

**Silo outflow of soft frictionless spheres**Ahmed Ashour,<sup>1,2</sup> Torsten Trittel,<sup>1</sup> Tamás Börzsönyi,<sup>3</sup> and Ralf Stannarius<sup>1</sup><sup>1</sup>*Institute for Experimental Physics, Otto von Guericke University, Magdeburg, Germany*<sup>2</sup>*Faculty of Engineering and Technology, Future University, End of 90th Street, New Cairo, Egypt*<sup>3</sup>*Institute for Solid State Physics and Optics, Wigner Research Center for Physics,**Hungarian Academy of Sciences, Budapest, Hungary*

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Outflow of granular materials from silos is a remarkably complex physical phenomenon that has been extensively studied with simple objects like monodisperse hard disks in two dimensions (2D) and hard spheres in 2D and 3D. For those materials, empirical equations were found that describe the discharge characteristics. Softness adds qualitatively new features to the dynamics and to the character of the flow. We report a study of the outflow of soft, practically frictionless hydrogel spheres from a quasi-2D bin. Prominent features are intermittent clogs, peculiar flow fields in the container, and a pronounced dependence of the flow rate and clogging statistics on the container fill height. The latter is a consequence of the ineffectiveness of Janssen's law: the pressure at the bottom of a bin containing hydrogel spheres grows linearly with the fill height.

DOI: [10.1103/PhysRevFluids.2.123302](https://doi.org/10.1103/PhysRevFluids.2.123302)**I. INTRODUCTION**

Granular materials have been handled by humans since ancient times in agriculture, mining, building and other activities, and intuitively we are all familiar with some of their unusual properties in typical everyday-life situations: when compacting the granular content of a bag by shaking, pouring salt or sugar from a muffineer, or dealing with clogging of the outflow from storage containers. Nevertheless, the comprehensive physical description is still incomplete today [1–4]. The flow of grains, even in the simplest form of hard monodisperse spheres, has retained some mysteries.

Hoppers are storages that exploit the ability of granular matter to flow liquid-like through sufficiently large outlets. Frequent problems with such storage devices involve, in particular, the formation of clogs that interrupt flow, and the redistribution of weight to the side walls by force chains, which may cause structural collapses. For hard spheres and, in two-dimensions (2D), hard disks, previous research activities brought important insights and quantitative laws that describe the discharge dynamics and static conditions reasonably well. Through sufficiently large orifices, such grains flow rather continuously. Quantitative predictions for flow rates were proposed (e.g., [5–8] and references therein). When the outlet diameter is smaller than about five particle diameters [9–13], clogs form after some time at the orifice and block further outflow, mostly unwanted. Spontaneous arch formation [14,15], the preceding kinetics [16], as well as the inherent force distributions [17,18] have been analyzed in the literature. In order to study the geometry and statics of such clogs, often 2D container geometries are chosen.

Identical hard spheres are idealized special cases: In practice one often deals with polydispersity, nonspherical grains, and/or elasticity of the material. Even small modifications of these features can have dramatic qualitative consequences [19]. However, studies of elastic particles, for example, are rather scarce. Experiments with dense air bubble arrangements in an “inverse” 2D Hele-Shaw silo model have been reported by Bertho *et al.* [20]. In this extreme case of incompressible but highly elastic constituents, outflow rates  $Q$  of bubbles with diameter  $D$  through an orifice with width  $W$  were found to obey a relation  $Q \propto g^{1.5}(W/D - k)^{0.5}$ , where  $g$  is the gravitational acceleration, and

