Spin liquids on the tetratrillium lattice

Matías G. Gonzalez

Bonn University

Quantum Spin Liquids 2025
Budapest, Hungary
October 7, 2025

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Thanks to collaborations with I. Zivkovic, Y. Iqbal, and many others.

Philosophy of the talk

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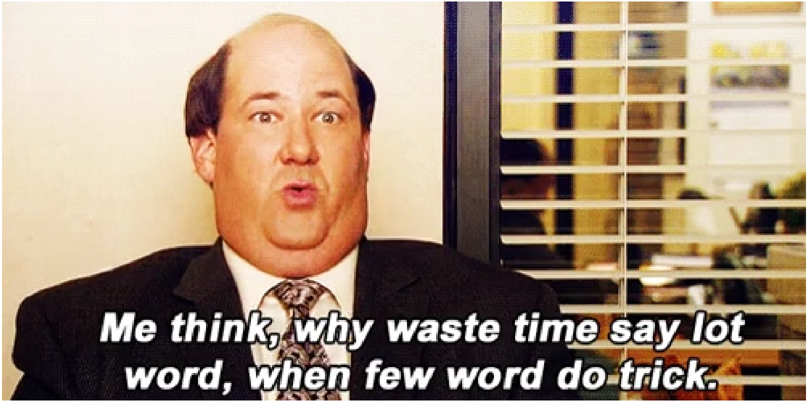
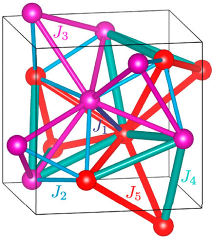


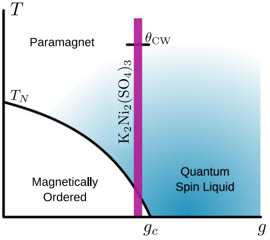
Motivation (see previous talk)

$S = 1$ Langbeinite $K_2Ni_2(SO_4)_3$



Ni^{2+} (red)
Ni^{2+} (magenta)

$J_1 = 0.364(2)$ K
$J_2 = -0.144(1)$ K
$J_3 = 0.798(2)$ K
$J_4 = 5.545(1)$ K
$J_5 = 2.657(1)$ K



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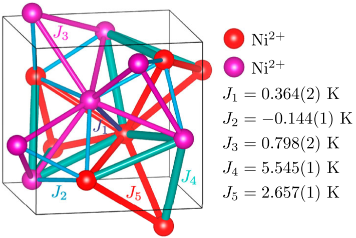
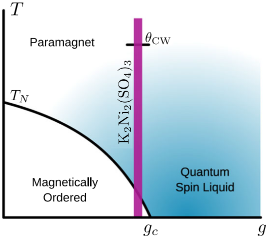
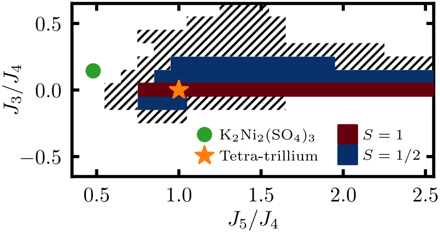




Disordered region



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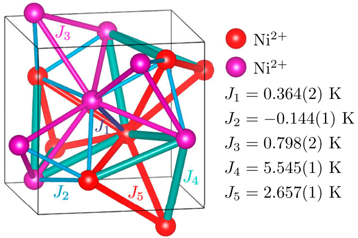

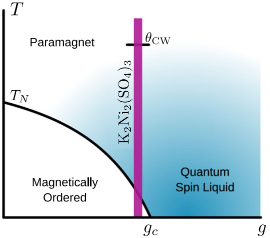

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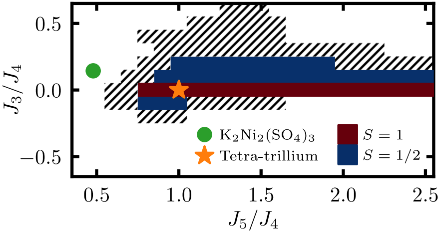


