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Arching in three-dimensional clogging

János Török[1,⋆], *Sára* Lévay[1], *Balázs* Szabó[2], *Ellák* Somfai[2], *Sandra* Wegner[3], *Ralf* Stannarius[3], and *Tamás* Börzsönyi[2]

[1] *Department of Theoretical Physics, Budapest University of Technology and Economics, H-1111 Budapest, Hungary*
[2] *Institute for Solid State Physics and Optics, Wigner Research Center for Physics, Hungarian Academy of Sciences, PO Box 49, H-1525 Budapest, Hungary*
[3] *Institute of Experimental Physics, Otto-von-Guericke-University, Universitatsplatz 2, D-39106 Magdeburg, Germany*

Abstract. Arching in dry granular material is a long established concept, however it remains still an open question how three-dimensional orifices clog. We investigate by means of numerical simulations and experimental data how the outflow creates a blocked configuration of particles. We define the concave surface of the clogged dome by two independent methods (geometric and density based). The average shape of the cupola for spheres is almost a hemisphere but individual samples have large holes in the structure indicating a blocked state composed of two-dimensional force chains rather than three-dimensional objects. The force chain structure justifies this assumption. For long particles the clogged configurations display large variations, and in certain cases the empty region reaches a height of 5 hole diameters. These structures involve vertical walls consisting of horizontally placed stable stacking of particles.

1 Motivation

Discharge of silos and the flow of granular material through orifices are important questions of everyday life and technological processes. Several experimental and numerical studies were carried out to understand the processes of arch formation, force chains and arch breaking [1–4]. From the technological point of view one is interested in the prevention of silo clogging which blocks the outflow. The jamming probability as the function of the diameter of the orifice (D) and the aspect ratio (Q) of the granulate is also an unsolved question [5–7].

The accurate analysis of the microscopic details of the clogged configuration in three-dimensional (3D) hoppers is a challenge. In 2D it is straightforward to define the blocking arch by finding a continuous chain of particles from one side of the orifice to the other. Here, one can visualize the force network and the structure of the arch experimentally by using photoelastic particles [3, 4]. With the help of discrete element (DEM) simulations one can quantitatively compare the morphology of clogging arches with experimental data, and accurately map the force distribution inside the silo [1, 2]. In the present work we identify the blocking structure in three-dimensional hoppers with DEM simulations focusing on particle configurations and forces and compare our finding to our measurements using X-ray computed tomography. We compare the case of spherical particles with elongated grains.

2 Methods

2.1 Discrete element simulation

Discrete Element Method (*DEM*) simulations were performed to study three-dimensional clogging. We used a modified version [8] of the molecular dynamics code *LAMMPS* [9] to generate a cylindrical hopper with an orifice of given size on the bottom wall. We used elongated particles with given aspect ratios ($Q = 2, 3.3, 5$) and spheres ($Q = 1$). Elongated particles were created by gluing spheres together with overlaps between them. We created polydisperse samples with the variation of particle diameters by ±20%, the length of particles by ±10% and the overlap of the spheres in elongated particles between $30-50\%$ in mean diameter units. The number of spheres required to build up a long particle was also varied to prevent linear gear effect. The initial state of simulations, the position of the particles was created using the *random sequential deposition* model in order to avoid the overlap of distinct particles. Particles were always created with average volume equivalent to a sphere with unit diameter.

The particles were put in a cylindrical container of 10 units radius, with a hole of diameter D in the bottom wall. The middle of the hole will be referred to as *center*. The height of the system was much larger than the height of the resulting static granular bed which was chosen to be about 25 particle diameters. After applying gravity, the system starts to flow out of the the container. A vertical cross-section of the simulated system is shown in Fig. 1. Periodic boundary conditions were used in the vertical direction in order to have the outflowing particles back in the

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[⋆] e-mail: torok@phy.bme.hu

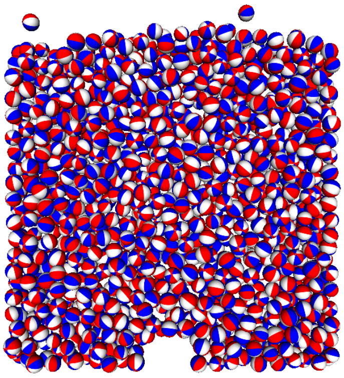

Figure 1. Sample cut of a clogged three-dimensional hopper with cylindrical orifice.

bulk again. The clogging states were defined by the lack of outflowing particles for a certain amount of time to allow for relaxation of the sample. In order to have several clogged states during simulations, we unclogged the system by raising every particle by 2 units. We measured the position and contact forces acting between particles. In the following we only deal with clogged states.