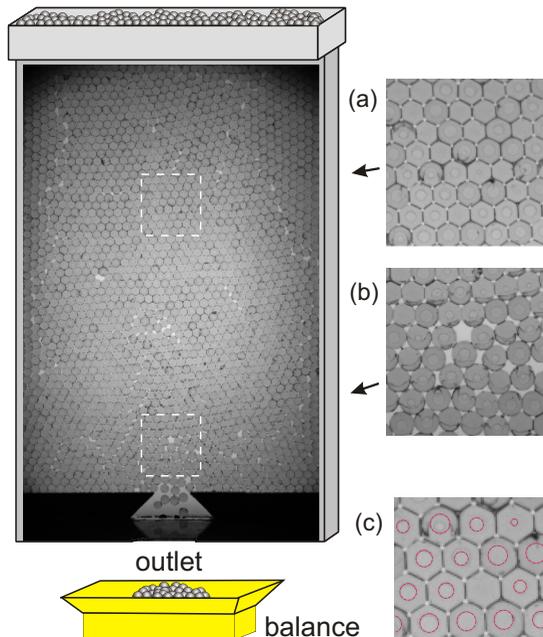


FIG. 1. Experimental setup: The small panels at the right (a),(b) show details of the hexagonal lattice formed by the soft spheres far from the container side walls in the upper half of the bin (a), and directly above the outlet (b); the positions are indicated by dashed squares. Panel (c) shows a detail of (a), where we have marked the contact areas of spheres with the front glass wall, as a measure of the force exerted on the beads by the front wall.

$k \approx 2/3$  is a constant empirically found in the experiment. The high deformability allowed passage of bubbles even when  $W < D$ ; no clogging was reported.

Recently, oily emulsions were studied in a 2D hopper geometry [21,22]. These ensembles of polydisperse oil droplets also have many similarities to granular material, but combined with shape elasticity. The authors reported considerable differences in the dynamic behavior; particularly, the critical ratio of orifice size and particle size is much smaller than for rigid grains, and nonpermanent, intermittent clogs were observed. Similar intermittent clogs are found with hard particles in a vibrated hopper [23] or in flocks of animals passing a gate [15].

Soft monodisperse hydrogel spheres [24,25] are incompressible, but deformable by moderate pressures so that they can adopt their shapes to local static conditions in the container. This changes the pressure profile and the discharge characteristic, as well as the flow field inside the container, and leads to a fill-height dependent outflow.

## II. EXPERIMENT

In the present study, we investigate hydrogel spheres of  $D \approx 9.2$  mm diameter, acquired at Happy Store, Nanjing in dry state and swelled with distilled water for at least 24 hours. Between experiments, the spheres were kept in distilled water. The spheres have only a very small polydispersity; their weights differed only within  $\pm 11\%$ , which corresponds to mean radius variations of about  $\pm 4\%$  (about 0.2 mm). The material differs in two aspects from the previously studied hard spheres: its softness and its small friction coefficient. For similar hydrogel spheres, a friction coefficient of  $\mu = 0.03$  has been estimated [26]. Dijkstra *et al.* [25] determined the Young modulus of hydrogel spheres to be  $\approx 20$  kPa. By measurement of Hertzian contact diameters, we have determined an

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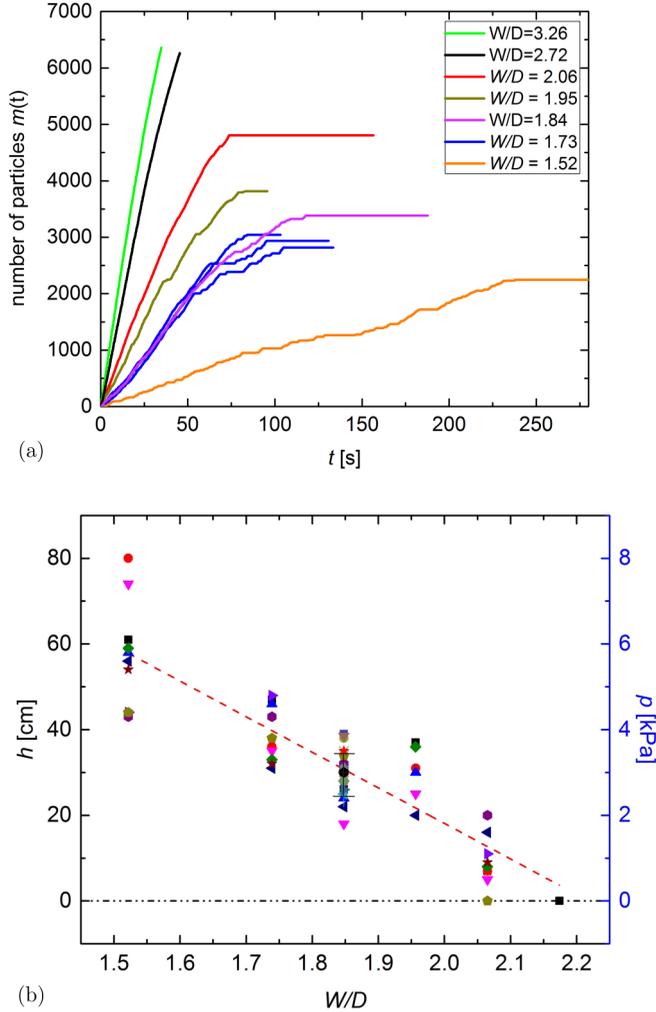