Key points

- There is a material close to a disordered region
- The disordered region exists around the Tetratrillium lattice

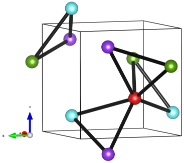
MGG, Y. Iqbal, I. Zivkovic, et al. Nat Commun 15, 7191 (2024)

The Tetratrillium Lattice

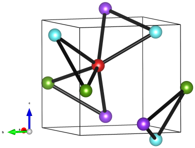
The trillium lattice

- Four sites per unit cell

site	position
1	(u, u, u)
2	$(-\frac{1}{2} + u, \frac{1}{2} - u, 1 - u)$
3	$(1 - u, -\frac{1}{2} + u, \frac{1}{2} - u)$
4	$(\frac{1}{2} - u, 1 - u, -\frac{1}{2} + u)$



u=0.336



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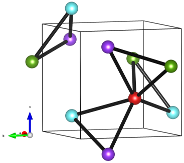

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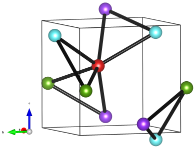

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- Large-N: subextensive SL
- Heisenberg: magnetic order

J Hopkinson, et al. Phys. Rev. B 74, 224441 (2006)
SV Isakov, et al. Phys. Rev. B 78, 014404 (2008)

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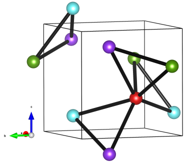

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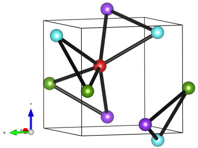

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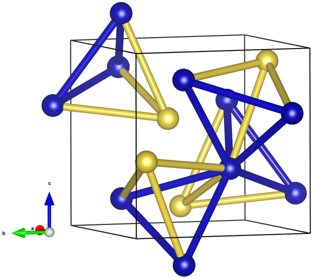

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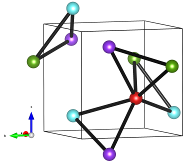

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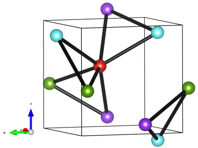

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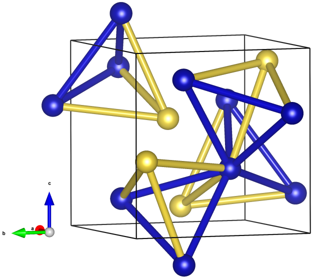

$J_1 = 0.48$ $J_{12} = 1$ $J_2 = 0.14$

MGG, Y Iqbal, I Zivkovic, et al. Nat Commun 15, 7191 (2024)

$J_1 = 0.55$ $J_{12} = 1$ $J_2 = 0.02$

W Yao, et al. Phys. Rev. Lett. 131, 146701 (2023)

We will use: $J_1 = 1$ $J_{12} = 1$ $J_2 = 0$

Large-N Theory

Spin Hamiltonian

$$\mathcal{H} = \frac{1}{2}\sum_{i,j}^{M}\sum_{\alpha,\beta}^{N_{\text{sub}}} J_{i_\alpha, j_\beta}\mathbf{S}_{i_\alpha}\mathbf{S}_{j_\beta}$$

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Constrainer Hamiltonian

$$\mathcal{H} \equiv \sum_{i=1}^{M}\sum_{\boxtimes=1}^{4} T_{i,\boxtimes}^2$$

with

$$T_{i,\boxtimes}^2 = (S_{i,\boxtimes_1} + S_{i,\boxtimes_2} + S_{i,\boxtimes_3} + S_{i,\boxtimes_4})^2$$

4 constraints and 8 sites per unit cell $\Rightarrow$ 4 flat bands

H Yan, et al. Phys. Rev. B 110, L020402 (2024)
H Yan, et al. Phys. Rev. B 109, 174421 (2024)

