Thermally Induced Optical Bistability in Thin Film Devices

ISTVAN JANOSSY, MOHAMMAD R. TAGHIZADEH, J. GORDON H. MATHEW, AND S. DESMOND SMITH

Abstract—A simple theoretical model describing thermally induced optical bistability is discussed. Results of experimental studies of optical bistability in ZnSe interference filters near the band edge are in good agreement with the predictions of this model.

I. Introduction

In solid-state systems, optical nonlinearities are significantly influenced at room temperature by thermal effects, especially for visible wavelength and steady-state operation. However, thermal effects themselves can be regarded as optical nonlinearities and can be utilized in nonlinear devices such as optically bistable elements.

There have been several publications recently on thermally induced optical bistability. The experiments carried out in this field can be divided into two groups. The first group is those in which the temperature dependence of the absorption coefficient (α) gives rise to the bistable operation. This requires that α increases more than linearly with the temperature, and this condition is usually satisfied near the fundamental absorption edge of semiconductors. This fact has experimentally been explored in a number of experiments [1]–[5]. Another possibility is to use the temperature shift of a sharp exciton line [6]. These systems are particularly simple as no cavity is required for bistability.

The second group consists of those experiments in which the absorbing layers were placed within thin-film interference structures (nonlinear interference filters) [7]–[9]. In these systems, although absorption is essential in order to achieve thermal effects, the bistable operation relies first of all on the temperature dependence of the refractive index n. The change in refractive index leads to the shift of the transmission peak of the resonator, and bistability is obtained in a rather similar manner as in the case of dispersive electronic bistabilities [10].

The aim of the present paper is to provide a simple description for both groups of thermal effects, based on the heat flow equation. We restrict ourselves to thin absorbing layers, which can be characterized by a temperature-dependent absorbance (or dissipation coefficient) $A(T)$. $A(T)$ gives the fraction of the energy of the laser beam that is absorbed in the layer at temperature T.

The difference between the two groups described above is that in the first group, $A(T)$ depends only on α ($A = 1 - e^{-\alpha L}$, L layer thickness), while in the second group, it depends on the refractive index too. A specific example is discussed in Section II.

One possible approach to the problem considered here was given by Rozanov [11]. He assumed a two-dimensional heat flow within the absorbing layer, and in addition introduced a term describing the heat transfer to the environment of the layer.

We believe that Rozanov's approach correctly describes the case of thin self-supporting layers used, e.g., in [2]. In a number of cases, however, the absorbing layer is evaporated on a substrate which is much thicker than the layer. In these cases, the role of the substrate in the heat flow is very important. To describe such a situation, we consider the limit of a very thin (typically a few micrometers) layer on a semi-infinite slab of the substrate (typically a few millimeters), and assume three-dimensional heat flow from the irradiated spot into the substrate.

In Section II, we present our theoretical model. The parameters entering the model are the $A(T)$ function, which characterizes the absorbing layer, the irradiated radius ρ_0, and the heat conductivity of the substrate κ. Knowing these parameters, the bistability curves can be calculated in absolute units and direct comparison to experiment is possible. Such a comparison for ZnSe interference filters is described in Section III where we found a satisfactory agreement. We also show that with this model, the qualitative features of the output beam shape can be understood. Finally, in Section IV, we point out some possible generalization of the present model.

II. Theoretical Description

We consider a thin absorbing layer, evaporated on a semi-infinite slab of a nonabsorbing substrate. A CW laser beam with input power P_{in} propagates through the layer where it is partially absorbed. In the following, we determine the steady-state temperature rise due to this absorption.

Manuscript received March 3, 1985; revised March 13, 1985. This work was supported in part by the U.K. SERC Joint Opto-Electronic Research Scheme and DCVD Funding Contract 2195/014 and was performed within the framework of the European Joint Optical Bistability Project (EJOB) of the Commission of the European Communities.

I. Janossy is with the Central Research Institute for Physics, H-1525 Budapest 114, Hungary, on leave at the Department of Physics, Heriot-Watt University, Riccarton, Edinburgh EH14 4AS, Scotland.

M. R. Taghizadeh, J. G. H. Mathew, and S. D. Smith are with the Department of Physics, Heriot-Watt University, Riccarton, Edinburgh EH14 4AS, Scotland.