FIG. 2. (a) Outflow through different orifice sizes. When  $W > 2.2D$ , no clogging is observed. With decreasing orifice size, the discharge rate decreases, the fluctuations of the outflow velocity increase, and intermittent clogs occur. At a certain fill height, the system clogs permanently [horizontal lines in the  $m(t)$  discharge characteristics]. (b) Remaining fill level  $h$  in the permanently clogged state for different outlet sizes  $W/D$ . A satisfactory linear fit yields  $h_c = (2.2 - W/D) \times 95$  cm, or, a critical pressure  $p_c = (2.2 - W/D) \times 6.6$  kPa. The bar at  $W = 1.83D$  indicates the standard deviation of the measurements. For the pressure scale at the right-hand side, see Fig. 4.

elastic modulus of our spheres between 30 and 50 kPa, where the outer part is slightly softer than the core.

Our experiments were performed in a quasi-2D bin with lateral dimensions of about  $400 \text{ mm} \times 800 \text{ mm}$ , and a thickness of 10 mm, slightly larger than the mean sphere diameters (Fig. 1). The outlet width  $W$  and its position can be controlled arbitrarily. In the experiments presented here, the position of the outlet is in the center of the container bottom. The approximate content of the filled container is  $m \approx 4700$  spheres. The container is filled with the soft spheres from the top, then the outlet is opened and the weight of the material that has passed the outlet is continuously monitored. We analyze the discharge characteristics, the formation of clogs and their stability, the velocity profiles in the bin, and static features.

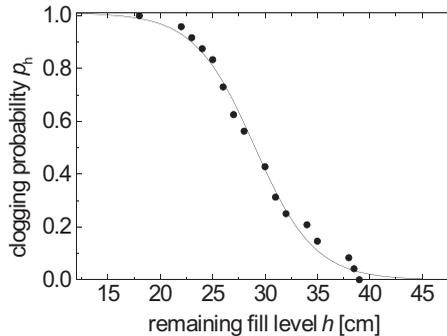


FIG. 3. Plot of the probability  $p_h$  that the system permanently clogs at a remaining fill height larger than  $h$  in a silo with an orifice size of  $W = 1.83D$  [see Fig. 2(b)]. The graph was constructed from 26 independent runs of the experiment. The mean height where the system clogged is 29 cm, the standard deviation of clogging heights is 5 cm. The solid hyperbolic tangent curve guides the eye.

### III. RESULTS AND DISCUSSION

When the outlet is closed, the monodisperse hydrogel spheres form an almost regular lattice in the container [cf. Figs. 1(a) and 1(b)]. Within this lattice, the hydrogel spheres are deformed towards a more or less hexagonal cross-sectional shape. Thus, the packing fraction becomes larger than the maximum  $\phi = \pi/\sqrt{27} \approx 0.605$  achievable for hexagonally packed hard spheres in a cell of thickness  $D$ . In fact,  $\phi$  is roughly 0.68. In Fig. 1(c), we have marked the contact areas of the beads with the front glass. The indentation can in principle be used to determine the forces acting on the individual spheres from the front wall. On average, these forces are within a narrow range; only very few individual spheres are found to be not in touch with the front wall. Thus, this sphere arrangement can be considered 2D as a good approximation.