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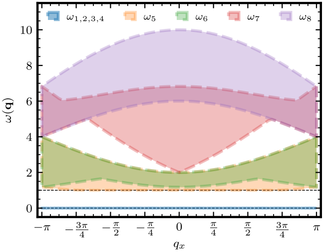
Spectrum of J(q)



Gap $\Rightarrow$ fragile spin liquid

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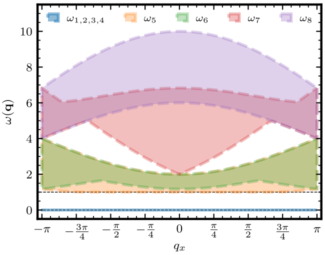

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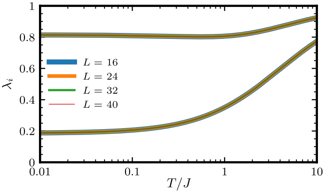
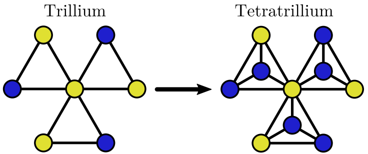
At finite temperatures...



Ising Model

Ground-state manifold



Trillium lattice: $2\uparrow 1\downarrow$ or $1\uparrow 2\downarrow$
Tetratrillium: turn them into $2\uparrow 2\downarrow$

Ising Model

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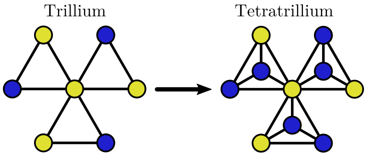

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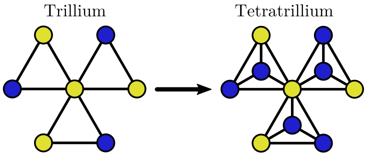
Pauling counting

unit	n	$s(T=0)$ trillium	tetratrillium
$\triangle$ or $\boxtimes$	6	0.4055	0.2027
$L=1$	6	0.4479	0.2240
$L=2$	314874	0.3956	0.1978

$$s_{\boxtimes} = 0.5\, s_{\triangle}$$

Ising Model

Ground-state manifold



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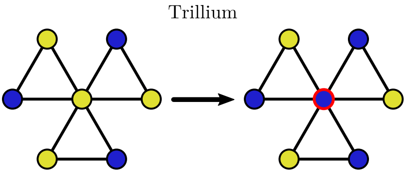
Navigating the manifold



TE Redpath, et al. Phys. Rev. B 82, 014410 (2010)

Ising Model

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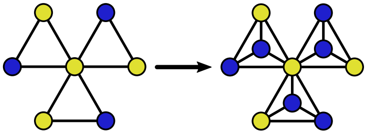

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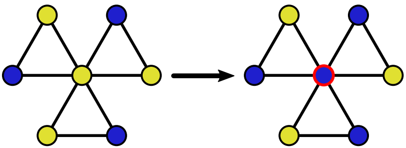

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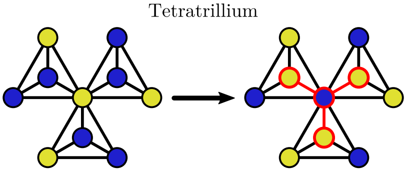
On the trillium lattice...

One can perform classical Monte Carlo simulations with Metropolis updates alone and keep the acceptance ratio finite while moving through the manifold.

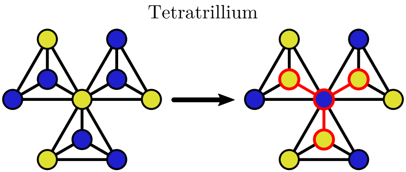
Ising Model

Navigating the manifold



Ising Model

Navigating the manifold



On the tetratrillium lattice...

One can perform classical Monte Carlo simulations with *star* updates to keep the acceptance ratio finite while moving through the manifold.

