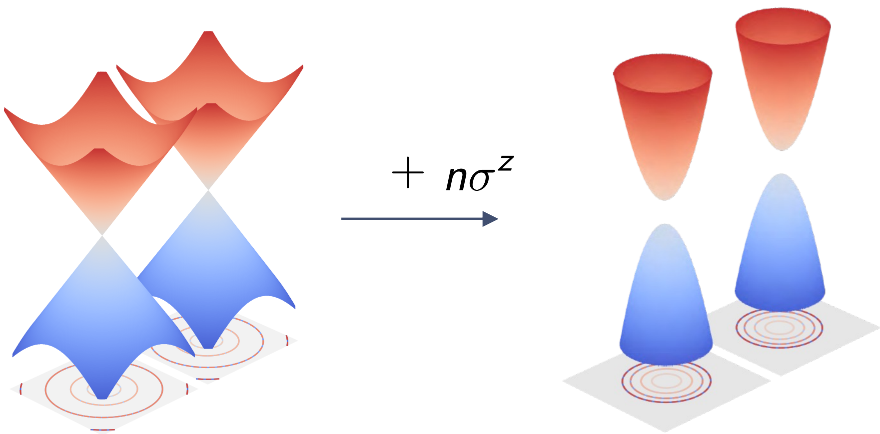
Note: we can write down *exactly solvable* spin-orbital models that realize H^{eff}!

Fermionic parton spectra in QSL, AF* and AM*



Quantum spin liquid
Z_2 gauge theory with
Dirac fermions

Fractionalized antiferromagnet (AF*)
gapped fermions $\langle s_i^z \rangle \sim (-1)^i n$,
and deconfined Z_2 gauge fields

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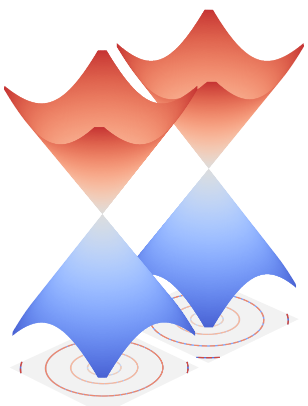
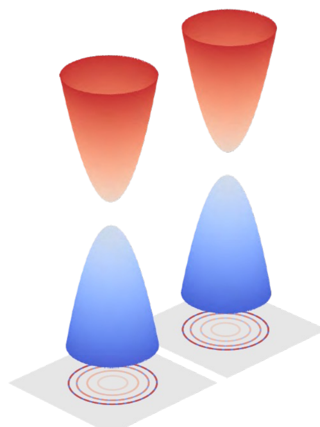
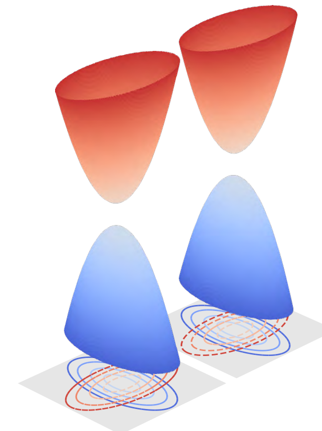


$$\xrightarrow{+\; n\sigma^z}$$



$$\xrightarrow{+\; n\kappa(q_x^2 - q_y^2)\, \mathbb{1}}$$



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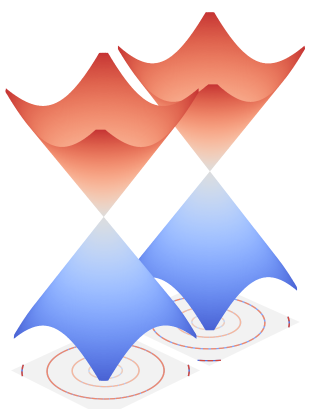

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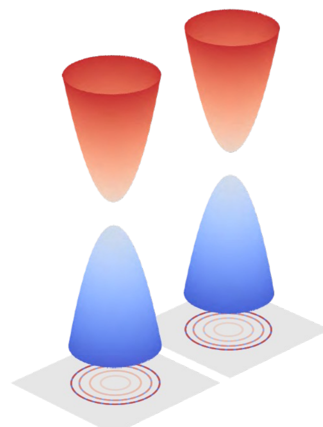