2.2 Experimental method

The experimental tests were realized using an axisymmetric nearly cylindrical container with the diameter of 19 cm and height of 21.4 cm [10]. The container was filled with peas which have nearly spherical shape with a mean diameter of 7.6 ± 0.23 mm. For the experiments presented here the orifice size was 23 mm, corresponding to $D = 3$. In order to determine the 3D configuration of the grains an X-ray tomogram (CT) was recorded using the robot-based flat panel X-ray C-arm system Siemens Artis zeego of the STIMULATE-lab, Otto von Guericke University, Magdeburg. The spatial resolution was 2.03 pixel/mm, with recorded volumes of 25.2 cm × 25.2 cm × 19 cm. A central vertical cross section of a sample tomogram is shown in Figure 2.

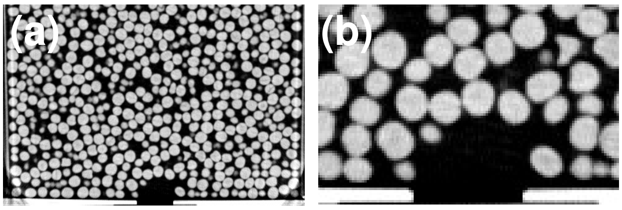

Figure 2. Vertical cross section of a tomogram covering (a) 188× 125 mm, and (b) 60 × 40 mm of the experimental system. The diameter of the hopper outlet was 24 mm.

2.3 Analysis methods

For characterizing the clogged state we used two methods: (i) *surface triangulation* of the clogged system and (ii) analysis of the average *packing density* (ϕ).

The clogged configuration is characterized by a cupola shaped cavity above the discharge hole. We need to identify the first layer of grains at the boundary of this cavity. Taking the center of mass of the particles, in principle this surface can be defined by a Delaunay triangulation, but this method unfortunately does not work well for such concave surfaces. Thus a novel method was developed in order to quantify the surface.

In our method triangles were identified by the edges between center of the particles, where the distance was not larger than 1.5 particle diameter. This ensures that all and only neighboring particles are connected. Using these triangles we determined the surface by projecting a grid from the orifice plane onto the triangles. We define the minimal $r_{\min}$ and maximal $r_{\max}$ radius from the points closest and farthest from the center.

The packing density was calculated by averaging several clogged states see Table 1. The spatial distribution in the $r - z$ plane is shown in Fig. 3 (a) for $Q = 1$ after averaging in the axial direction [10]. In these runs the the diameter of the outlet was $D = 3.4$. Approaching the orifice from the top along the vertical axis we find a decreasing packing density. Near the outlet there is an empty region, which is the region below the dome. The upper border of this region corresponds to the dome which holds the material above this clogged configuration.

3 The dome for spherical particles

The primary question we would like to answer is whether the dome is a complete three-dimensional structure, or it is rather an assembly of two-dimensional arches, similar to those observed in 2D hoppers?

The average density profile indicates a nice three-dimensional self supporting dome like structure which is taller than a hemisphere (Fig. 3 (a)). A comparison of the average density fields obtained in experiments and simulation are shown in Fig. 4 (a)-(b) for spheres, while numerically obtained average density fields for elongated grains are presented in Fig. 4 (c)-(d). Individual samples show similar features but often have large empty or sparser regions. The example shown in Fig. 3 (b) indicates empty regions above and sideway above the cupola. This structure with low density regions can be better seen in Fig. 3 (c), where only those particles are shown which are closer to the center than 2.9 particle diameter. We see a large hole in the cupola which is visible also on the force network Fig. 3 (d), where the red lines indicate the forces with line widths proportional to the normal force. One can observe a nice two-dimensional arch composed of 6 particles, which form a bridge above the outlet. Most probably the hole in Fig. 3 (c) is closed by an other smaller cupola placed on the hole of the first one.

We used the triangulation surface detection method to find these details. In Fig. 3 (e) a cut of the triangulation result is shown from the side with the hole facing the observer. The surface triangles indicate small auxiliary cupolas on the hole. The irregularity of the primary dome can be demonstrated by plotting the surface generated by the triangulation method, see Fig. 3 (f).

