

Effect of particle shape on silo discharge

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Abstract. Silo discharge rate of rod-like and ellipsoidal particles is compared in laboratory experiments. We focus on how the flow rate is changing with particle aspect ratio a/b . Our experimental investigations for ellipsoids are complemented by numerical simulations using the Discrete Element Model (DEM). For ellipsoids the flow rate is a non-monotonic function of a/b with slightly elongated or slightly flat ellipsoids discharging the fastest. DEM simulations show that both the packing fraction as well as the particle velocity has a non-monotonic aspect ratio dependence, thus both contribute to the non-monotonic tendency of the flow rate. When comparing the flow rate of rods with ellipsoids, we get very similar values for similar aspect ratios. On the contrary, when comparing the packing fraction we find a noticeable difference: it is about 10% lower for rods than for ellipsoids. This suggests that the particle velocity for rods should be larger by about the same amount, so that the product of these two factors results in the same flow rate for the two types of grains.

1 Introduction

Most natural or industrial granular materials consist of nonspherical grains. Slightly elongated or flat particles show interesting rheological behavior as a function of grain shape [1–5]. This is because of the fact that in a shear flow these particles rotate, and the neighboring particles hinder each other’s rotation. Statistically this leads to a shear induced alignment since the ensemble averaged rotation speed depends on the orientation of the particles: faster rotation when long axis is parallel to the flow gradient and slower rotation when long axis is nearly parallel to the flow direction. Thus the particles spend the longest time nearly parallel to the flow direction, and consequently the system is orientationally ordered [6, 7].

When a granulate is discharged from a container we find that the flow rate is constant (independent of the fill height) [8] and it increases with the orifice size as a power law (Beverloo law [8, 9]) and also depends on the internal friction of the granular material, which is changing with the surface roughness as well as the shape of the grains. Increasing surface roughness leads to decreasing flow rate, but the grain shape dependence is much less obvious [10–13]. In a recent work we have analyzed the silo flow rate as a function of grain aspect ratio for moderately anisometric ellipsoidal particles [11]. We found a non-monotonic dependence of the flow rate for both rice-like and lentil-like ellipsoids, where slightly elongated or flat grains discharged faster than beads or more anisometric ones.

In the present work, we compare the silo flow rate and the packing fraction of the samples for rod like particles

(with sharp edges) and ellipsoids experimentally. We also recall the results of our numerical analysis for ellipsoids.

2 Experimental and numerical procedures

The silo discharge experiments were performed using an acrylic cylinder with an inner diameter of $D_c = 172$

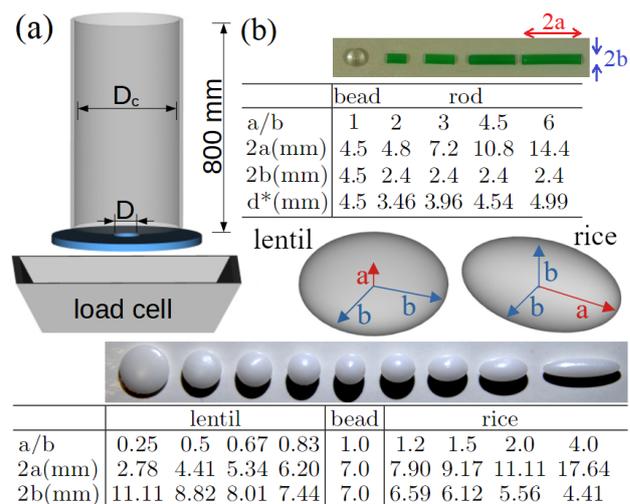


Figure 1. (a) Schematic diagram of the experimental setup with a cylindrical acrylic silo with the inner diameter of $D_c = 172$ mm and a height of 800 mm. (b) Photographs of the particles used in the experiments: rods, glass beads and ellipsoids. The dimensions of the particles are given in the tables, the equivalent diameter is $d^* = 7$ mm for all ellipsoids, this corresponds to the diameter of a sphere with the same volume as the given particle.

