Stability of critical spin liquids & deconfined criticality

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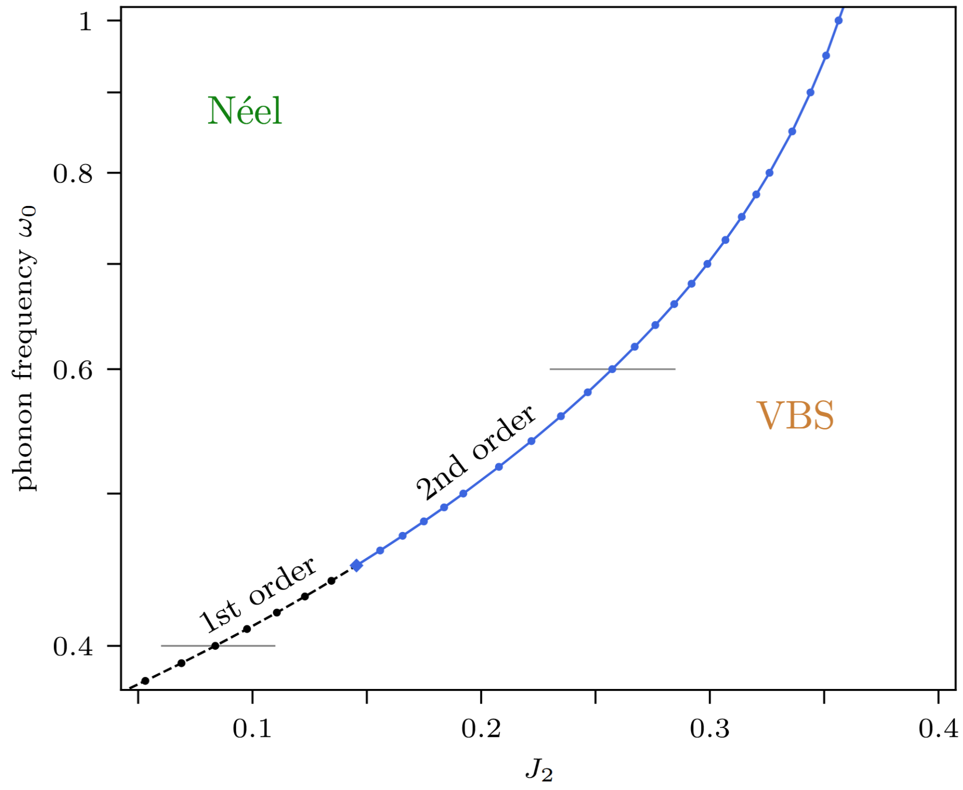

In collaboration with:
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Quantum Spin Liquids 2025: Experimental Enigmas & Theoretical Challenges
October 10th, 2025

Deconfined Quantum Criticality

Proposed continuous phase transition

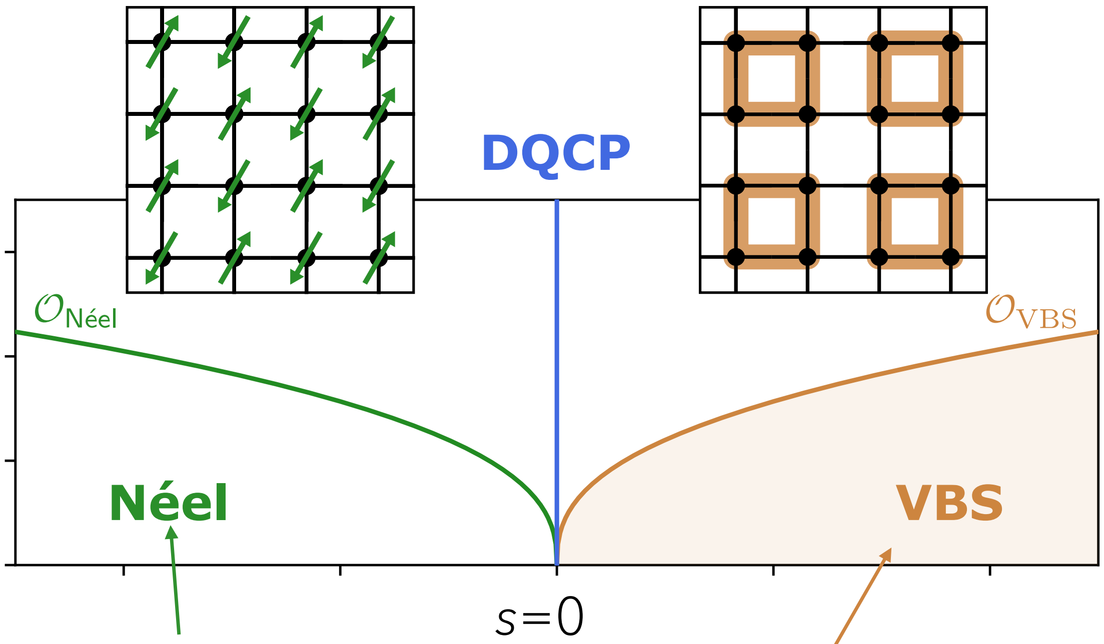
- Between incompatible ordered phases
- Fine-tuned in Landau paradigm

[Senthil, et al. Science 2004]

Emergent CP¹ gauge theory

- Emergent U(1) gauge theory
- *Deconfined* at transition point s=0

$$\mathcal{L}_c = \sum_{\alpha=1}^{2} |(\partial_\mu - i a_\mu) z_\alpha|^2 + s|z|^2 + u(|z|^2)^2 + \lambda_4 \left[\Phi^4 + (\Phi^\dagger)^4\right]$$



$$\boldsymbol{n_j} = \eta_j \langle \boldsymbol{S_j} \rangle \qquad v_j^a = \langle \vec{S}_j \cdot \vec{S}_{j+\delta_a} \rangle$$