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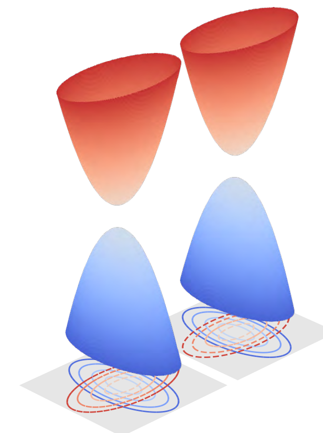
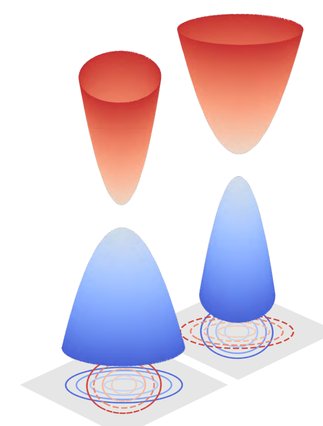


Fractionalized altermagnets (AM*)

$+\ \lambda q_x q_y \tau^z$



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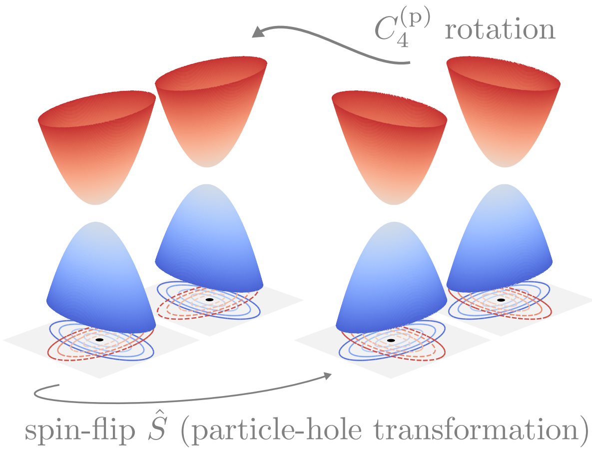
Parton spectrum and physical observables in AM*



d-wave **particle-hole asymmetry** of parton bands:

$$\varepsilon_\tau^{\pm}(\mathbf{q}) = \pm|n| \pm \frac{|\mathbf{q}|^2}{2m_{\text{eff}}} - \kappa n(q_x^2 - q_y^2)$$

(encodes momentum-dependent spin splitting)

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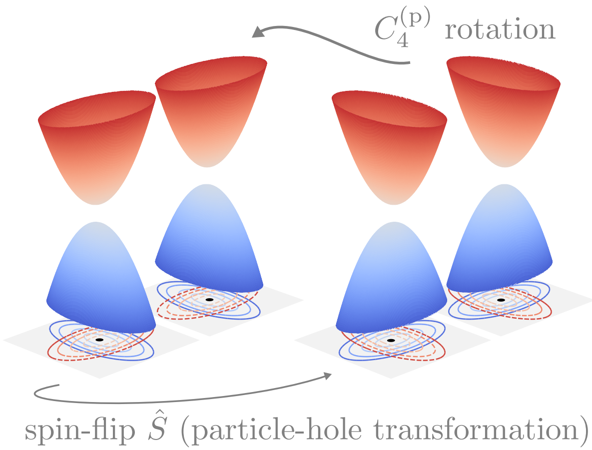

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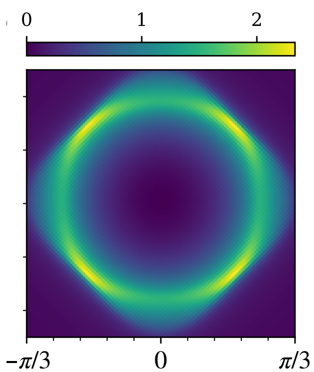
What are physical observables in AM*? Recall: $s_i^z = 1 - 2f_i^\dagger f_i$

Longitudinal structure factor

(No single-parton spectra observable!)

$\langle s^z s^z \rangle(\mathbf{k}, \omega)$



Spin transport in a field

Applied field h breaks $C_4 R_\pi(y)$ -symmetry
⇒ Anisotropic spin conductivity!

