Method for the evaluation of the effects of film nonlinearities on the holographic image

István Bányász

Institute of Isotopes of the Hungarian Academy of Sciences, P.O. Box 77, Budapest H-1525, Hungary

Received December 3, 1992

An exact method for the evaluation of the images reconstructed from holograms recorded in nonlinear media is reported for what is to the author's knowledge the first time. The method is based on the use of the nonlinear holographic characteristics of the recording material without any limitation on the range of bias exposures and visibilities. These characteristics have been obtained by fitting analytical functions to experimental data describing practical recording materials. Numerical calculations have been carried out for high-numerical-aperture thin amplitude holograms of a five-element Ronchi ruling. The contrast of the reconstructed image as a function of the bias exposure and beam ratio is computed.

Although many valuable papers have been published on the theory of holography and especially on the effects of nonlinear recording since the late 1960's,[1–12] the results of the majority of them are not applicable to the solution of the real problems of practical holography. The authors of these papers applied binomial and polynomial expansions of various forms to the nonlinear transmittance–exposure function of the recording material such as an expansion on a set of Chebyshev polynomials[7] or expansion into Fourier series.[10] Owing to the approximations applied, none of the above studies could give accurate methods with universal applicability.

The aim of this Letter is to develop an exact method for the evaluation of the direct effects of film nonlinearity on the reconstructed holographic image. The philosophy of the following calculations is similar to those presented in two earlier papers by the author.[13,14]

The amplitude transmission of the photographic material in the case of amplitude (absorption) holograms is given by the following relationship:

$$t(\xi) = t[E(\xi)], \tag{1}$$

where ξ is the spatial coordinate along the one-dimensional hologram and E is the exposure. If we want to obtain exact results that are valid for strongly modulated object waves (i.e., in a broad range of exposures) we cannot rely on the expansion of the $t(E)$ function since it does not apply to high exposures. The experimental t–E curves of the standard holographic recording materials are readily available in holography textbooks.[9–12] These curves can easily be incorporated into numerical calculations either in the form of a look-up table or by fitting an empirical formula to them. However, the numerical quadrature of the diffraction integral involves a high-density sampling of the complex amplitude transmittance of the hologram. The local spatial frequency of the space-dependent total exposure and hence that of the transmission in off-axis holography is mainly determined by the carrier spatial frequency. Its maximum value in the hologram plane is typically in the range of 1000–3000 lines/mm. According to the sampling theorem, a sampling frequency of at least double this maximum frequency is required for the correct representation of the t–E function. In the case of the holograms studied here this means a minimum of approximately 160,000 sampling points per hologram. Consequently a large storage capacity and computing time are required, even with powerful computers. Calculations based on this direct method are in progress.

Instead of applying the direct approach, the author has developed the following method. Let us denote the first-order diffraction efficiency of a holographic grating by η and the square root of it by σ. For elementary holograms (produced by plane waves) the first-order diffraction efficiency as a function of the bias exposure and the fringe visibility can be measured easily. The amplitude diffraction efficiency versus the bias exposure and fringe visibility function, $\sigma(E_0, V)$, gives a complete description of the nonlinearity of the holographic recording material

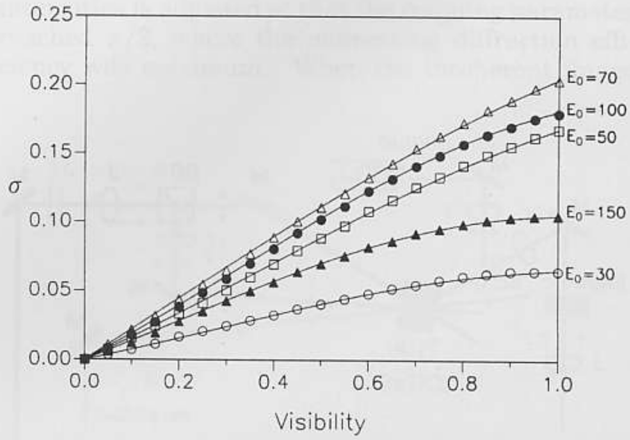

Fig. 1. $\sigma(V)$ function of Kodak 649f. Symbols: experimental data from Ref. 15. Solid curves: values fitted by Eq. (3) with the parameters shown in Table 1. The parameter E_0 is in units of μJ/cm^2.