The outflow of the hydrogel spheres has a unique orifice size dependence. When  $W/D$  is larger than  $\approx 2.2$ , the bin empties without clogging. This is in sharp contrast to hard spheres where clogging already sets in when  $W/D \lesssim 5$  [9, 11, 13, 27, 28]. Another striking difference to hard grains is that there are only very small fluctuations of the flow velocity field inside the container for those outlet sizes. For outlet widths smaller than  $2.2D$ , the soft-sphere flow undergoes a qualitative change. Typical discharge characteristics for  $W$  between  $2.1D$  and  $1.5D$  are shown in Fig. 2(a). With decreasing ratio  $W/D$ , the fluctuations of the outflow rate increase. Temporary clogs occur which can last for several seconds; they are followed by a permanent clog at a certain fill level  $h_c$ . Repeating the measurement with fixed outlet size  $W/D$ , the fill heights where permanent clogs occur were found to vary by about  $\pm 30\%$ . The average critical height  $h_c$  depends on the outlet size; a clear trend is seen in Fig. 2(b). In first approximation, this critical fill height  $h_c$  is a linear function of the orifice size.

For one fixed outlet size,  $W = 1.83D$ , we show exemplarily the cumulative clogging probability  $p_h$  as a function of the container fill height in Fig. 3. It was constructed from the size distribution of 26 avalanches, and the solid line is an empirical fit function: a hyperbolic tangent  $p(h) = 0.5 + 0.5 \tanh[(h - 29 \text{ cm})/5.7 \text{ cm}]$ . The mean value and standard deviation of the corresponding distribution are indicated in Fig. 2(b) by a vertical bar.

It seems obvious to assume that such behavior is related to hydrostatic pressure characteristics in the container. We confirmed this by measurements of the pressure  $p_0$  at the bottom for different fill heights  $h$ . A horizontal plate with dimensions  $6 \times 1 \text{ cm}^2$  was connected to the balance and inserted at the bottom as a weight sensor, and  $p_0$  was extracted from the measured weights (Fig. 4). There are statistical variations ( $\approx 10\%$ ) for each individual filling, but in general  $p_0$  was found to grow linearly with  $h$ . This appears to be a consequence of the low friction coefficient. No saturation was observed, in contrast to the pressure in the container filled with hard particles. As a demonstration,

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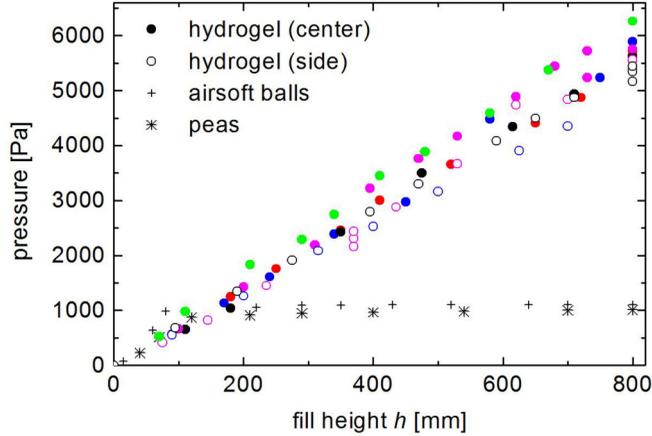