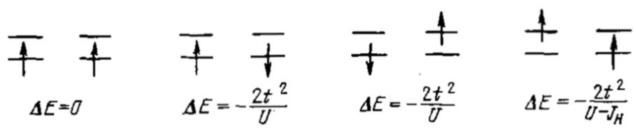
$$j_a^{\mathrm{spin}} = \sigma^{a;b} \nabla_b h^z$$

$$\uparrow$$

$$\sigma^{x,x} - \sigma^{y,y} \propto \kappa n h^z$$

Fractionalized symmetry-breaking orders

So far: fractionalized phases (AF* and AM*) in Mott insulators



$$\Rightarrow \quad H = \sum_{\langle i,j \rangle} (J_1 \mathbf{S}_i \mathbf{S}_j + J_2 \boldsymbol{\tau}_i \boldsymbol{\tau}_j + 4 J_3 \mathbf{S}_i \mathbf{S}_j \boldsymbol{\tau}_i \boldsymbol{\tau}_j),$$

Kugel, Khomskii, Sov. Phys. Usp. 1983

spin-1/2 $\otimes$ orbital-1/2

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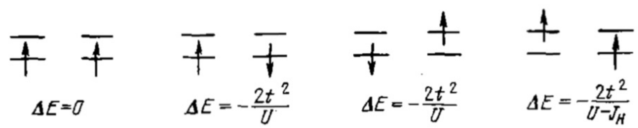

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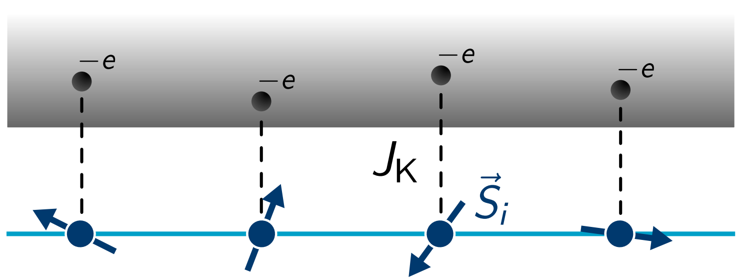
What about states (metals, superconductors) of itinerant electrons?

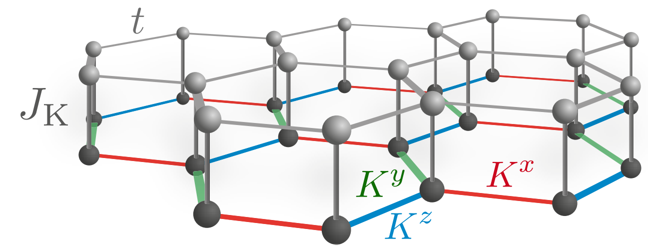
⇒ Consider a band of electrons coupling to lattice of S=1/2 moments



Kitaev Kondo lattice: fractionalized Fermi liquid

Couple electrons to Kitaev spin liquid via Kondo interaction

$$H = -t \sum_{\langle ij \rangle} \left(c^\dagger_{i,\sigma} c_{j,\sigma} + \text{h.c.} \right)$$
$$+ J_\text{K} \sum_i \vec{S}_i \cdot \frac{1}{2} c^\dagger_{i,a} \vec{\sigma}_{ab} c_{i,b} - K \sum_{\langle ij \rangle_\alpha} S^\alpha_i S^\alpha_j$$