Ising Model

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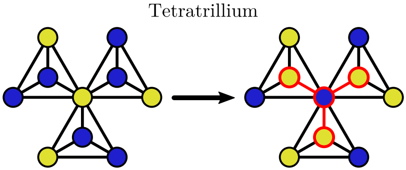

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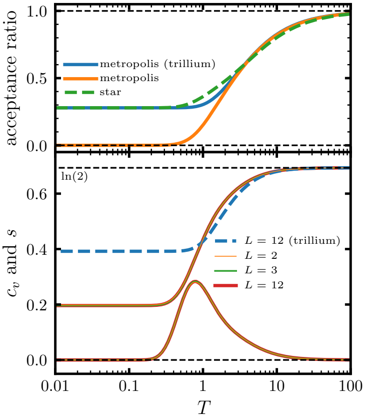

Ising Model

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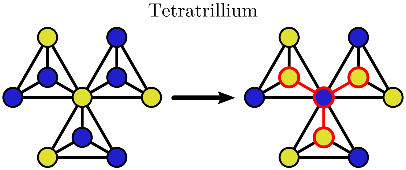
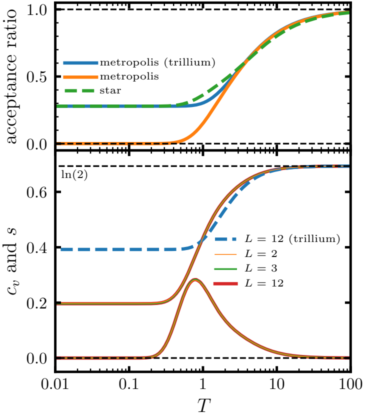




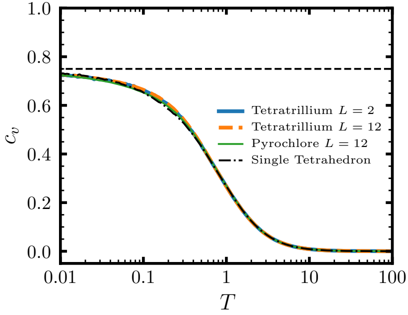
On the tetratrillium lattice...

One can perform classical Monte Carlo simulations with *star* updates to keep the acceptance ratio finite while moving through the manifold.

No sign of a phase transition and little to no finite-size effects.
A classical spin liquid with fast-decaying correlations.

Heisenberg Model

Classical Monte Carlo



Heisenberg Model

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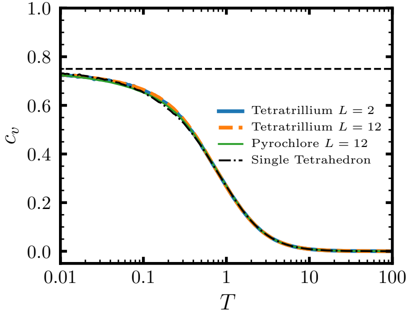
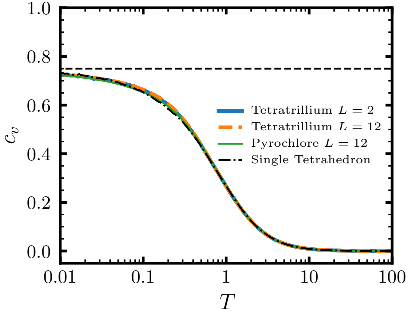


Key points

- No sign of a phase transition
- $c_v(T \to 0) \to 0.75$ as for the pyrochlore lattice

Heisenberg Model

Classical Monte Carlo



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Classical Summary

Trillium Lattice

- Large-*N*: subextensive SL
- Heisenberg: ordered
- Ising: SL

Heisenberg Model

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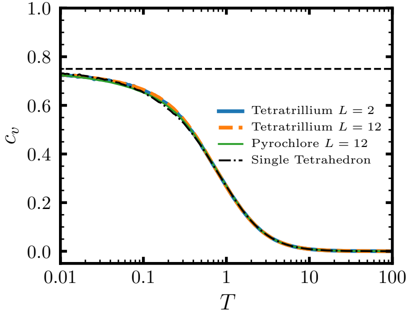

