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THICKNESS DEPENDENCE OF THRESHOLD FIELD FOR INSTABILITIES IN CHOLESTERICS

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An instability of the cholesteric planar texture is investigated under superposed dilatation and external field. The oscillatory behaviour of the threshold field versus thickness is explained using the coarse-grained continuum theory.

When a planar cholesteric sample is subjected to an electric or magnetic field, parallel to the helical axis, a periodic deformation occurs above a threshold field. The theory of this deformation have been given by Helfrich[1] and Hurault[2]. Their theory applies to the case, when in the absence of field the pitch of the cholesteric helix is equal to its equilibrium value P_o, i.e. the sample thickness (L) is an integral multiple of $\frac{P_o}{2}$. In this case the threshold magnetic field is proportional to $(LP_o)^{-\frac{1}{2}}$ and for electric field the threshold voltage is proportional to $\left(\frac{L}{P_o}\right)^{\frac{1}{2}}$.[2]

Recently Belyaev and Blinov[3] measured the threshold voltage as a function of the thickness in a wedge shaped sample. A similar experimental result measured by us is shown in Fig. 1. The vertical dotted lines correspond to the Grandjean lines,

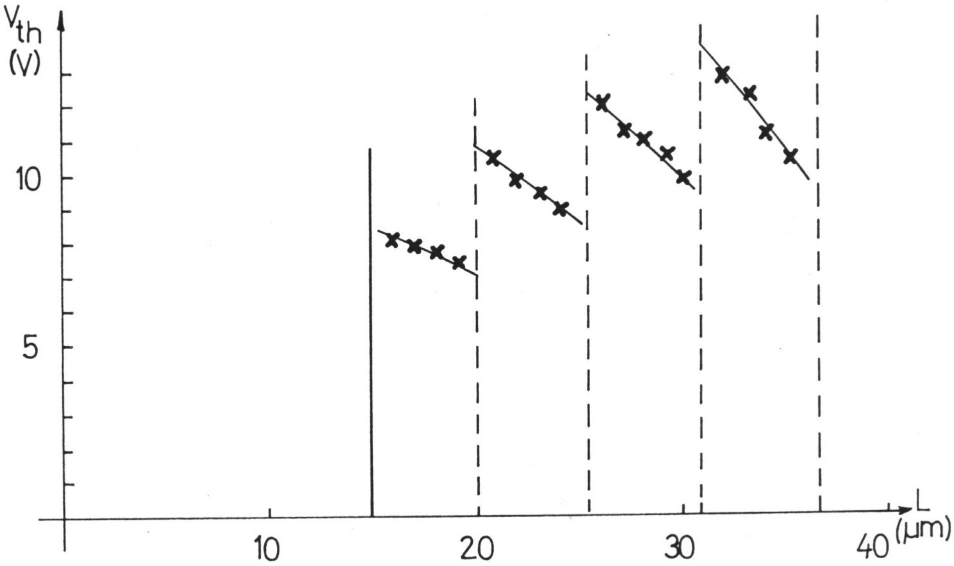

FIG. 1. Thickness dependence of the threshold voltage measured on methoxy-butyl-azoxy-benzene doped with 1 % cholesteryl chloride using AC electric field (Resulting pitch P_o = 11.5 µm)

i.e. where the number of half turns of the cholesteric helix (n) changes. At the middle of each region, belonging to a fixed n, the actual pitch of the cholesteric is equal to its equilibrium value, P_o. The threshold voltages belonging to these points follow rather well the predicted $\sqrt{L}$ dependence. Inside the regions however the threshold voltage decreases as the thickness increases; at the Grandjean lines the threshold changes discontinuously.

The aim of the present letter is to provide a simple explanation of this observed thickness dependence. The main points of the explanation can be summarized as follows.

At thicknesses, where L is not an integral multiple of $P_o/2$, the helix is in a dilated or compressed state. Let us first consider a part of the sample, where the helix is dilated. As known, high enough dilatation even itself can lead to an instability of the planar texture. (Similarly to the case of undulation instability in smectics[4].) So two destabilizing forces are superposed at the same time, dilatation and external field. Although in the wedge, used in the experiment, dilatation is nowhere high enough to produce a periodic deformation in itself, it reduces the field strength required to induce instability. On the other hand, at other parts of the sample, where the helix is compressed, the threshold field increases.

To make this consideration quantitative, we use the coarse-grained version of the continuum theory of cholesterics[4]. The free energy can be written in terms of the displacement of the cholesteric planes (u) as

$$F=\frac{1}{2}B\left[\frac{\partial u}{\partial z}-\frac{1}{2}\left(\frac{\partial u}{\partial x}\right)^2\right]^2+\frac{1}{2}\tilde{K}\left(\frac{\partial^2 u}{\partial x^2}\right)^2-\frac{1}{4}\chi_a H^2\left(\frac{\partial u}{\partial x}\right)^2 \qquad /1/$$

with $B=K_2q^2$; $\tilde{K}=\frac{3}{8}K_3$; $q=\frac{2\pi}{P}$

(z is the direction of the helical axis in the unperturbed state, the periodicity is along x). We take

$$u(x,z)=u_o(z)+u_1(x,z) \qquad /2/$$

where $u_o(z)$ describes the constant dilatation (or compression) of the helix owing to the boundary conditions, i.e.