Senthil, Sachdev, Vojta, PRL 2003
UFPS, Vojta, Meng, PRB 2018

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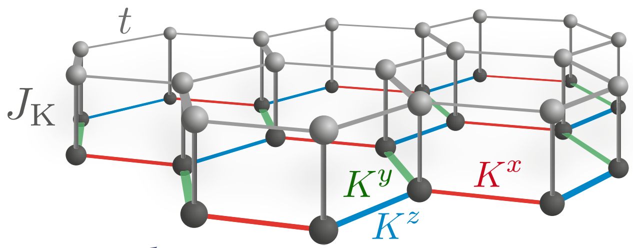
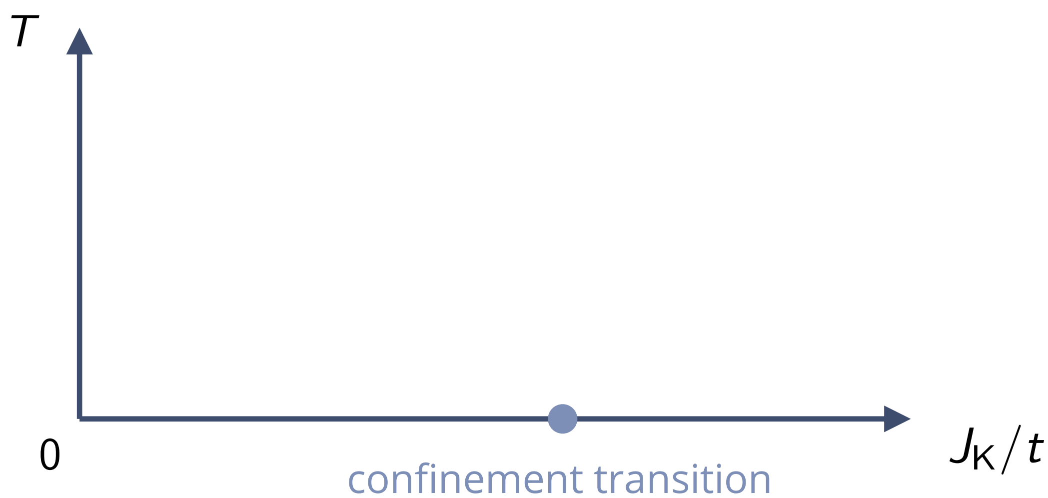


Topological order: spin liquid **stable** against weak Kondo couplings!



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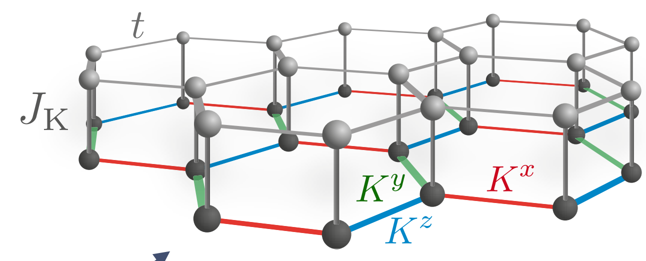

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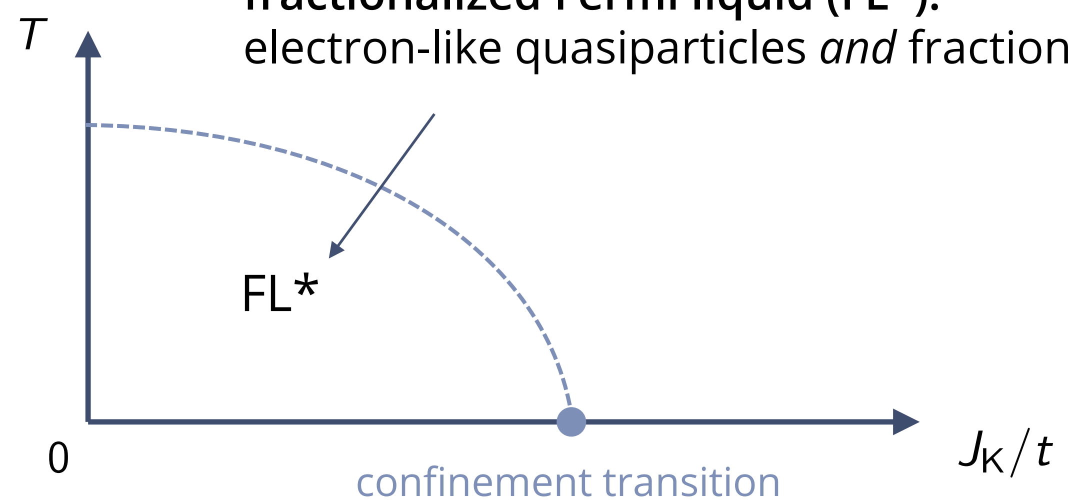
fractionalized Fermi liquid (FL*):
electron-like quasiparticles *and* fractionalized excitations



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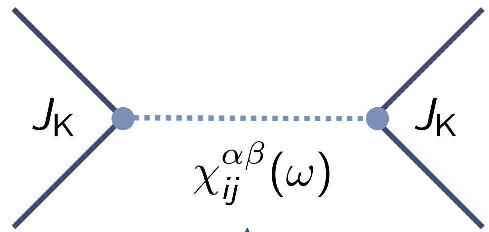
Quasiparticle interactions in FL*