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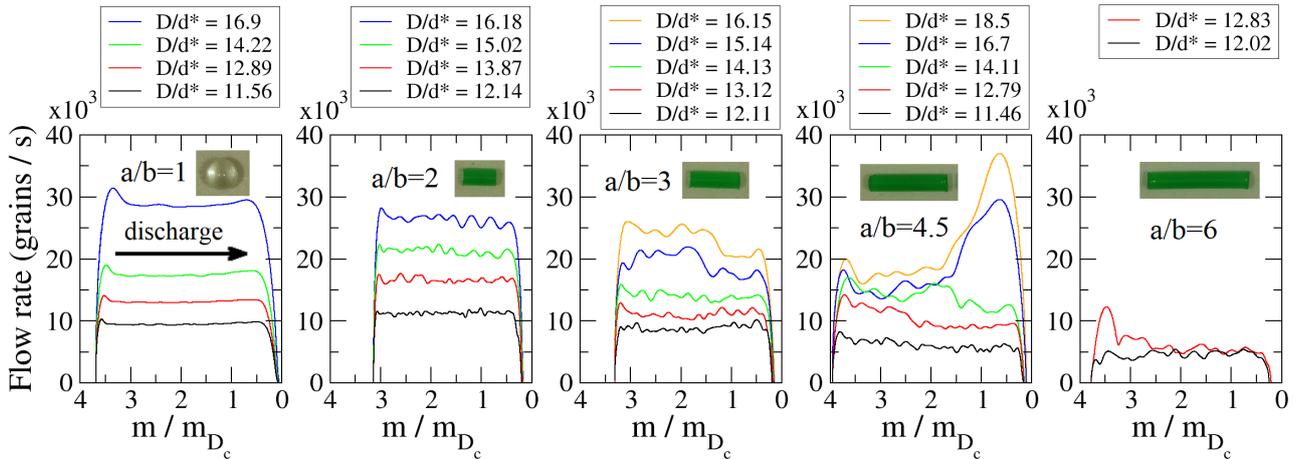


Figure 2. Silo flow rate as a function of the mass in the cylinder for beads and plastic rods with aspect ratios of $a/b = 2, 3, 4.5$ and 6 . The mass m in the silo is normalized by the mass m_{D_c} corresponding to the filling height of one silo diameter D_c . The curves represent the average of 2 measurements and correspond to different normalized orifice sizes D/d^* , where D is the orifice size and d^* is the equivalent grain diameter, i.e. the diameter of a sphere with the same volume as the particle.

mm and a height of 800 mm with an orifice at the bottom with adjustable diameter D (see Fig. 1(a)). After filling the cylinder manually, the orifice was opened, and the flow rate was recorded by measuring the weight of the discharged mass with a load cell. We used 2 sets of samples: ellipsoids and rods the pictures of which are shown in Fig. 1(b) together with their characteristic dimensions. The Polyoxymethylene (POM) ellipsoids were produced by injection moulding by Yuyao Strong Co., China [14]. The equivalent diameter of all ellipsoids was $d^* = 7$ mm, i.e. their volume was equal to the volume of a $d = 7$ mm bead. This is convenient, since for these samples one can directly compare the silo flow rate for a given orifice size D . The rods were produced by ourselves by cutting $d = 2.4$ mm nylon trimmer lines (Oregon) with a cable cutting machine (Model GlobalCut 100, Navia International GMBH, Germany). Thus the volume as well as the equivalent diameter d^* of the rods is changing with their length (see Fig. 1 for the data). In order to compare the flow rate for these samples we performed experiments with two orifice sizes slightly above and below $D/d^* = 12.8$, and interpolated the data to $D/d^* = 12.8$. Similarly to our earlier measurements with these plastic rods in a narrow silo, we included datasets taken with spherical glass beads of the diameter of $d = 4.5$ mm for comparison [15].

In the numerical investigations we performed Discrete Element Model calculations aiming for a direct comparison with the experimental data for ellipsoids which are modeled using the superquadric equation [16]. This is easier numerically than the case of rods with sharp rims. We employed a self-written GPU-NVIDIA implementation as a parallelization procedure [17] based on [18, 19], which allowed the examination of system sizes comparable with the experiments. A superquadric is defined by the length of its half-axes a, b, c , and the blockiness parameters n_1 and n_2 . We used $n_1 = n_2 = 2$, representing ellipsoids,

and the values of $a, b \equiv c$ were the same as those of the experimental particles. In all the cases, the system is composed of $N = 50000$ mono-disperse superquadrics, of the same equivalent diameter d^* . POM has relatively low surface friction, therefore in the simulations presented here we use an inter-particle friction coefficient of $\mu = 0.3$. After defining the particle-particle and particle-wall contact forces, the DEM computes the movement of each particle, see more details in [11]. For the computation of the density, velocity and stress macroscopic fields, we have taken advantage of a useful coarse-graining technique, described in [20, 21].