Parton gauge theory

$$\boldsymbol{n} = z^\dagger \boldsymbol{\sigma} z$$

$$v_1 \sim \mathrm{Re}\,\Phi = \Phi_1$$

$$v_2 \sim \mathrm{Im}\,\Phi = \Phi_2$$

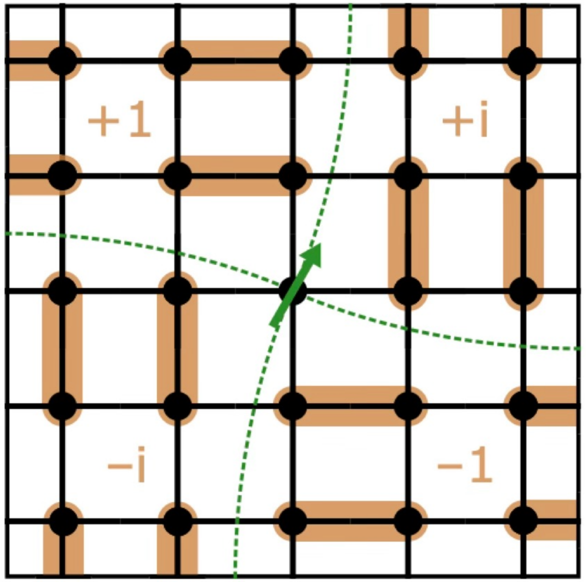
Deconfined Quantum Criticality

Emergent SO(5) symmetry at DQCP

- Of the super-order parameter $\tilde{\boldsymbol{n}} = (\boldsymbol{n}, \boldsymbol{v})$
- Proposed CFT with algebraic correlations controlled by one exponent [Senthil, et al. Science 2004]

Intertwinement of order parameters

- Defects of one order carry quantum numbers of the other [Haldane, PRL 1988]; [Read & Sachdev PRB 1990]
- Defect proliferation leads to a competing ordered phase



[Levin and Senthil, PRB 2004]

Alternative description: SO(5) non-linear sigma model

- Intertwinement is due to WZW topological term [Wang, et al. PRX 2017]; [Zou et al. PRX 2021]

$$S_2^{\mathrm{DQCP}}[\tilde{\boldsymbol{n}}] = S_2^{\mathrm{kin}}[\tilde{\boldsymbol{n}}] + \int \mathrm{d}^3x\, s(\boldsymbol{n}^2 - \boldsymbol{v}^2) + \Gamma_2^{\mathrm{SO}(5)}[\tilde{\boldsymbol{n}}]$$

- Fuzzy sphere method suggests non-unitary CFT & fixed point annihilation + weakly first order transition [Zhou, et al. PRX 2024]; [Nahum PRB 2020]

1D: XXZ Chain

With spin-1/2 Hamilonian:

$$\mathcal{H}_{\mathrm{DQCP}} = \sum_i J_{ij}\left(S_i^x S_j^x + S_i^y S_j^y + \Delta S_i^z S_j^z\right)$$

[Jiang, Motrunich PRB 2019],
[Roberts, et al. PRB 2019]
[Mudry, et al. PRB 2019]

Small $\frac{J_2}{J_1} \approx 0$:

Tune J_2

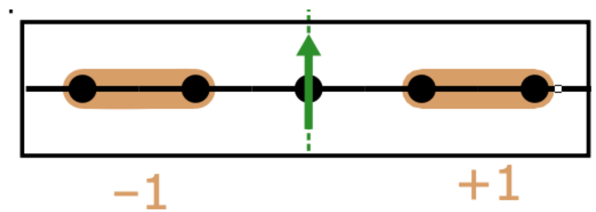
Large $\frac{J_2}{J_1} \approx 1$:

Antiferromagnet (Néel)

Dimerized phase (VBS)

Why might we expect DQCP in this 1D model?

- Known to have have a description in terms of a SO(4) NLSM and a $SU(2)_1$ WZW critical theory
  [Affleck & Haldane PRB 1987]; [Tanka & Hu PRL 2005]; [Senthil & Fisher PRB 2006]
- Also shows intertwinement of VBS and Néel orders
  [Sachdev 2011]; [Mudry et al. PRB 2019]