Inside FL*, Kondo coupling generates electron-electron interactions
⇒ integrate out Kitaev spin liquid for $J_\mathrm{K} \ll K, t$

$$\mathcal{S}_\mathrm{int} = \sum_{ij} s_i^\alpha(\omega) V_{i,j}^{\alpha\beta}(\omega) s_j^\beta(\omega) =$$



$+ \ldots$

spin density of itinerant electrons

spin susceptibility of Kitaev QSL
(exactly known)

Knolle, Kovrizhin, Chalker, Moessner, PRL 2014

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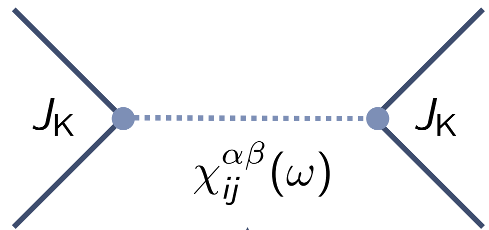

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Static approximation: drop frequency dependence

$$\mathcal{H}_{\mathrm{int}} = J_\mathrm{K}^2 |\chi_0| \left[\sum_i n_{i,\uparrow} n_{i,\downarrow} + \frac{2|\chi_1|}{3|\chi_0|} \sum_{\langle ij \rangle_\alpha} \left(\frac{1}{2} c_i^\dagger \sigma c_i \right) \left(\frac{1}{2} c_j^\dagger \sigma^\alpha c_j \right) \right]$$

⇒ Study instabilities of $\mathcal{H} = \mathcal{H}_t + \mathcal{H}_{\mathrm{int}}$ (honeycomb lattice)!