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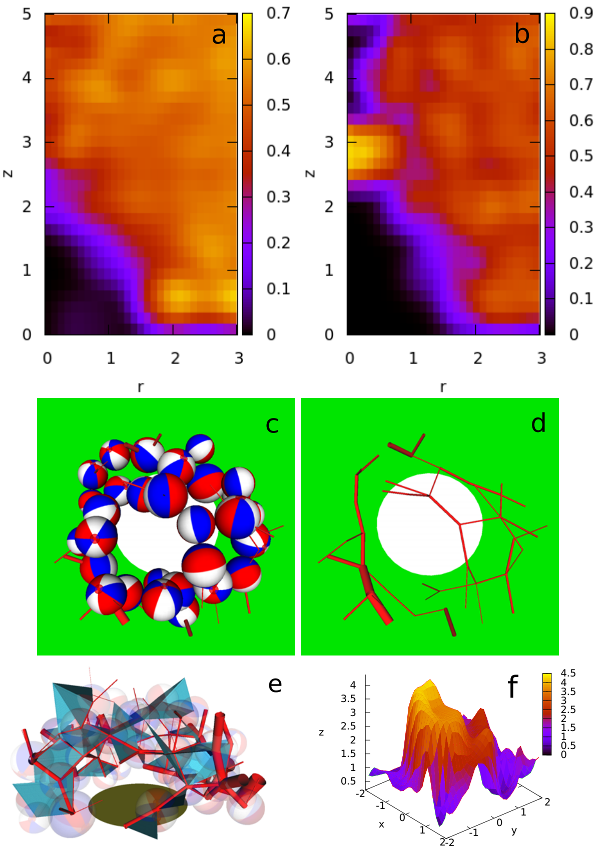

Figure 3. (a) Average density field for spheres. All the other figures are for the same single clogged configurations with spheres (b) two-dimensional average density field, (c) particles closer than 2.9 particle diameter to the center, (d) forces to latter particles, (e) middle 2 particle diameter cut of the surface triangulation, (f) detected surface from the triangulation.

The above example provides a strong hint that clogging in three-dimensional hoppers is actually a result of the formation of two-dimensional arches which eventually block the outlet. For a more general characterization we analyzed the statistics of $r_{\max} - r_{\min}$ the distance of the surface point closest and farthest from the outlet center. As can be seen in Table 1 the value of $(r_{\max} - r_{\min})/r_{\min} \simeq 2.5$ indicating the presence of large holes in the cupola. Data for different values of D are very consistent, there is only little variation of both the minimal and maximal radii which affirms that different examples of clogged cupolas have essentially the same structure.

We may conclude that it is very likely that the clogged state of a three-dimensional hopper is basically composed of a complex structure of two-dimensional arches. For a three-dimensional system these arches consist of 5-6 particles, as in two-dimensional systems [11]. The three-dimensional system is thus composed of these small two-dimensional units. In order to better quantify the above observations we are currently working on the analysis of the force network in our three-dimensional system.

Table 1. Minimal and maximal distances of the surface from the center, their variances $\sigma_{\min}$ and $\sigma_{\max}$, N is the number of configurations.

Q	$D/2$	$r_{\min}$	$r_{\max}$	$\sigma_{\min}$	$\sigma_{\max}$	N
1	1.5	1.52	3.53	0.20	0.12	21
1	1.6	1.71	3.62	0.10	0.12	8
1	1.7	1.80	5.02	0.11	0.43	5
2	1.5	1.21	4.74	0.34	0.55	8
2	1.6	1.36	4.47	0.18	0.35	14
2	1.7	1.54	6.77	0.23	0.3	22
2	1.9	1.69	6.45	0.19	1.71	13
2	2.1	1.8	5.61	0.22	1.11	19
3.3	2.0	1.01	4.39	0.23	0.37	40
3.3	2.3	1.23	5.05	0.28	0.87	44
3.3	2.6	1.49	5.96	0.28	1.65	31

4 The dome for elongated particles

It was shown [12] that elongated particles align themselves in shear flow. Our recent observations on hopper flows with elongated grains show that the particles approach the outlet with approximately their smallest cross section (they are aligned nearly parallel to the flow lines) [10]. Focusing on the position of a given particle near the outlet, clogging may happen at any instant, thus we expect that the fluctuations in the resulting cupola surface will be larger for longer particles. On the other hand, particles close the space with their circular surface, so we expect to see similar structures as in the case of the spheres.

Table 1 summarizes the numerical findings for different systems. The surface starts at around the orifice perimeter for spheres, but $r_{\min}/D$ decreases with elongation, indicating that trapped particles hanging inside reduce the effective orifice size. This indicates that the force chain arch is at about the same distance from the center but the inner end of the particles is closer.