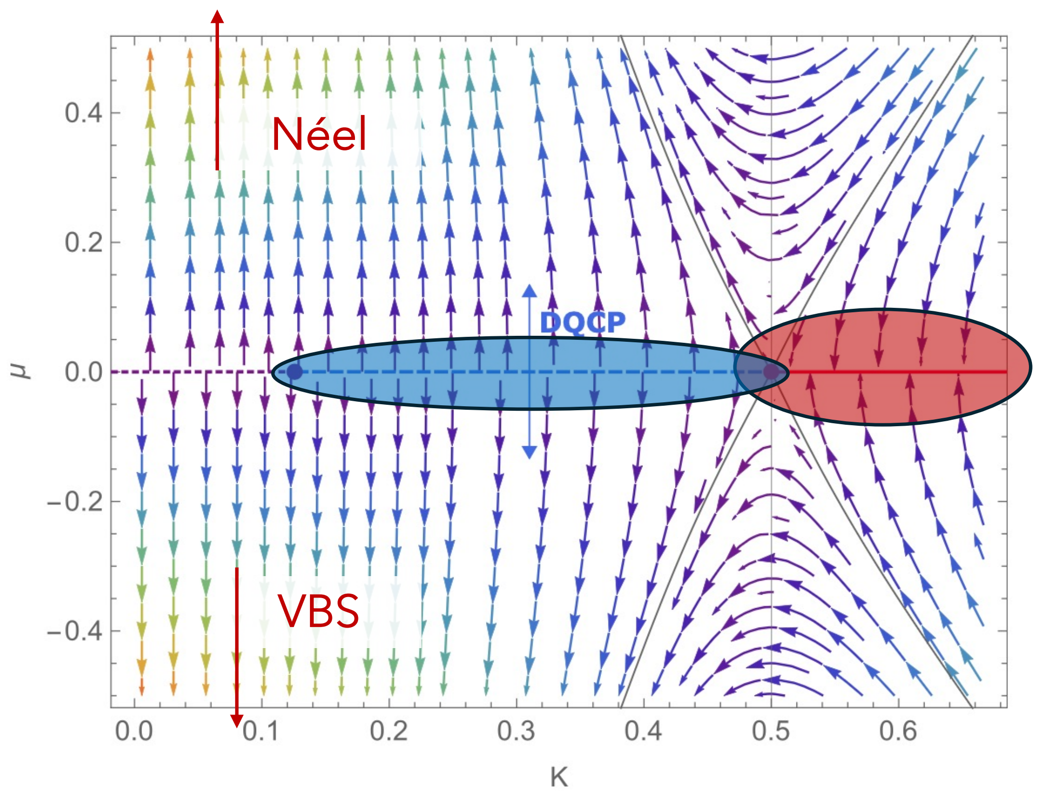
1D: XXZ Chain

Bosonise to get Sine-Gordon action

$$S_{\mathrm{SG}} = \int \mathrm{d}x\,\mathrm{d}\tau \left[\frac{1}{2\pi K} \left((\partial_x \phi)^2 + (\partial_\tau \phi)^2 \right) - \mu \cos(4\phi) \right]$$

1/8<K<1/2
DQCP

$\Delta > 1$

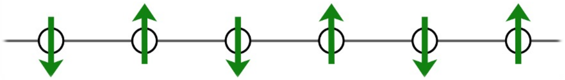


K>1/2
Luttinger liquid

$\Delta \leq 1$

1D Deconfined Quantum Criticality

Small $\frac{J_2}{J_1} \approx 0$:

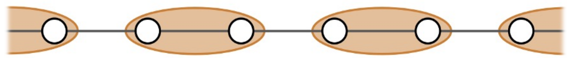


Antiferromagnet (Néel)

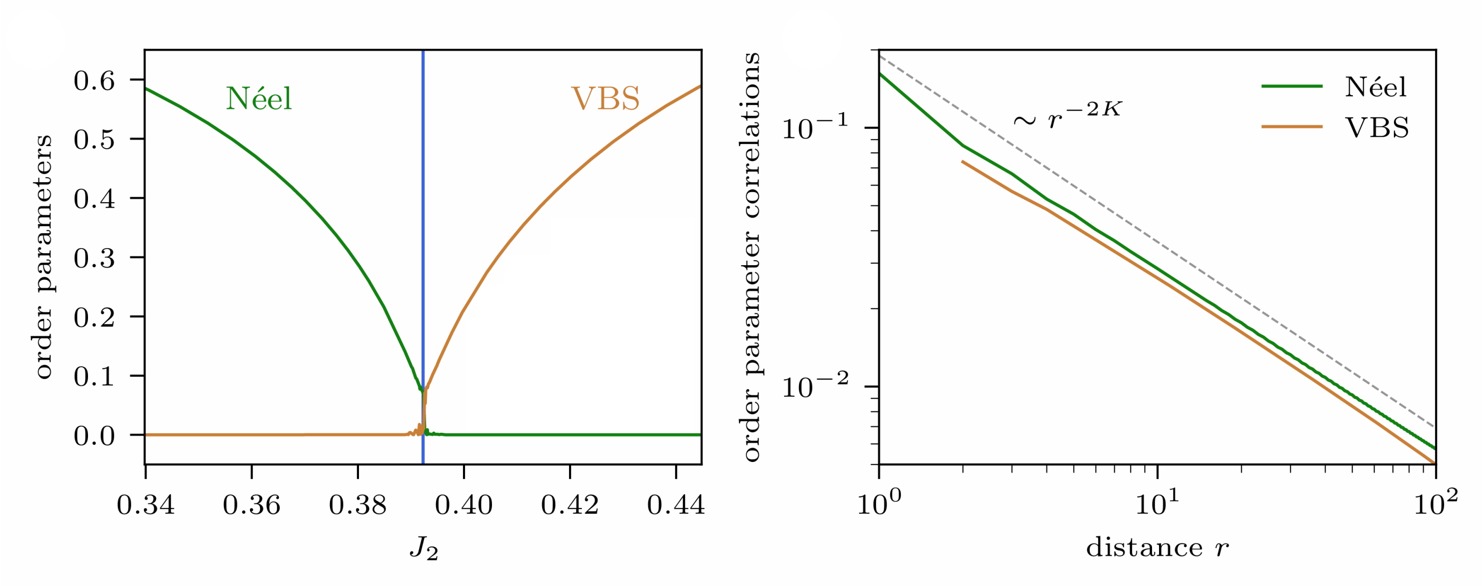
Tune J_2

$\Delta = 4$

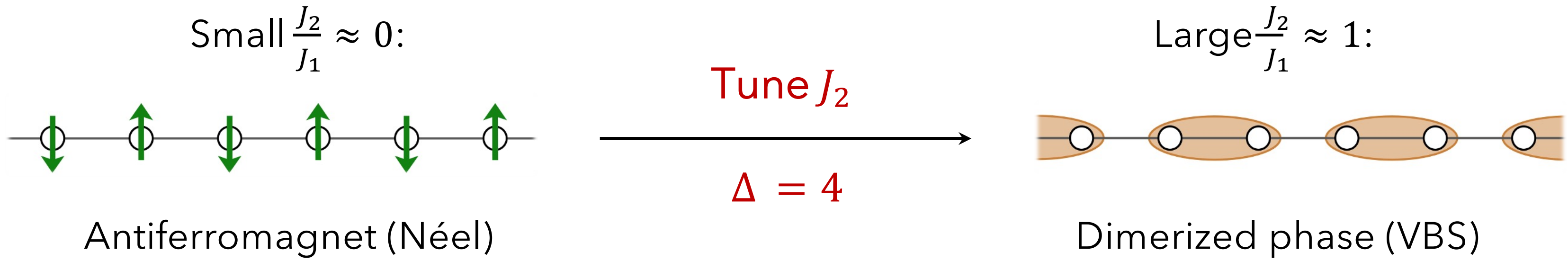
Large $\frac{J_2}{J_1} \approx 1$:



Dimerized phase (VBS)



1D Deconfined Quantum Criticality



- In disordered spin chains, coupling to the lattice can induce a spin-Peierls instability towards a dimerized state [Cross, Fisher PRB 1979]

Is deconfined criticality stable in the presence of lattice distortions?