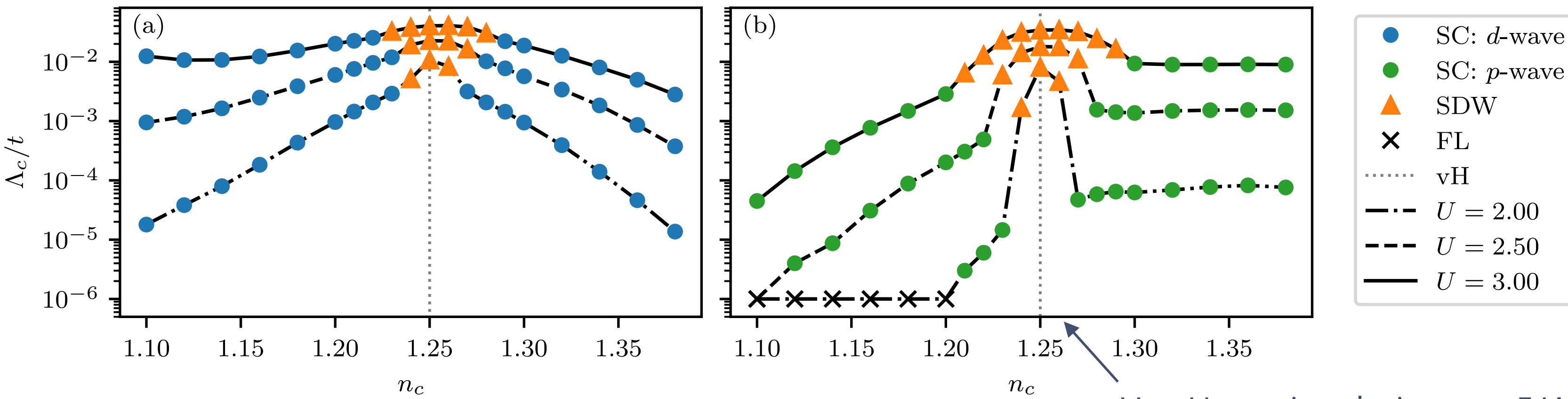
FRG results for effective model

Bunney, UFPS, Rachel, Vojta, PRL 2025

Functional renomalization group results for effective model

Antiferromagnetic Kitaev exchange

Ferromagnetic Kitaev exchange



Van Hove singularity at n=5/4

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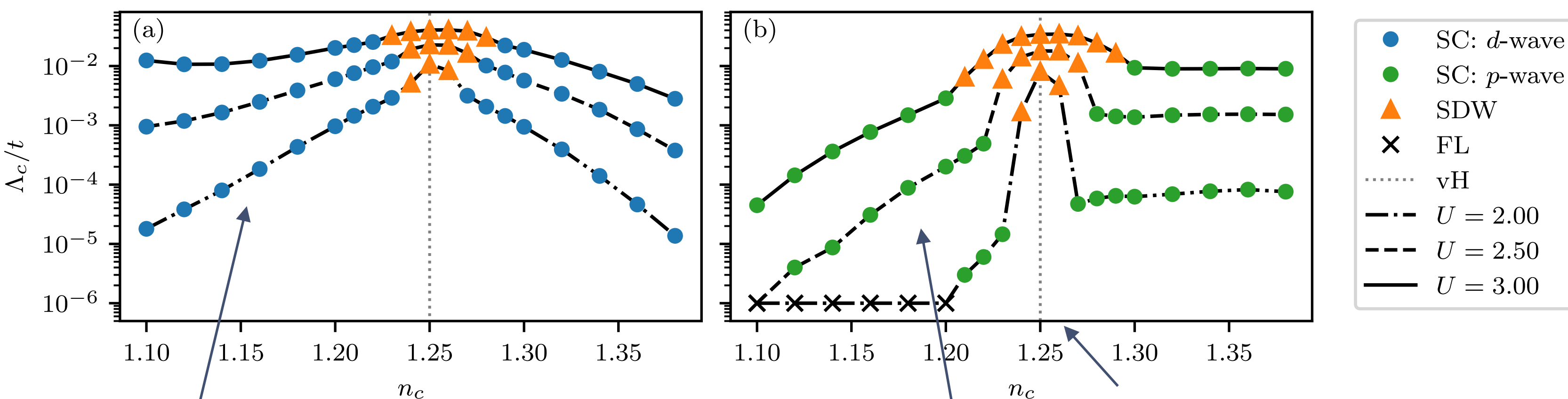
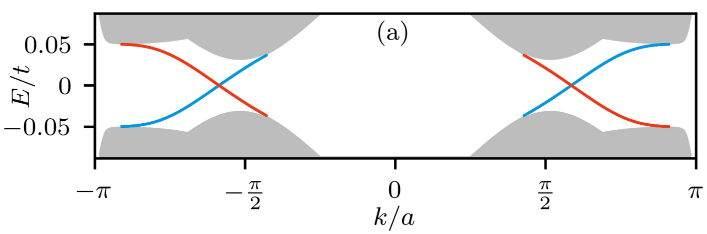
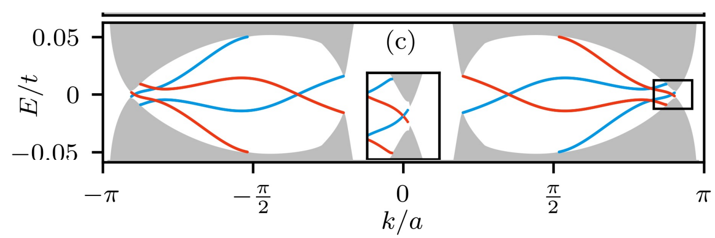