FIG. 4. Static pressure  $p_0$  at the container bottom as a function of the fill height in the center of the bottom (solid symbols) and near the side of the silo bottom (open symbols). Different colors symbolize independent runs of the experiment. The scattering of the data reflects natural variations of the individual experiments. The experimental error is smaller than the size of the symbols. The pressure at the bottom of the container filled with hydrogel spheres increases with fill height as  $p(h) \approx h \times 7 \text{ kN/m}^3$ . Pressure measurements for peas and airsoft balls (details see text) are shown for comparison.

we included in Fig. 4 measurements with peas (mean diameter 7.1 mm, density  $1.35 \text{ g/cm}^3$ , friction coefficient 0.35 determined from the angle of repose) and with airsoft balls (diameter 5.88 mm, density  $1.05 \text{ g/cm}^3$ ). There, the pressure saturates at fill heights of about 10 cm as a consequence of the Janssen effect [29,30].

Further increasing the hydrogel fill height leads to fracturing of the hydrogel beads before a noticeable deviation from the linear pressure increase would be observed. The pressure in the center of the silo bottom (solid symbols) is slightly higher than near the sides (open symbols). The density  $\rho_h$  of the hydrogel is 1.03 times that of water. The static pressure averaged over the bottom amounts to about  $\approx 68\%$  of the hydrostatic pressure in a water-filled vessel. The difference to a water column originates primarily from the packing fraction  $\phi$ . The weight of the material would

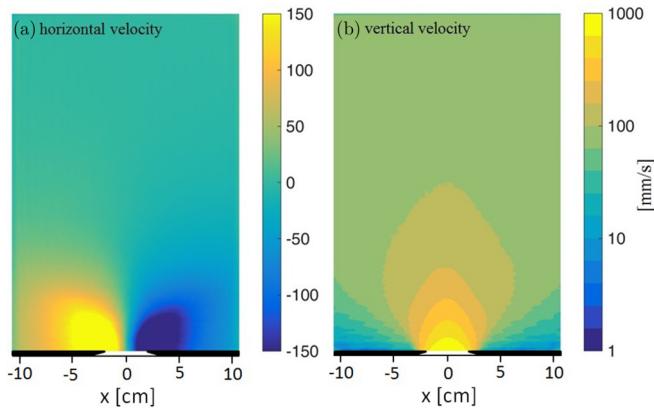


FIG. 5. Flow profiles (mm/s) at an orifice size  $W = 60 \text{ mm}$  ( $W/D = 6.5$ ) in the completely filled silo ( $h = 80 \text{ cm}$ ): (a) horizontal flow, the yellow (bright) region indicates flow to the right, the dark (blue) region to the left, green indicates zero horizontal flow; (b) vertical flow. The images show a  $23 \text{ cm} \times 34 \text{ cm}$  region of the container above the orifice.