- The Luttinger liquid has the same instability for all K < 1, so the DQCP is also unstable

[Hofmeier*, **JW***, Seifert, Knolle PRB 2024]

Instability to Static Distortions

$$H_s = J_1 \sum_j (1 + (-1)^j \delta)(\vec{S}_j \vec{S}_{j+1} + \Delta S_j^z S_{j+1}^z) + J_2 \sum_j (\vec{S}_j \vec{S}_{j+2} + \Delta S_j^z S_{j+2}^z) + L\kappa\delta^2$$

New relevant coupling [Hofmeier, **JW**, Seifert, Knolle PRB 2024]

- Breaking translational symmetry allows coupling λ for any $\delta > 0$

$$S_u = S_{\mathrm{SG}} + \int \mathrm{d}x\, \mathrm{d}\tau\, \lambda \sin(2\phi)\,,$$

- This destroys the line of fixed points corresponding to the DQCP regime

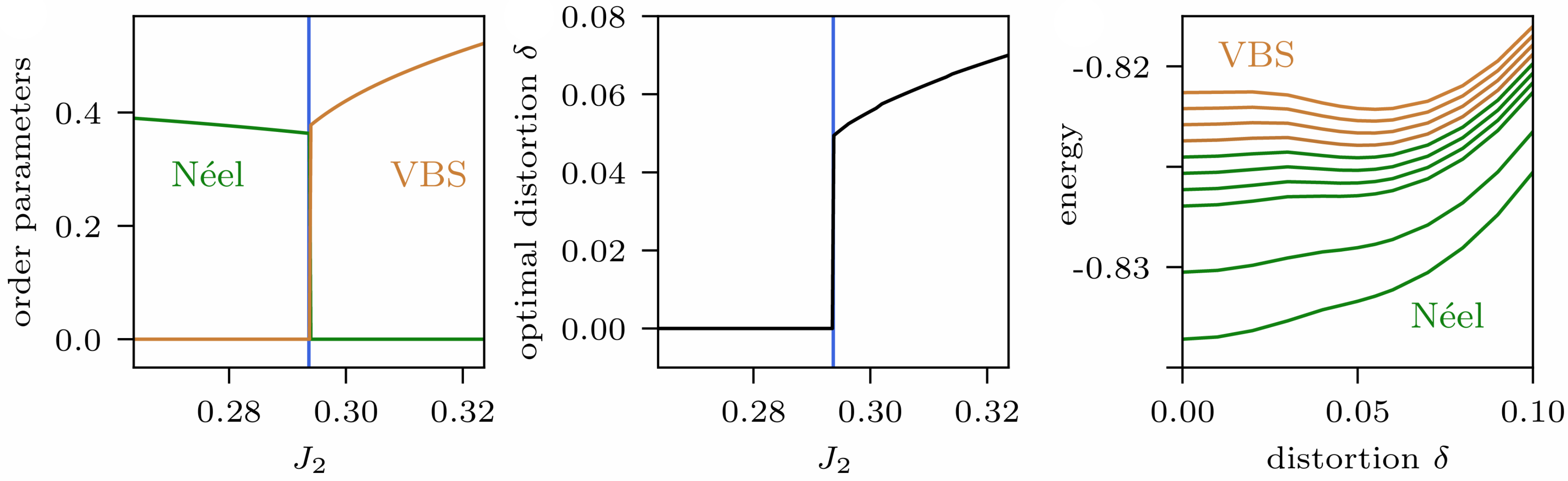
Energetically favourable to generate a distortion at the critical point

Instability to Static Distortions

Numerical calculation

- Simulate XXZ Hamiltonian for various distortions and minimise the energy

$$H_s = J_1 \sum_j (1 + (-1)^j \delta)(\vec{S}_j \vec{S}_{j+1} + \Delta S_j^z S_{j+1}^z) + J_2 \sum_j (\vec{S}_j \vec{S}_{j+2} + \Delta S_j^z S_{j+2}^z) + L\kappa\delta^2$$



Dynamical Phonons

Dynamical lattice vibrations with an energy scale ω_0

- High-energy phonons should stabilise the deconfined criticality
- Spin-lattice Hamiltonian given by

$$H_s = J_1 \sum_j (1 + (-1)^j \delta)(\vec{S}_j \vec{S}_{j+1} + \Delta S_j^z S_{j+1}^z) + J_2 \sum_j (\vec{S}_j \vec{S}_{j+2} + \Delta S_j^z S_{j+2}^z) + L\kappa\delta^2$$