Van Hove singularity at n=5/4

Chiral spin-singlet pairing, C=4　　Chiral spin-triplet pairing, C=4



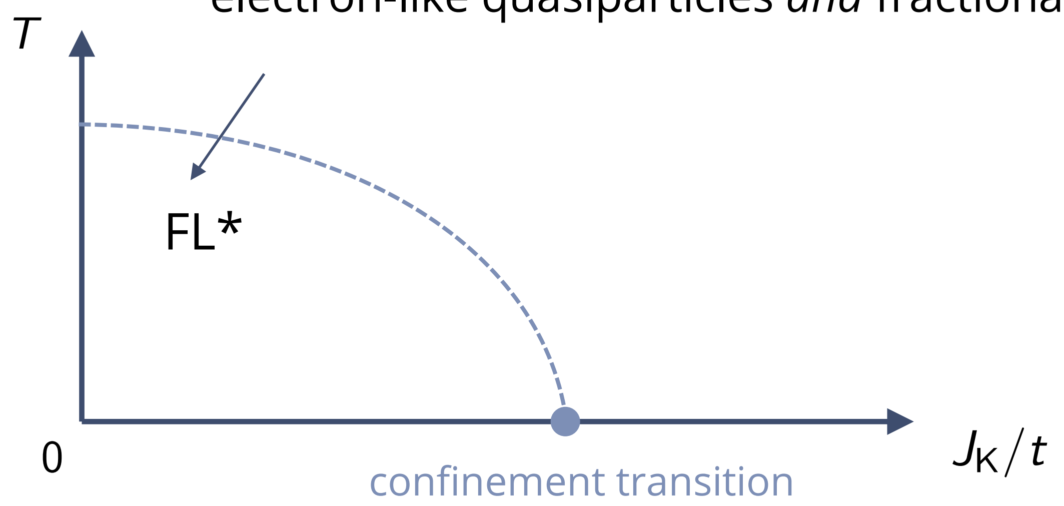


⇒ Interactions induce superconductivity (with non-trivial band topology)!

Fractionalized superconductivity

Bunney, UFPS, Rachel, Vojta, PRL 2025

fractionalized Fermi liquid (FL*):
electron-like quasiparticles *and* fractionalized excitations



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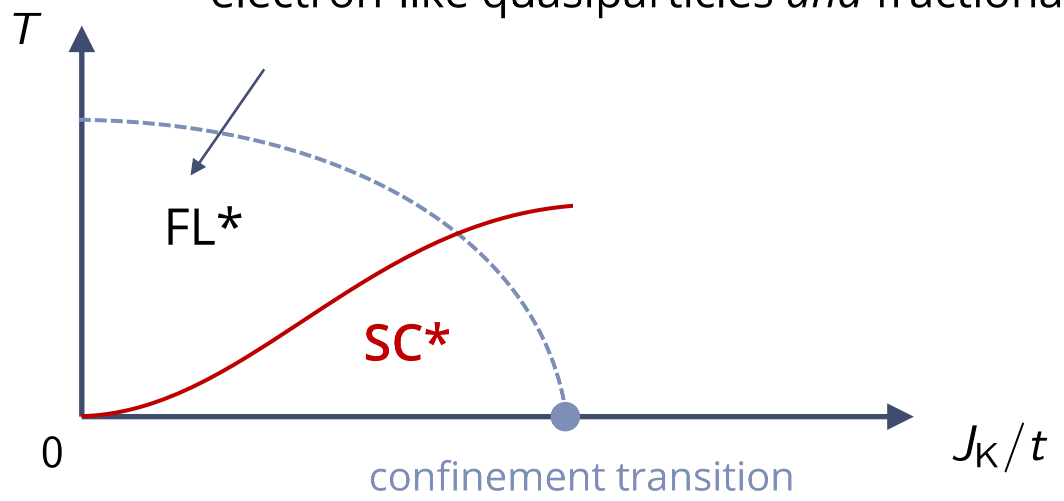

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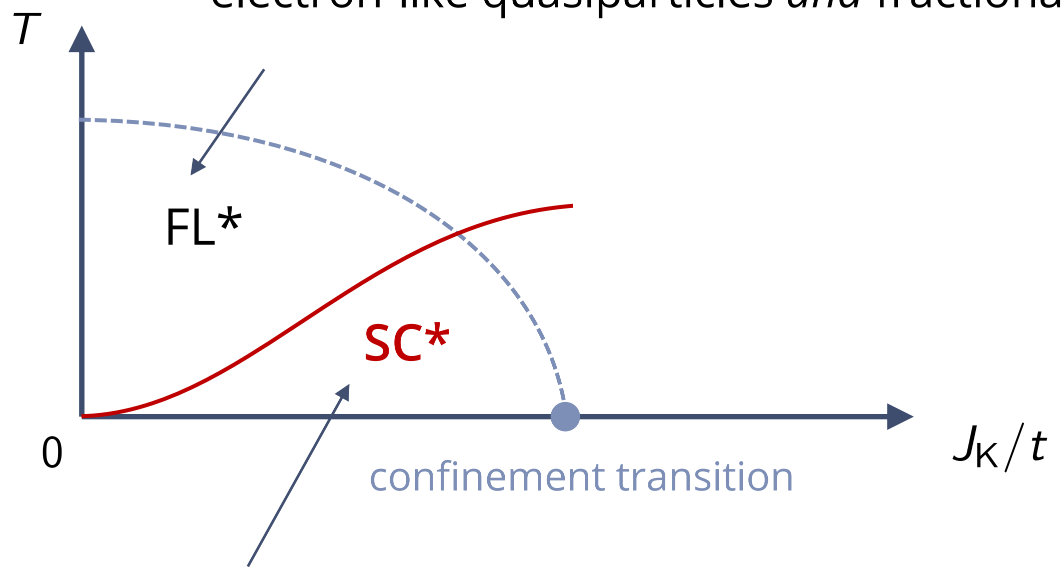
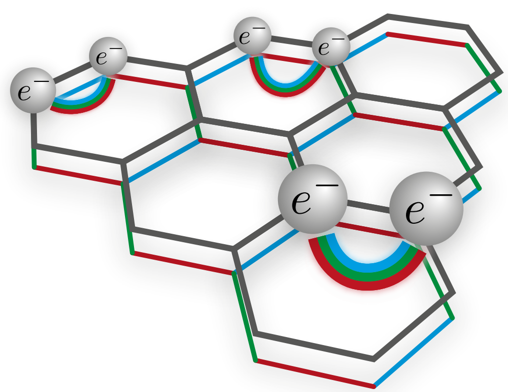
fractionalized Fermi liquid (FL*):
electron-like quasiparticles *and* fractionalized excitations