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Classical Summary

Trillium Lattice

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- Heisenberg: ordered
- Ising: SL

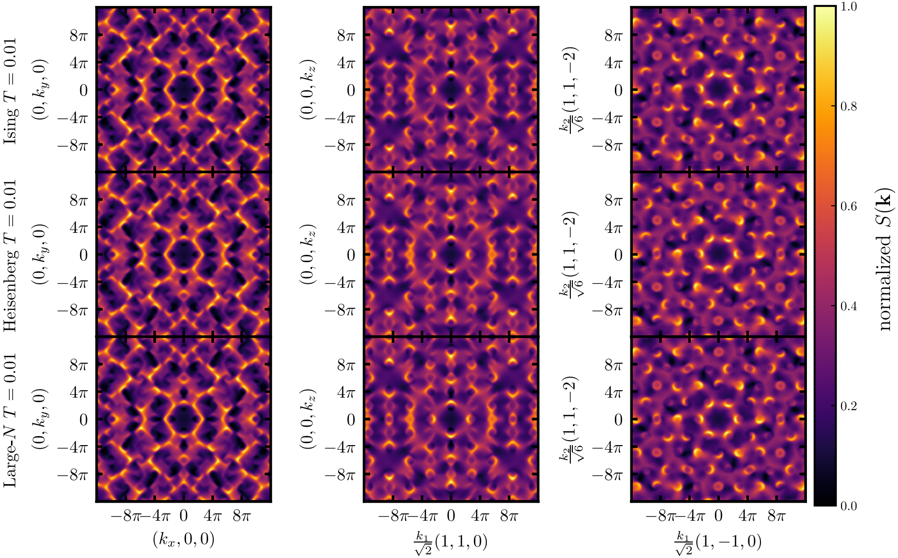
Tetratrillium Lattice

- Large-N: fragile SL with exponentially-decaying correlations
- Heisenberg: SL
- Ising: SL

Effect of decorating a lattice while keeping the number of constraints constant.

Spin structure factor

Spin structure factor



Quantum Heisenberg Model

Pseudo-Majorana FRG

$$S_i^x = -i\eta_i^y \eta_i^z, \; S_i^y = -i\eta_i^z \eta_i^x, \; S_i^z = -i\eta_i^x \eta_i^y$$

with $\{\eta_i^\mu, \eta_j^\nu\} = \delta_{i,j}\delta_{\mu,\nu}$

- No unphysical states
- T can be used as a cutoff

N Niggemann, et al. Phys. Rev. B 103, 104431 (2021)
B Schneider, et al. Phys. Rev. B 109, 195109 (2024)

Quantum Heisenberg Model

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Advantages

- Exact at large temperatures
- Can handle complicated 3D lattices
- Access to correlations at low temperatures
- Detect phase transitions through finite-size scaling
- Can handle Vesta files

https://github.com/NilsNiggemann/PMFRG.jl/tree/TemperatureFlow

Quantum Heisenberg Model

Pseudo-Majorana FRG

$$S_i^x = -i\eta_i^y\eta_i^z,\ S_i^y = -i\eta_i^z\eta_i^x,\ S_i^z = -i\eta_i^x\eta_i^y$$

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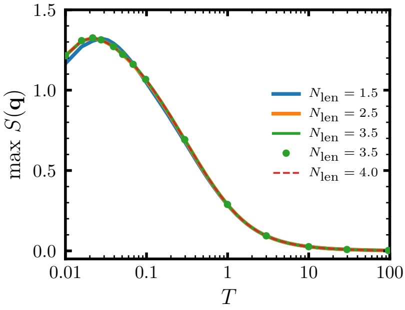
Maximum $S(\mathbf{q})$



Key points

No sign of a phase transition.
Fast convergence with system size.