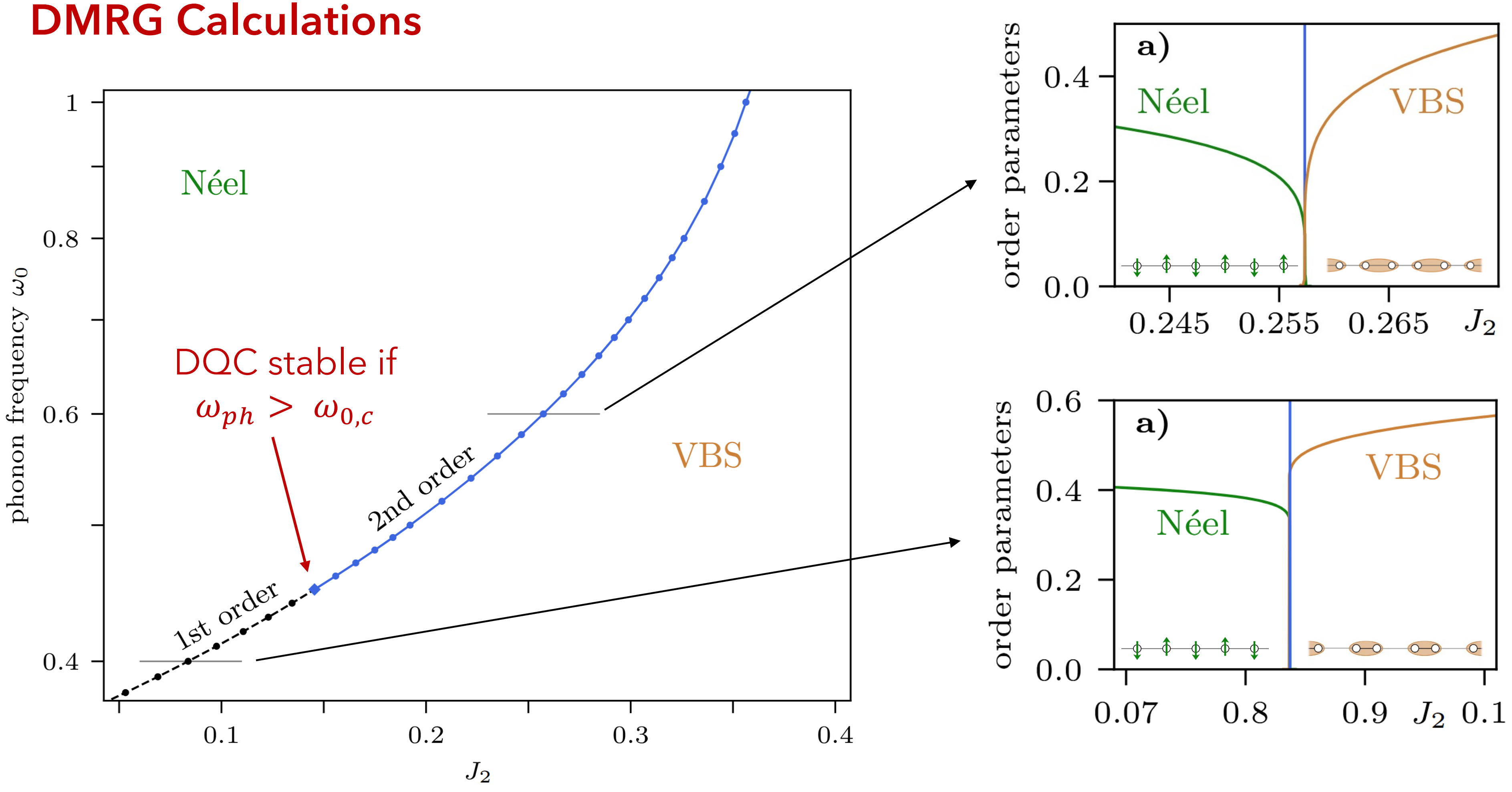
$$H_{sp-ph} = g \sum_j \underbrace{(a_j^\dagger + a_j - a_{j+1}^\dagger - a_{j+1})}_{displacement\ \ u(j,t) - u(j+1)} \vec{S}_j \vec{S}_{j+1}$$

$$H_{ph} = \omega_0 \sum_j a_j^\dagger a_j$$

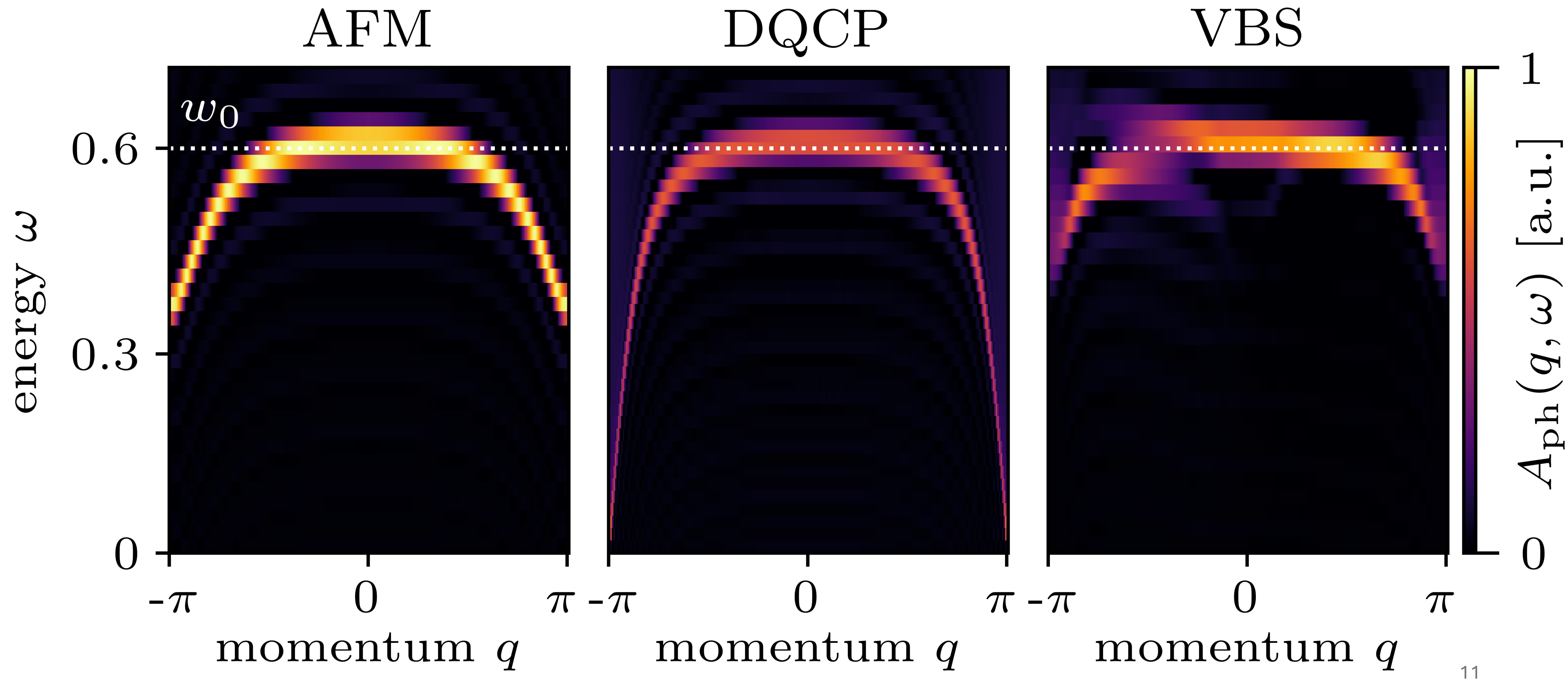
Stable DQCP transition above critical frequency $\omega_{0,c}$