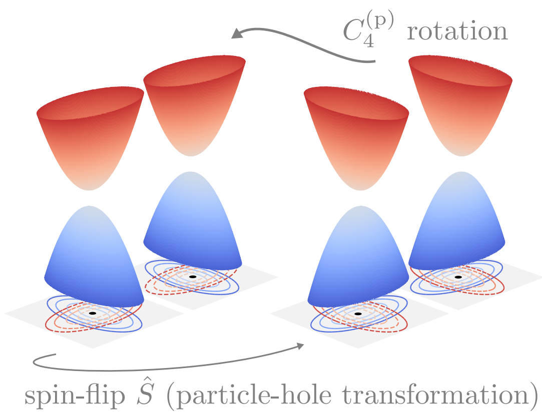




fractionalized superconductivity (SC*)
⇒ “Majorana glue”: pairing mediated by QSL

Fractionalized phases: insulating and itinerant

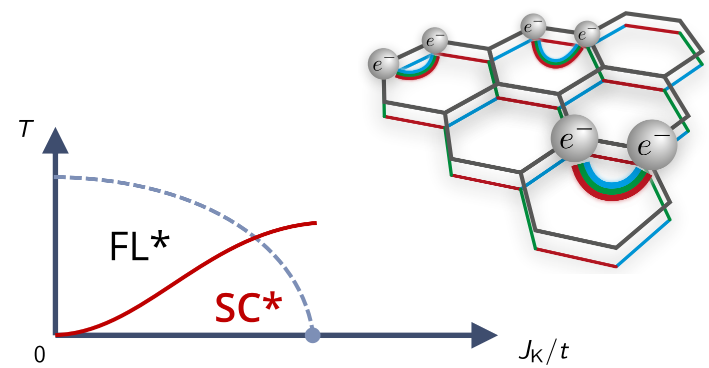
Fractionalized Altermagnetism (AM*)



- Exactly solvable models for fractionalized quasiparticles (partons) with altermagnetic splittings
- Symmetries act projectively, e.g.: TRS encoded in particle-hole symmetry

Neehus, Rosch, Knolle, UFPS, arXiv:2025.12298

FL* and SC* in Kitaev Kondo lattice



- Kitaev Kondo lattice: realizes fractionalized Fermi liquid (FL*)
- Pairing instability in FL* leads to superconductivity (SC*) without confinement

Bunney, UFPS, Rachel, Vojta, PRL 2025

Appendix

Results from symmetry analysis

Allowed terms

	C_4	$R_y(\pi)$	$\mathcal{T}$	$T_{x,y}$	
$H_0(\mathbf{q})$	$+1$	$+1$	$+1$	$+1$	~ Bare parton dispersion
$\mathbb{1}$	$+1$	-1	-1	$+1$	~ Magnetization m^z
σ^z	-1	-1	-1	-1	~ Néel antiferromagnetism n
$(q_x^2 - q_y^2)\mathbb{1}$	-1	-1	-1	$+1$	~ $n\times$ (staggered lattice distortion κ)
$q_x q_y \tau^z$	$+1$	$+1$	-1	-1	~ Orbital antiferromagnetism