0146-9592/93/080658-03$5.00/0

Table 1. Parameters of the $\sigma(E_0,V)$ Function of Kodak 649f

Indices (xx)	cf_{xx}	cV_{xx}	cw_{xx}
01	0.56	1	0.9
11	41.7	35	61
12	11	18.3	8.5
13	0	0.36	1.22
21	104	81.4	77
22	62.5	16	11.4
23	0	0.47	0.58
31	24	13	15.8
32	1.5	25	22.5
33	0	0	0

(see, e.g., Ref. 9, Chap. 10, paragraphs 6–8), hence it can be used for the evaluation of the nonlinear effects instead of the $t(E)$ curve.

Based on previous experience in numerical modeling, the author succeeded in fitting the following empirical function to a representative $\sigma(E_0, V)$ curve of a standard silver halide recording material, Kodak 649f (Ref. 15):

$$\sigma(E_0, V) = f(E_0)[1 - \exp(-V)]\exp\left\{-\frac{[V - V_0(E_0)]^2}{w^2(E_0)}\right\}, \tag{2}$$

where E_0 is the bias exposure, V is the visibility of the interference fringes, and $f(E_0)$, $V_0(E_0)$, and $w(E_0)$ are parameter functions of the following form:

$$\mathrm{Par}(E_0) = ci_{01}\left[\frac{1}{\exp\left(\dfrac{ci_{11} - E_0}{ci_{12}}\right) + 1} + ci_{13}\right] \times \left[\frac{1}{\exp\left(\dfrac{E_0 - ci_{21}}{ci_{22}}\right) + 1} + ci_{23}\right] \times \left[\frac{1}{\exp\left(\dfrac{ci_{31} - E_0}{ci_{32}}\right) + 1} + ci_{33}\right] \tag{3}$$

where Par stands for f, V_0, and w, and ci_{xx} represents the three sets of constants ($i = f, V, W$). This $\sigma(E_0, V)$ function, fitted to the experimental characteristics of the Kodak 649f photomaterial,[15] is presented in Fig. 1. The parameters of the actual function are listed in Table 1. This function describes the nonlinear behavior of all the silver halide holographic recording materials, and the evaluation of its parameters for further particular materials is in progress. The use of an empirical analytical function fitted to experimental data offers two advantages over that of look-up tables made from the same data. First, it is easy to handle and much faster in a computer program. Second, one can easily create hypothetical characteristics by modifying one or more parameters of the analytical function and establish relationships between these parameters and the quality of the holographic image.

Let us denote the complex amplitude transmittance of the one-dimensional amplitude object by $\mathbf{s}(u)$, where u is the local object coordinate. In the case of a perpendicular plane-wave illumination the complex amplitude distribution of the object wave in the hologram line can be described by the Fresnel–Kirchhoff integral[13]

$$\mathbf{S}(\xi) = \int_{u_1}^{u_2} \mathbf{s}(u)\frac{\cos\theta}{r_1}\exp(ikr_1)\mathrm{d}u, \tag{4}$$

where $r_1(u, \xi)$ is the separation of the actual source and observation points and $\theta(u, \xi)$ is the inclination angle.