0018-9197/85/0900-1447$01.00 © 1985 IEEE

The temperature rise in the layer is determined by the heat flow equation

$$\kappa_1 \nabla^2 T = -Q(r) \tag{1}$$

where κ_1 is the heat conductivity of the layer and Q is the quantity of heat absorbed/unit volume/unit time at the position $\boldsymbol{r}$. $Q(\boldsymbol{r})$ can be written as

$$Q(\boldsymbol{r}) = I(\boldsymbol{r})\, \alpha(T(\boldsymbol{r})) \tag{2}$$

where I is the irradiance at $\boldsymbol{r}$, and it is assumed that α is a function of the local temperature only.

The boundary conditions are that at the interfaces, the normal component of the heat current is continuous and far away from the absorbing spot, the temperature approaches the ambient temperature T_F. We neglect the heat transfer to the air, which implies that the normal component of the heat current at the absorbing–air interface ($z = 0$ plane) is zero.

We first consider the case when the heat conductivity of the layer κ_1 and that of the substrate κ are equal. Under such circumstances, the solution of (1) is [12]

$$T(\boldsymbol{r}) - T_F = \frac{1}{4\pi\kappa} \int_{r'} \frac{Q(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}\, d^3\, \boldsymbol{r}' \tag{3}$$

where the integration is over the whole irradiated spot and includes the imaginary "mirror sources" too, which are obtained by reflecting the real heat sources to the $z = 0$ plane.

In the limit when the layer thickness is much smaller than the laser spot size ($L << \rho_0$), there is no significant temperature gradient in the normal direction across the layer, so we can assume that $T(\boldsymbol{r}) \approx T(\boldsymbol{\rho})$ where $\boldsymbol{\rho}$ is the component of $\boldsymbol{r}$ in the $z = 0$ plane. Replacing, furthermore, $\boldsymbol{r} - \boldsymbol{r}'$ by $\boldsymbol{\rho} - \boldsymbol{\rho}'$ and neglecting the distance between the real and mirror sources, (3) becomes

$$T(\rho) - T_F = \frac{1}{2\pi\kappa} \int_{\boldsymbol{\rho}'} \frac{I_{\text{in}}(\rho')\, A(T(\rho'))}{|\boldsymbol{\rho} - \boldsymbol{\rho}'|}\, d^2\, \rho' \tag{4}$$

where the integration is over the $z = 0$ plane and $I_{\text{in}}(\rho')$ is the irradiance of the incoming beam at ρ'.

As explained in the Introduction, $A(T)$ gives the fraction of the input intensity which is absorbed in the layer at a fixed temperature T. If the optical parameters of the layer (α and n) and the properties of the interfaces are known, $A(T)$ can be calculated, e.g., there are no reflections at the interfaces, then

$$A(T) = 1 - e^{-\alpha L}.$$

For a Fabry–Perot resonator with finesse F,

$$A(T) = \frac{A_0}{1 + F \sin^2 \Phi}$$

where A_0 and F depend on the αL product; $2\Phi = 4\pi L n/\lambda$ (λ wavelength of light) is the cavity roundtrip phase change in the resonator. As can be seen, in the first case, $A(T)$ is determined only by α, while in the second case, the refractive index is important too, and it can contribute significantly to the temperature dependence of the absorbance.

We note further that $A(T)$ can be determined experimentally as well by measuring the transmission and reflection coefficients of the layer (τ and R) at sufficiently low laser powers as a function of T. A is given by

$$A = 1 - \tau - R. \tag{5}$$

In the following, we consider a Gaussian input beam, i.e.,

$$I_{\text{in}}(\rho) = \frac{P_{\text{in}}}{\pi\rho_0^2}\, e^{-\rho^2/\rho_0^2}.$$

Using the dimensionless variables $\boldsymbol{\Omega} = \boldsymbol{\rho}/\rho_0$ and $\boldsymbol{\Omega}' = \boldsymbol{\rho}'/\rho_0$, (4) can be rewritten as

$$T(\boldsymbol{\Omega}) - T_F = \frac{P_{\text{in}}}{2\pi\kappa\rho_0} \int_{\boldsymbol{\Omega}'} \frac{e^{-\Omega'^2}}{|\boldsymbol{\Omega} - \boldsymbol{\Omega}'|}\, A(T(\boldsymbol{\Omega}))\, d^2\, \boldsymbol{\Omega}'. \tag{6}$$