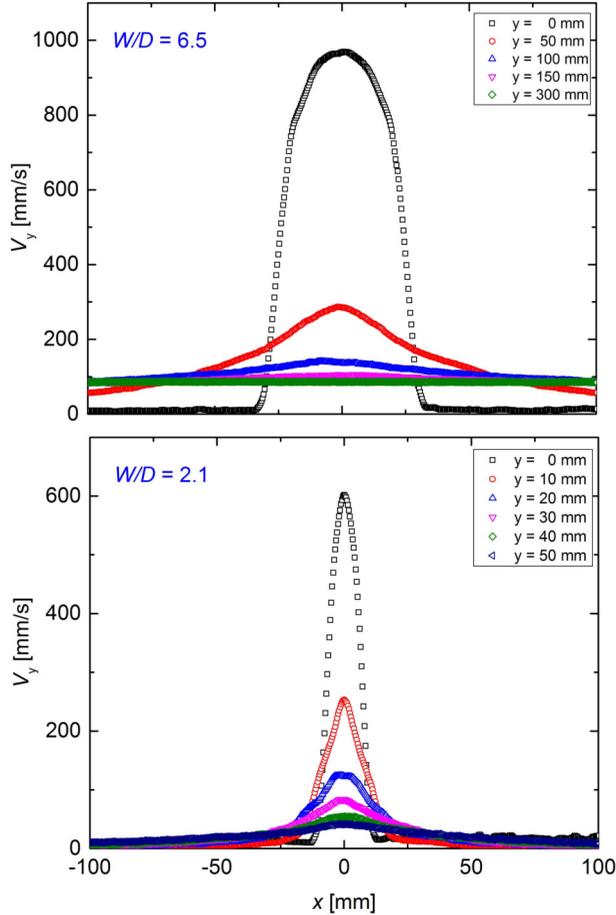


FIG. 6. Flow profiles at orifice sizes  $W = 60$  mm and  $20$  mm in the completely filled silo,  $h = 80$  cm. In levels  $y$  higher than  $\approx 2.5W$  above the orifice, the vertical flow profile becomes smooth. In the upper part, we find simple plug flow. (Only  $20$  cm of the full bin width is shown.)

provide a maximum pressure coefficient of  $p(h)/h = \rho_h g \phi = 6.9$  kN/m<sup>3</sup>. The measured value in the center of the silo bottom is slightly higher than this average, and the pressure at the lateral sides is somewhat lower, as seen in Fig. 4. From these considerations, it seems reasonable to conclude that there is practically no vertical stress relaxation by force chains towards the lateral walls. Note that when the bin discharges, the pressure  $p$  on the bottom plate drops by up to 50% compared to  $p_0$  at same momentary fill heights.

The flow field was measured by means of particle tracking, and a coarse-graining technique (comparable to Ref. [31]) was used to smoothen the flow profiles. All flow data presented below were obtained with the full silo, so that the fill height corresponds to slightly more than  $80$  cm. It turns out that these profiles are also essentially different from that of hard particles. Figure 5 shows the horizontal and vertical flow components. It is evident that the flow is not restricted to a funnel-shaped region above the outlet, but that the particles also move sideways along the bottom of the container [Fig. 5(a)]. In this respect, the hydrogel-filled container differs from a silo filled with hard grains where one usually finds stagnant zones at the bottom. The vertical flow component becomes rather uniform at levels approximately  $3W$  above the outlet [Fig. 5(b)]. Figure 6 shows the mean vertical flow velocity at different height levels determined from particle tracking.

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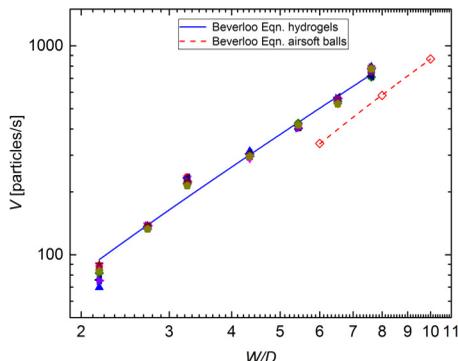


FIG. 7. Comparison of the flow rates vs  $W/D$  for soft hydrogel spheres and hard airsoft balls in the completely filled silo,  $h = 80$  cm. It is evident that both can be fitted with Eq. (1), but the soft particles show a substantially higher flow rate at given relative orifice sizes, and the coefficient  $k = 0.3$  for the soft spheres is extremely small.

The upper part of the silo performs simple plug flow. The hexagonal lattice may be the primary reason for this feature; a study of bidisperse or polydisperse hydrogel spheres is required to test this hypothesis.

Figure 7 shows the average discharge rate  $V$  in spheres per second for different  $W/D$  ratios. The solid line is a fit with Beverloo's equation,