In order to be able to unclog the system the outlet must be made larger with increasing particle elongation in spite of the fact that the particle diameter decreases as the volume is kept constant. Unfortunately our data are not good enough to determine any quantitative correspondence between equivalent D as a function of Q.

The other striking feature of the long particles is the strongly increased value of $\sigma_{\max}$ for larger D which indicates a large variation of the possible clogged configurations. The average density plots of these systems yield a large oval-shaped low density region which is compatible only with a fluctuating cupola height. Sometimes we get hemisphere like cupolas but other times very elongated ones. An example for an elongated cupola is shown in Fig. 5, where we observe a vertical wall composed of horizontally placed elongated particles. It seems that if the outlet is large and if it takes a long time to form a clogged state, particles can be placed upon each other to form a stable packing around the hole. This packing can take very important sizes and can reach five times D, even at moderate aspect ratios.

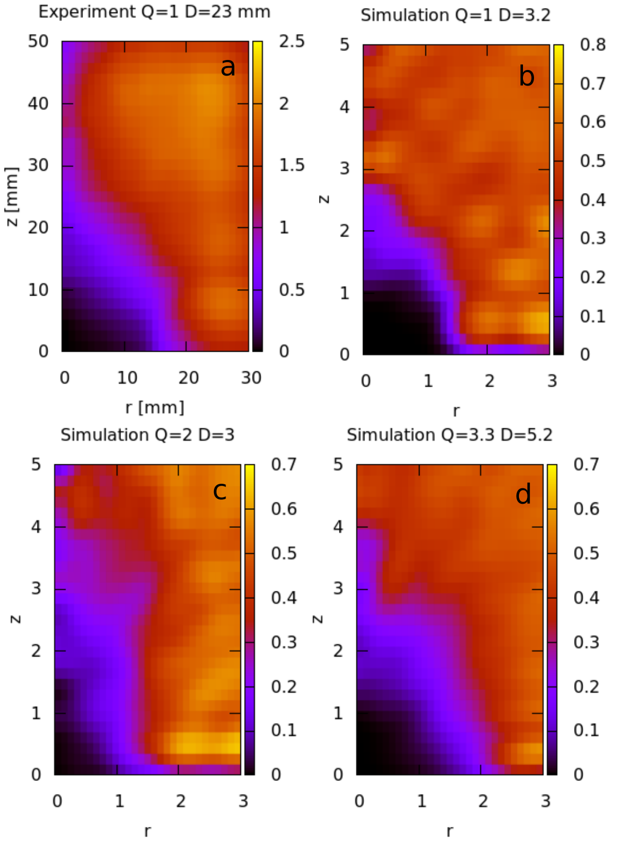

Figure 4. Average density profile for different particle aspect ratios. (a) spheres (experiment), (b) spheres simulation, (c) Q=2, (d) Q=3.3

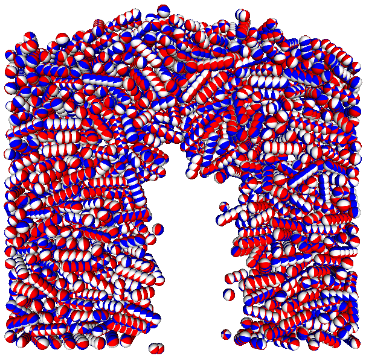

Figure 5. A sample snapshot cut for elongated clogged state. $Q = 3.3$, $D = 2.6$.

Systems consisting of long grains are characterized by large fluctuations, i.e. after shaking the above described structure collapses and the subsequent clogged state can be a small, hemisphere-like object.

5 Summary

Discrete Element Simulations and experiments were performed to study three-dimensional clogging of granular materials in a flat bottom hopper. The clogged state is characterized by a cupola-shaped configuration of particles around the outlet. Two methods were used to study the structure of the clogged material: We have developed a triangulation method which works for concave objects.

The inner shape of this object is close to a hemisphere but with large holes in it, indicating that it is essentially composed of two-dimensional arches which eventually fill the space. The above statement is justified by the fluctuations seen both on the individual density plots and on the variation of the maximal distance of the surface from the center of the outlet. Further studies are planned to characterize the clogging structures using the force network.

The clogging of elongated particles was also studied. In some cases, similar sphere-shaped cupolas were found, but in many cases, especially for long particles and large orifice, the cupola was standing on the top of a stable structure of horizontally placed long particles. These structures can grow up to five times the orifice diameter, even at the moderate aspect ratios considered here.

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