The complex amplitude of an off-axis plane reference wave at the hologram line is

$$\mathbf{P}(\xi) = P\exp(-ik\xi\sin\alpha_r), \tag{5}$$

where P is a constant and α_r is the angle of incidence of the reference wave. If the intensity distribution of the object wave in the hologram plane is a slowly varying function compared with the carrier fringe spacing, a sufficiently small surrounding of each hologram point can be regarded as a plane-wave hologram recorded at a well-defined bias exposure and visibility. The local bias exposure at point ξ is

$$E_0(\xi) = |\mathbf{S}(\xi)|^2 + |\mathbf{P}(\xi)|^2. \tag{6}$$

The local fringe visibility at point ξ is

$$V(\xi) = 2\frac{\sqrt{\dfrac{|\mathbf{P}(\xi)|^2}{|\mathbf{S}(\xi)|^2}}}{1 + \dfrac{|\mathbf{P}(\xi)|^2}{|\mathbf{S}(\xi)|^2}}. \tag{7}$$

We can evaluate the actual $\sigma[E_0(\xi), V(\xi)]$ function by substituting $E_0(\xi)$ and $V(\xi)$ into Eqs. (2) and (3).

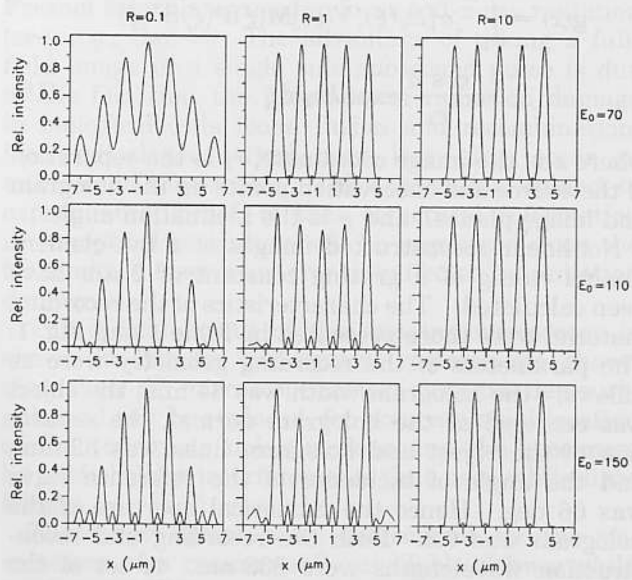

Fig. 2. Calculated intensity distributions of images reconstructed from nonlinear holograms. The beam ratios R are indicated above the corresponding columns, while the bias exposures E_0 (μJ/cm^2) are to the right of the corresponding rows.

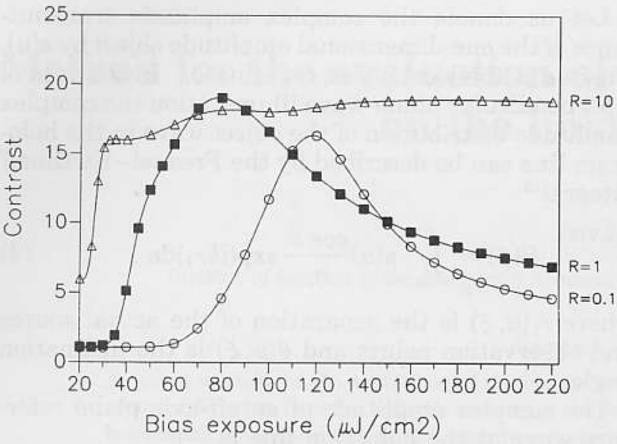

Fig. 3. Calculated contrast versus bias exposure function of the holographic image at three beam ratios. Solid curves: spline functions fitted to the calculated points.

The complex amplitude of the diffraction-limited first-order real image at the hologram line is

$$\mathbf{G}(\xi) = \mathbf{M}(\xi)\mathbf{P}(\xi)\mathbf{S}^*(\xi), \tag{8}$$

where the asterisk denotes complex conjugation and for perfect reconstruction the complex amplitude of the reconstruction wave is the complex conjugate of the reference wave:

$$\mathbf{M}(\xi) = \mathbf{P}(\xi)^*. \tag{9}$$

In case the different diffraction orders do not overlap in the image position, the complex amplitude of the reconstructed nonlinear first-order real image is readily obtained by multiplying the diffraction-limited complex amplitude [Eq. (8)] by the $\sigma[E_0(\xi), V(\xi)]$ function and performing the second Fresnel–Kirchhoff integration:

$$\mathbf{g}(x) = \int_{\xi_1}^{\xi_2} \sigma[E_0(\xi), V(\xi)]\mathbf{M}(\xi)\mathbf{P}(\xi)\mathbf{S}^*(\xi) \times \frac{\cos\rho}{r_2}\exp(ikr_2)\mathrm{d}\xi\,, \tag{10}$$

where x is the image coordinate, r_2 is the separation of the source and observation points (in the hologram and image planes), and ρ is the inclination angle.