3 Results

We present the evolution of the flow rate as a function of the mass in the cylinder during the discharge process in Fig. 2 obtained in experiments for rods and glass beads. The mass in the cylinder is normalized by the mass corresponding to the filling height of one silo diameter D_c . Similar graphs obtained for the case of ellipsoids can be seen in [11]. From these measurements we get the average flow rate using the data in the middle part of the discharge process, corresponding to the range of $1.4 < m/m_{D_c} < 2.8$. We mention that for a narrow cylinder, a surge is observed at the end of the discharge process for a certain range of the orifice diameter [15]. Two curves in Fig. 2 obtained for rods with $a/b=4.5$ show this characteristics for the two largest value of the normalized orifice diameter: $D/d^* = 16.7$ and 18.5 . For most of our measurements which we will analyze in the following, there was no surge, for those where a surge occurs, we average the flow rate before the surge.

In order to compare the silo flow rate for different particle shapes we plot the average flow rate (defined above) as a function of grain aspect ratio a/b for both rods and

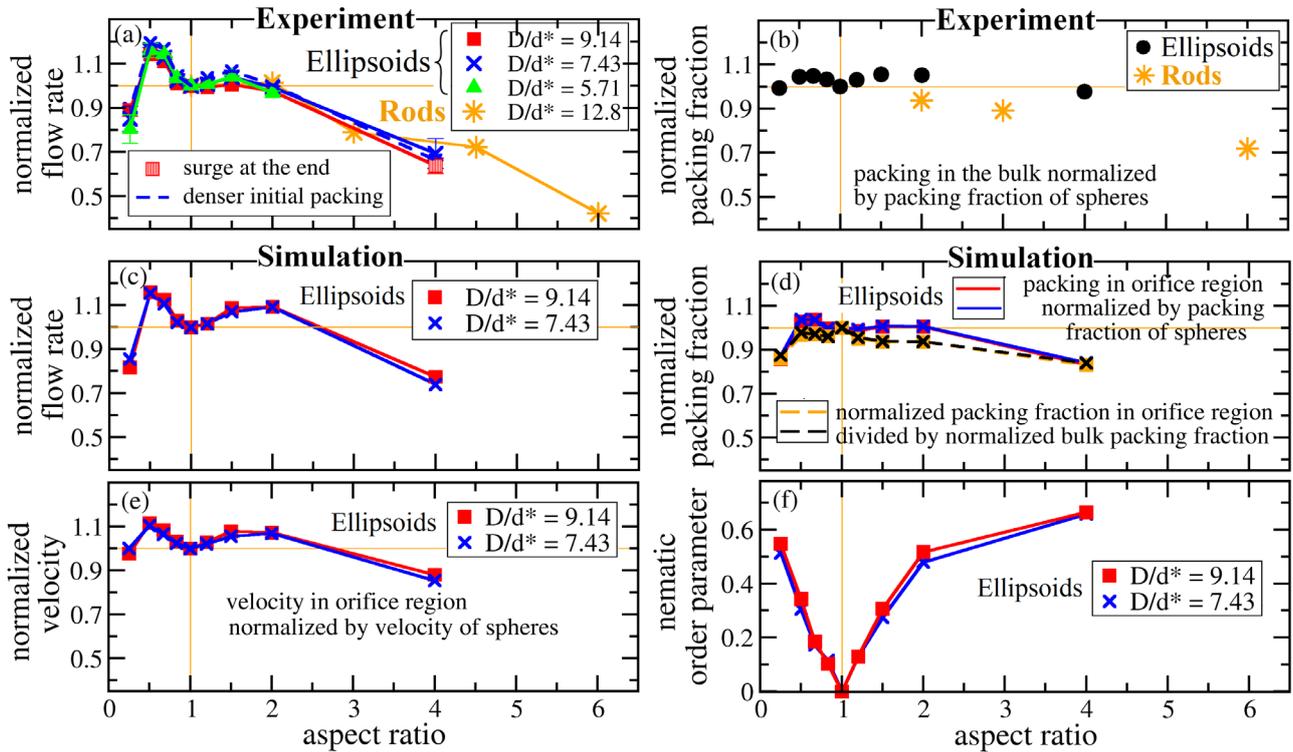


Figure 3. Experimental results for rods and ellipsoids: (a) average flow rate and (b) static packing fraction as a function of particle aspect ratio. All quantities normalized by the value obtained for beads. The average flow rate was obtained by averaging the data in the range of $1.4 < m/m_{D_c} < 2.8$. (c-e) Numerical results for ellipsoids: (c) average flow rate, (d) packing fraction, (e) velocity and (f) nematic order parameter as a function of aspect ratio.