$$U_0(z) = \frac{P-P_0}{P_0} z \quad ; \quad P = \frac{2L}{n}$$

For a perturbation of the form

$$U_1(x,z) = U_1 \sin\frac{\pi}{L} z \sin k_x x \qquad /3/$$

the average of F over the sample becomes

$$\langle F \rangle = F_0 + \frac{1}{8} U_1^2 k_x^2 \left[B \frac{\pi^2}{L^2} \frac{1}{k_x^2} + \tilde{K} k_x^2 - B\left(\frac{P-P_0}{P_0} - \frac{1}{2} \chi_a H^2 \right) \right] \qquad /4/$$

The unperturbed texture becomes instable at the smallest field when there exists a k_x value at which the coefficient of U_1^2 in the square bracket becomes negative. This occurs for $k_x^2 = \sqrt{\frac{B}{\tilde{K}}} \frac{\pi}{L}$ so the threshold field is

$$H_{th}^2 = \frac{2}{\chi_a} \left\{ 2\sqrt{B\tilde{K}} \frac{\pi}{L} - B \frac{P-P_0}{P_0} \right\} \qquad /5a/$$

or

$$H_{th}^2 = \frac{1}{\chi_a} \frac{\pi^2}{L^2} \sqrt{6K_2K_3}\, n \left\{ 1 - n\sqrt{\frac{2K_2}{3K_3}} \frac{P-P_0}{P_0} \right\} \qquad /5b/$$

For $P=P_0$ this formula gives Hurault's expression for the threshold field[2] namely

$$H_{th}^0 = \frac{\sqrt{2}}{\sqrt{LP_0}} \pi \sqrt{\frac{(6K_2K_3)^{1/2}}{\chi_a}}$$

For the general case eq./5b/ can also be written in the form (neglecting terms proportional to L^{-2})

$$H_{th} = H_{th}^0 \left(1 - \frac{2\Delta L}{P_0} \sqrt{\frac{2K_2}{3K_3}} \right)^{1/2} \qquad /6/$$

where $\Delta L = L - n\frac{P_0}{2}$.

For electric field a similar generalized formula can be derived

$$V_{th} = V_{th}^0 \left(1 - \frac{2\Delta L}{P_0} \sqrt{\frac{2K_2}{3K_3}} \right)^{1/2} \qquad /7/$$

where V_{th}^{o} is the threshold voltage given by Hurault[2]

$$V_{th}^{o} = \frac{\sqrt{2}}{\xi}\sqrt{\frac{L}{P_o}}\,\pi\sqrt{\frac{(6K_2K_3)^{1/2}}{\varepsilon_o\varepsilon_a}}$$

(ξ is a factor depending on the dielectric constant and conductivities).

Our formula /7/ is displayed in Fig. 2 compared with the previous theory. It describes well the oscillating behaviour of the threshold voltage.

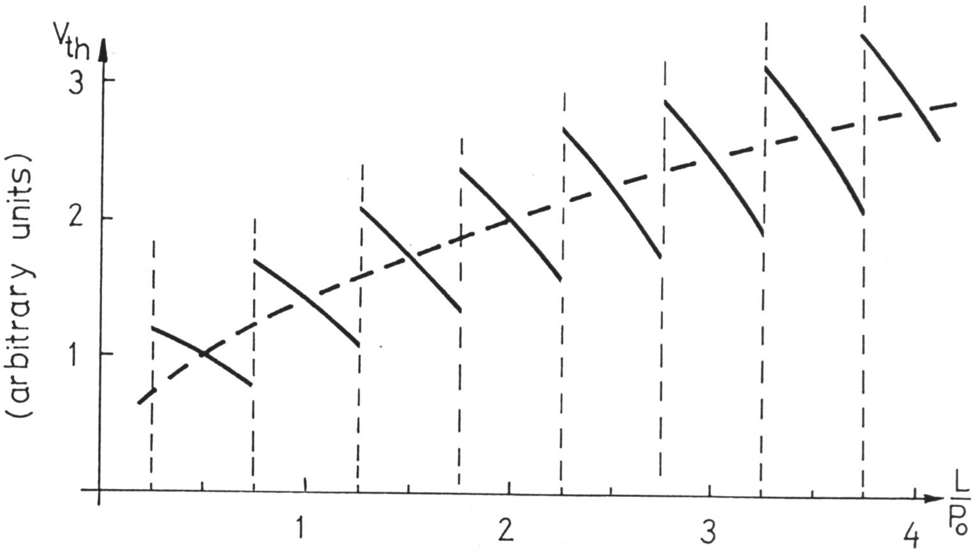

FIG. 2. Threshold voltage versus thickness calculated theoretically for $K_2=K_3$ (--- Hurault's theory[2]; —— our formula /7/)

From a Grandjean line to the next one ΔL varies from $-\frac{P_o}{4}$ to $\frac{P_o}{4}$. This means a modulation of ~ 35 % of V_{th} for $K_2=K_3$. Note that this modulation does not depend on the thickness, it does not become negligible even for very thick samples although the change in the pitch becomes very small. This conclusion is in accord with the experimental results[3].

The effect discussed here confirms the strong analogy between the elastic properties of the cholesteric and smectic A phases. The formula /5a/ is applicable for smectic A also. In this case usually it is not possible to induce instability with field alone, as the threshold is too high. However, according to /5a/, the threshold could be reduced considerably by dilating the sample. Observations of this kind could provide a simultaneous determination of the two elastic constants, B and $\tilde{K}$.

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