$$V = \dot{N} = V_0(W/D - k)^{1.5} \quad (1)$$

with the parameters  $V_0 = 37$  particles/s = 15.5 g/s and  $k = 0.3$ . Compared to hard particles, where the parameter  $k$  is of the order of 1.5, this value is very small; this means that the flow velocity through narrow orifices is much more effective. For hard spheres, Thomas *et al.* [12] have shown that Beverloo's equation fits the continuous flow of hard particles at high opening sizes as well as the flow within avalanches through narrow openings, and that there is no discontinuity in the dynamic parameters. This was confirmed in our earlier experiments with nonspherical hard grains [19]. For the soft spheres, the validity of Beverloo's equation cannot be tested in the limit of small orifice sizes. The reason is that the flow rate fluctuates considerably when  $W < 2D$  (Fig. 2), and even nonpermanent clogs can be observed around  $W \approx 2D$  (the flow pauses for up to several seconds and restarts after a nonpermanent arch above the orifice has resolved). Moreover, in our setup the influence of the fill height on the flow rate becomes non-negligible. As is seen in Fig. 2, the flow rate slightly decreases even in the  $W \approx 2.06D$  geometry when the container is half empty. Therefore, the lowest datum points in Fig. 7 ( $W \approx 2.18D$ ) deviate systematically from the prediction.

## IV. CONCLUSIONS AND SUMMARY

The outflow of soft, practically frictionless hydrogel spheres is distinguished qualitatively in several aspects from that of previously investigated hard particles. First, the extremely low friction coefficient leads to hydrostatic pressure conditions in the 2D bin. Second, the softness of the particles leads to a pressure dependence of the clogging probability at small orifice widths. The combination of these two effects results in a complex dependence of the clogging characteristics on the fill height of the container. When  $W/D$  is larger than 2.2, the silo empties without clogging (only a few particles in a single layer remain at the horizontal container bottom). When the ratio  $W/D$  becomes smaller, the flow fluctuates, and nonpermanent arches may form and block the outflow up to several seconds. Permanent arches block the outlet when the pressure at the bottom becomes lower than a certain critical value, i.e., when the fill level of the silo reaches a critical height. At the smallest openings where we found avalanches in our container,  $W \approx 1.5D$ , the clogs formed when the pressure at the

orifice became less than about 5 kPa. We assume here that the pressure measured in a stationary state is comparable to the pressure during slow outflow. The pressure at the container bottom suffices to deform the spheres by at least 20% of their radii. This allows the spheres to adopt their shapes under the action of force chains, to leave intermittent clogs, and to pass through narrow outlets as long as a certain pressure is maintained. For hard particles, there is practically no influence of the fill level on the outflow unless the hopper is emptied to a very shallow layer. The outflow is practically pressure independent, even though the absolute pressure for comparable particle densities is much lower than for the hydrogel spheres (Fig. 4).

The flow profile inside the container distinguishes from that of hard spheres with friction in that the flow involves all particles, even those in the first layer on a flat horizontal bottom. There is *no* stagnant zone in our bin. The downward flow in the upper part (approximately  $2.5W$  above the bottom) is constant, practically independent of horizontal and vertical positions. This is primarily due to the monodispersity of the material, which generates a nearly perfect hexagonal packing, so that the material far above the opening is displaced downward as a crystallized structure. We assume that this may be different with bidisperse or polydisperse hydrogel spheres, it is not necessarily related to softness or low friction of our material. Finally, the equation derived by Beverloo for the discharge rate can be used to satisfactorily describe the outflow as a function of the relative opening width  $W/D$ . The equation becomes meaningless, however, when  $W/D < 2$  because the flow is neither constant nor pressure independent when the outlet becomes comparable to one or two particle diameters.

Weeks and collaborators [21,32] recently reported hopper experiments with hydrogel spheres. Their observations were in qualitative agreement with our findings. In their study, the focus was on very small numbers of particles, so that a pressure dependence could not be tested, and the hopper geometry differed from ours in that the bottom was trapezoid shaped.

Compared to most of the previously investigated materials, the hydrogel spheres differ in two aspects: they are soft and they have very low friction. In the first respect, they are between air and oil bubbles and hard materials; much closer to hard spheres. In the second respect, they are much closer to the fluid inclusions. In view of a general understanding of silo discharge, it will be of primary importance to discriminate the individual influences of both features on the outflow characteristics. In addition, a study of hydrogels in 3D hoppers will be needed to allow a better comparison with realistic practical situations.

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