ellipsoids in Fig. 3. Here the flow rate data are normalized by the average flow rate measured for beads. For the case of ellipsoids there is an interesting non-monotonic behavior, i.e. slightly elongated rice-like ellipsoids or slightly flat lentil like ellipsoids discharge faster than spherical particles or very elongated or very flat grains. We will shortly explain our main findings regarding this feature below, but a more detailed analysis can be found in [11]. More importantly, here we compare the aspect ratio dependence of the flow rate of ellipsoids with rods. Focusing on this aspect, we see that the normalized flow rate data for rods with $a/b=2$ and 3 nicely fall on the curves obtained for rice-like ellipsoids, and the two other points for rods (for $a/b=4.5$ and 6) continue to follow the decreasing trend with reaching a normalized flow rate of below 0.5 for $a/b=6$.

Before we continue the comparison of rods and ellipsoids we summarize our findings for the case of ellipsoids. For that case our numerical simulations allow us to get more insight into the process. Namely, the flow rate can be written as the product of the packing fraction and the grain velocity through the orifice. In Fig. 3(c-e) we plot these three quantities (flow rate, packing fraction and grain velocity) as a function of aspect ratio. We see that both, the packing fraction as well as the grain velocity shows a non-monotonic trend, so they are both contributing to the non-monotonic trend of the flow rate. In other words the slightly nonspherical particles (either lentil-like

or rice-like) flow out of the silo faster because (i) their packing fraction is slightly larger and (ii) their falling velocity through the orifice is also larger than those quantities for spherical particles. And we also see that taking more anisometric grains (i.e. longer rice or flatter lentils) both the packing fraction and the grain velocity drops leading to a decrease in the flow rate. Interestingly, the degree of orientational ordering, characterized by the nematic order parameter, does not follow the above described non-monotonic trend (Fig. 3(f)).

Returning to the comparison between rods and ellipsoids, we show the packing fraction for these materials as a function of aspect ratio in Fig. 3(b). These data were obtained experimentally using a cylindrical container (with similar dimensions as the silo) and measuring the weight of a sample of the volume of 4 liters first dry and then immersed in water. Thus these data correspond to a random packing in a static configuration. As we see the packing fraction of rods is about 10% lower than the packing fraction of rice-like ellipsoids with the same aspect ratio. This means that in order to get the same flow rate as for ellipsoids the grain velocity through the orifice should be about 10% larger. That would mean, that the region from where the particles are accelerating (free fall arch) is somewhat higher for rods than for ellipsoids. This is an experimental prediction based on our data. Unfortunately in our current numerical system it is not possible to model the behavior

of rods with sharp edges. Such numerical investigations can be a subject of future work.

In summary, based on our experimental data we compared the aspect ratio (a/b) dependence of the silo flow rate for two types of non-spherical grains: rods and ellipsoids. We find that for aspect ratios $a/b > 2$ the normalized flow rate decreases the same way for these two grain types. For the case of ellipsoidal grains our experimental investigations were complemented by numerical simulations based on the Discrete Element Model. The numerical simulations allowed us to decompose the aspect ratio dependence of the flow rate into two factors: packing fraction and grain velocity. This analysis showed that both factors are non-monotonic as a function of a/b , and contribute to the non-monotonic tendency of the flow rate in a similar way. Experimentally, we can determine the packing fraction of the material in the static state. The aspect ratio dependence of the packing fraction for ellipsoids is non-monotonic (as expected). The packing fraction of the rods decreases with a/b , but more interestingly it is noticeably (by about 10%) below the packing fraction of rice-like ellipsoids with the same aspect ratio. Since the normalized flow rate was found to be very similar for ellipsoids and rods, we speculate that the grain velocity of rods through the orifice should be larger for rods by about the same amount ($\approx 10\%$), meaning that the region from where the grains are freely falling is expected to be slightly higher.

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