Dynamical Phonons: Phase Diagram

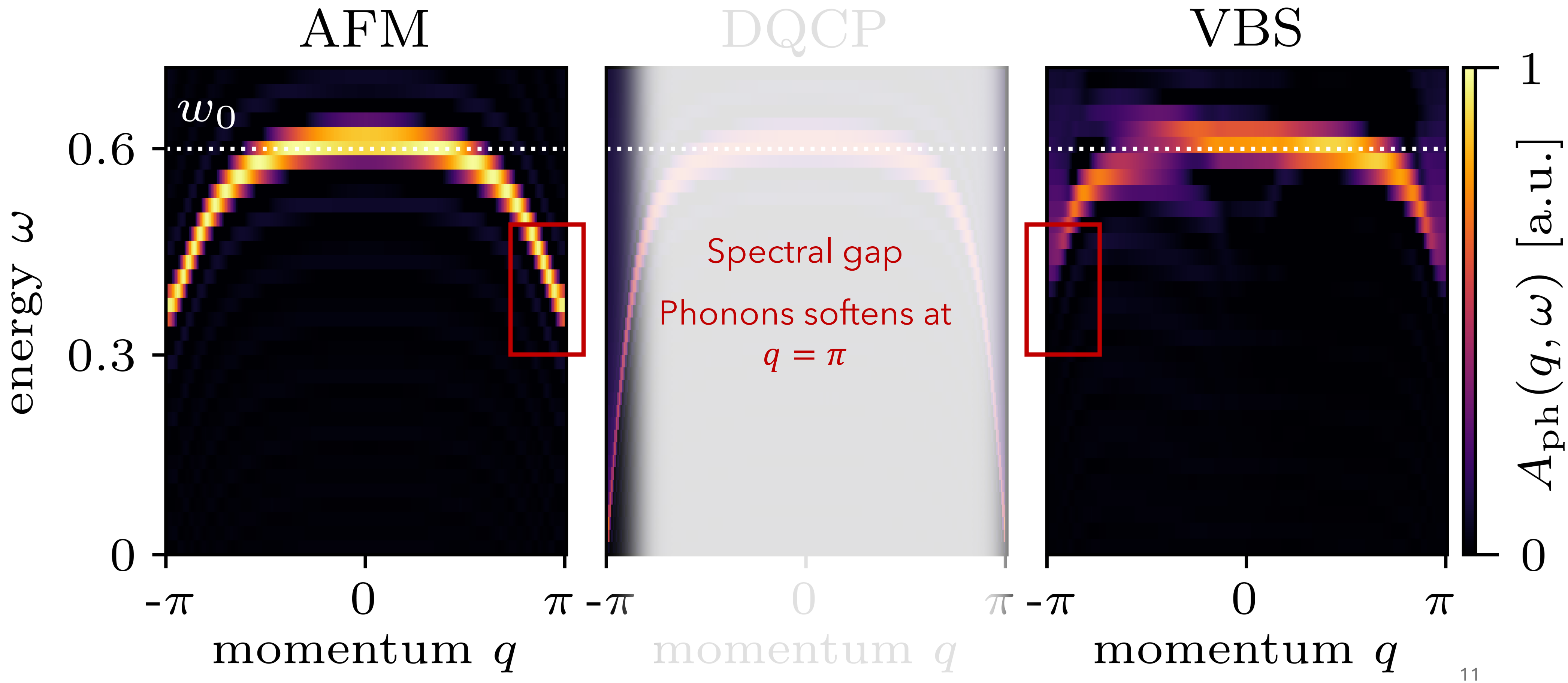
DMRG Calculations



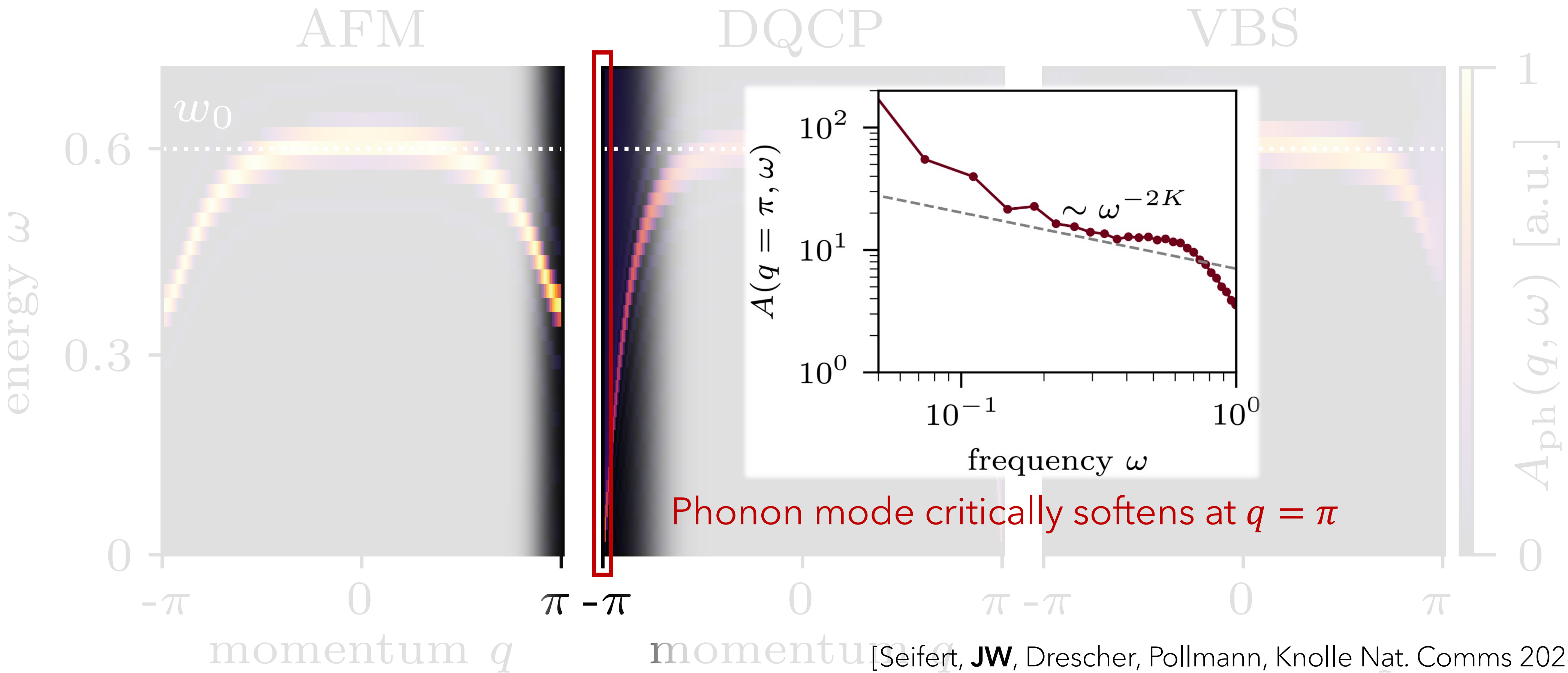
Phonon Spectral Function at DQC



Phonon Spectral Function at DQC



Phonon Spectral Function at DQC



[Seifert, **JW**, Drescher, Pollmann, Knolle Nat. Comms 2024]
[Lee, You, Sachdev, Vishwanath PRX 2019]

2D Deconfined Quantum Criticality

Expect the same instability in 2D

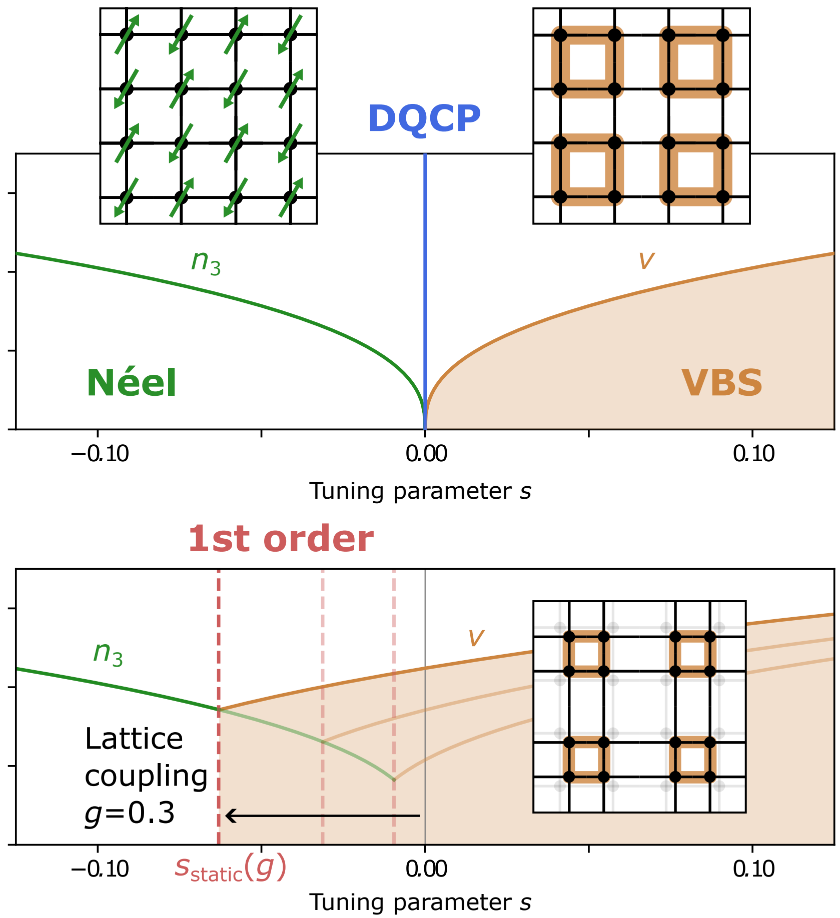
- Monopoles in the NCCP[1] U(1) gauge theory will couple to the lattice the same way as the U(1) Dirac spin liquid

  [Seifert*, **JW***, Drescher, Pollmann, Knolle Nat. Comms 2024]
  [Ferrari*, **JW***, Seifert, Valentí, Knolle arXiv:2410.26376]

- It is a 2D sibling of the XXZ spin chain's DQCP; they have the same type of WZW critical point

- Intertwined SO(N) orders are generically fragile to phonons since lattice only couples to VBS sector

  [Hofmeier*, **JW***, Seifert, Knolle PRB 2024]