Quantum Heisenberg Model

Pseudo-Majorana FRG

$$S_i^x = -i\eta_i^y \eta_i^z, \ S_i^y = -i\eta_i^z \eta_i^x, \ S_i^z = -i\eta_i^x \eta_i^y$$

with $\{\eta_i^\mu, \eta_j^\nu\} = \delta_{i,j}\delta_{\mu,\nu}$

- No unphysical states
- T can be used as a cutoff

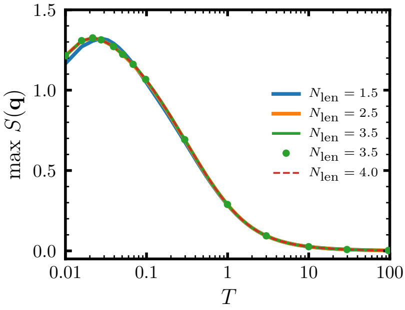
N Niggemann, et al. Phys. Rev. B 103, 104431 (2021)
B Schneider, et al. Phys. Rev. B 109, 195109 (2024)

Advantages

- Exact at large temperatures
- Can handle complicated 3D lattices
- Access to correlations at low temperatures
- Detect phase transitions through finite-size scaling
- Can handle Vesta files

https://github.com/NilsNiggemann/PMFRG.jl/tree/TemperatureFlow

Maximum $S(\mathbf{q})$

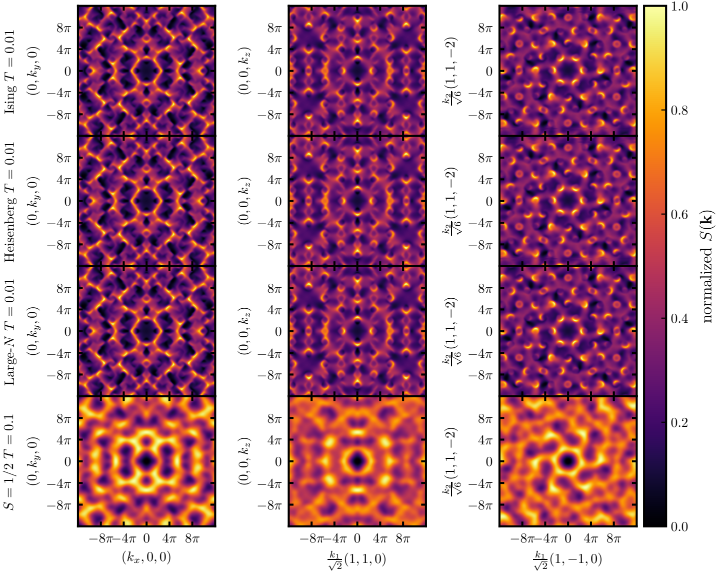


Key points

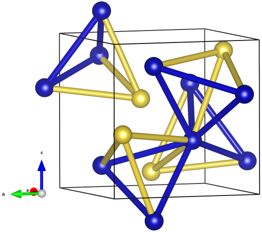
No sign of a phase transition.
Fast convergence with system size.

Is it a Quantum Spin Liquid?

Spin structure factor



Conclusions and perspectives



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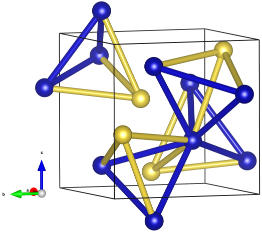

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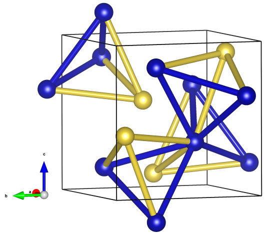

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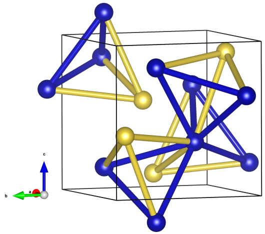

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Thank you!