Nonlinear reconstructed images of a five-element Ronchi ruling of a grating constant of 2 μm have been calculated. The characteristics of the recording material were those presented in Table 1 and Fig. 1. The parameters of the recording geometry were as follows: the hologram width was 84 mm, the object was centered at the hologram normal, the separation of the object and hologram lines was 32 mm, and the angle of incidence of the reference wave was 66 deg. Hence the numerical aperture of the hologram was 0.8. Both the recording and reconstruction wavelengths were 633 nm. A set of the intensity distributions of the reconstructed images is shown in Fig. 2, where E_0 is the maximum bias exposure and R is the minimum reference-to-object beam ratio through the hologram.

In order to get a more quantitative picture of the nonlinear effects, the contrast of the reconstructed image (defined as the integral of the intensity distribution on the transparent lines divided by that on the opaque lines, including the two at both ends) has been evaluated for a range of bias exposures and beam ratios. The results are presented in Fig. 3. As one can expect, the higher the beam ratio the broader the range of the bias exposures that result in images of acceptable contrast. Although the maximum achievable contrasts do not differ significantly for the three values of the beam ratio, a glance at the first two images of the middle row of Fig. 2 reveals that, in spite of having the same value of contrast, they are far from being of the same quality. This implies that, for establishing more accurate relationships, other characteristics of the image, e.g., the fluctuation of the peak intensities, are to be calculated, too.

It has been proved that this new method, based on the use of the experimental $\sigma(E_0, V)$ function of the recording material, is applicable in predicting the quality of the holographic image and establishing practical rules for optimizing the holographic recording of different sets of objects.

References

1. A. A. Friesem and J. S. Zelenka, Appl. Opt. **6**, 1755 (1967).
2. K. Biedermann, Optik **28**, 160 (1968).
3. J. W. Goodman and G. R. Knight, J. Opt. Soc. Am. **58**, 1276 (1968).
4. G. R. Knight, Appl. Opt. **7**, 205 (1968).
5. A. A. Friesem, A. Kozma, and F. G. Adams, Appl. Opt. **6**, 851 (1967).
6. J. C. Wyant and M. Parker Givens, J. Opt. Soc. Am. **59**, 1650 (1969).
7. F. J. Tischer, Appl. Opt. **9**, 1369 (1970).
8. Yu. N. Denisyuk, G. V. Semenov, and N. A. Savostianenko, Opt. Spectrosc. **29**, 994 (1970) (in Russian).
9. R. Collier, K. Burckhardt, and L. Lin, *Optical Holography* (Academic, New York, 1971), Chaps. 10 and 12.
10. F. T. S. Yu, *Introduction to Diffraction, Information Processing, and Holography* (MIT Press, Cambridge, Mass., 1973), Chaps. 11 and 12.
11. H. M. Smith, ed., *Holographic Recording Materials* (Springer-Verlag, Berlin, 1977), pp. 51–53.
12. P. Hariharan, *Optical Holography* (Cambridge U. Press, Cambridge, 1984), pp. 81–83.
13. I. Bányász, G. Kiss, and P. Varga, Appl. Opt. **27**, 1293 (1988).
14. I. Bányász, Proc. Soc. Photo-Opt. Instrum. Eng. **1574**, 282 (1991).
15. R. Collier, K. Burckhardt, and L. Lin, *Optical Holography* (Academic, New York, 1971), Chap. 10, Paragraph 8, Fig. 10.9.