2D Deconfined Quantum Criticality

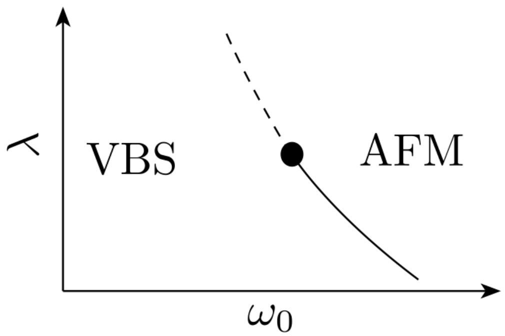
Dynamical phonons

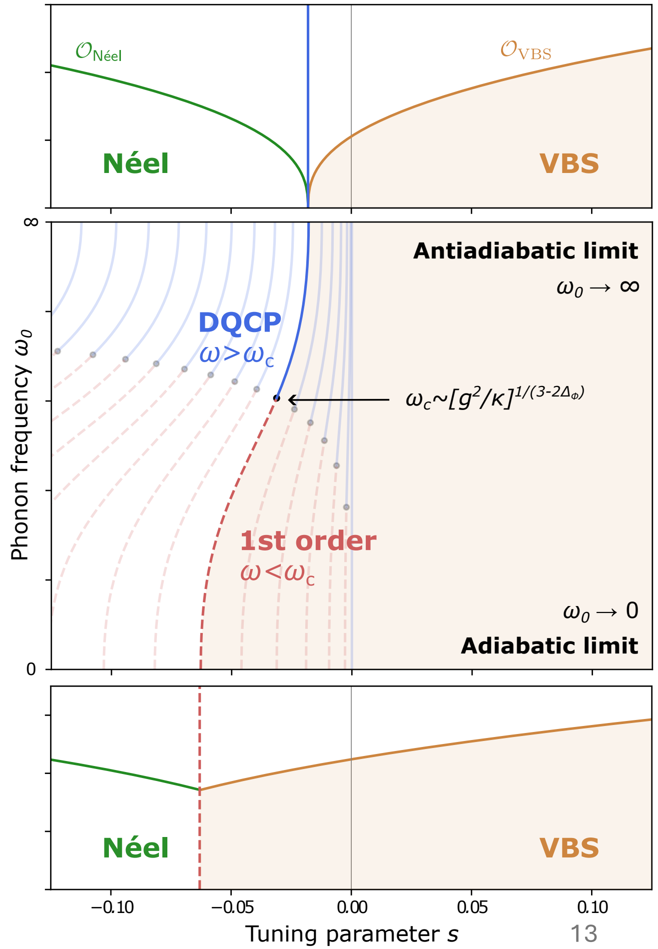
- DQCP also potentially ‘stable’ for large phonon frequencies
- Critical coupling scales as $\omega_{0,c} \sim (g^2/\kappa)^{1/(3-2\Delta_\Phi)}$

2D SSH model

[Götz, Hohenadler, Assaad PRB 2025]
[Götz, Assaad, Costa arXiv:2412.17215]

- DQCP tuned by varying phonon frequency
- 2nd order transition becomes 1st order when transition pushed below $\boldsymbol{\omega}_{0,c} \simeq 2.6\, t$

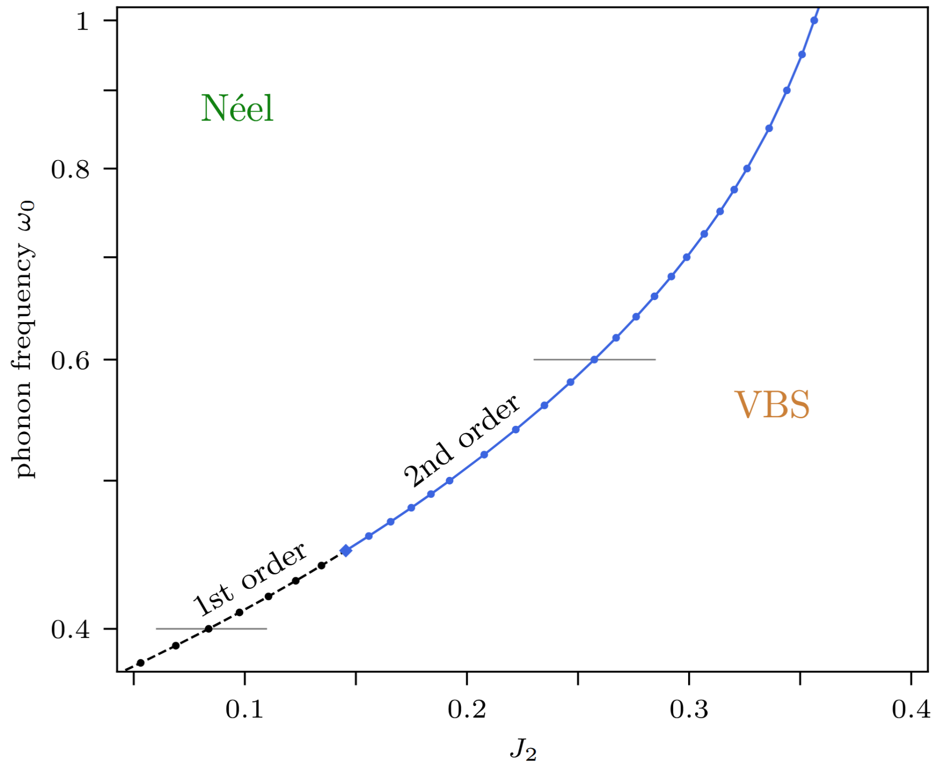




Stability of deconfined quantum criticality

Spin-Peierls a generic instability of 'algebraic' liquid states

- U(1) Dirac spin liquid & Luttinger liquid
- Deconfined quantum critical points in 1D & 2D
- Numerics confirm it's a strong instability in test systems



Spin-Peierls instability of deconfined quantum critical points
Hofmeier*, **JW***, Seifert, Knolle
Phys. Rev. B **110**, 125130 (2024)

& ongoing numerical work with
Anton Romen, Michael Knap

Thank you for your attention!