Equation (6) represents a nonlinear integral equation for the temperature. It can be solved, e.g., by iteration. As a first approximation, in the integral we may consider a constant temperature within the spot and replace $T(\boldsymbol{\Omega}')$ by $T_S = T(0)$ ("spot temperature"). In this approximation, from self consistency we get

$$T_S - T_F = \frac{P_{\text{in}}}{2\kappa\rho_0\sqrt{\pi}}\, A(T_S). \tag{7}$$

This equation is equivalent to the one used in a number of papers [2]–[4], [13] where the assumption was made that the heat loss from the illuminated spot is proportional to the temperature rise at the center of the spot. Here we presented a more solid foundation of this assumption, and we related the proportionality factor to the spot size and the heat conductivity.

The solutions of (7) can be found graphically in the same way as used in other cases of bistability [10]. If the curve $A(T)$ has an inflexion point in the region where it is increasing, then in a certain range of input power and below a certain value of T_F there are three intersections of $A(T)$ and the straight line $(2\kappa\rho_0\sqrt{\pi}/P_{\text{in}})\,(T - T_F)$. This is the power range where we have optical bistability. Two of the intersections correspond to stable solutions of (7), determining the temperature rises in the "on" and "off" states, respectively.

Using the same approximation as before, i.e., replacing $A(T)$ by $A(T_S)$ on the RHS of (6), a first approximation of the temperature profile can be calculated. We note that in this approximation, the shape of the profile is independent of the specific form of $A(T)$. We present it in Fig. 1 where, for comparison, the Gaussian beam profile is shown as well. From the fact that the temperature drops much less steeply than the intensity, we believe that even this first approximation is a good qualitative description.

The second approximation could be obtained by using on the RHS of (6) the temperature distribution given in

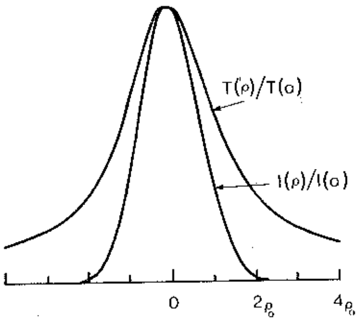

Fig. 1. Calculated temperature profile due to Gaussian input beam.

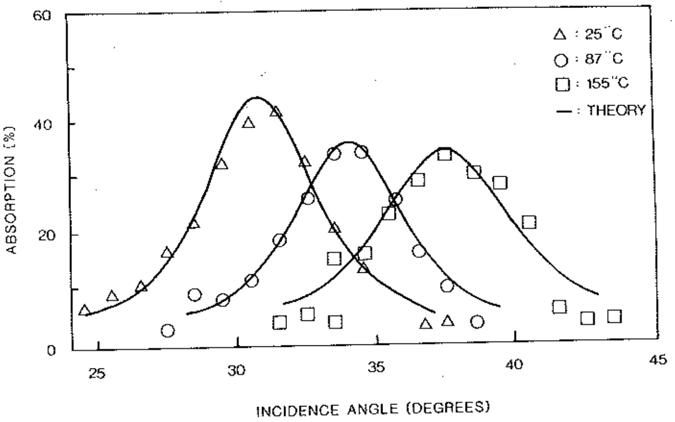

Fig. 2. Absorption as a function of incident angle at different temperatures. Solid line indicates a Lorentzian fitting.

Fig. 1. In the present paper, we only investigate the first approximation. We note, however, a general property of the solution of (6): the temperature rise at any point **(Ω)** depends only on the combination $P_{in}/\kappa\rho_0$. From this fact, it follows that the power necessary to produce a given spot temperature, and consequently the critical powers for the bistable range, is proportional to the heat conductivity and the spot size ($P_{cr} \sim \kappa\rho_0$).

Up to now we have assumed that the heat conductivities of the absorbing layer (κ_1) and the substrate (κ) are equal. The case when κ_1 and κ differ is more complicated, and we plan to investigate it in a further publication. However, in the limit of very thin absorbing layers, we think that it is good approximation to neglect the heat conduction within the layer. In this case, (6) is still valid, κ representing the heat conductivity of the *substrate*. A simple consideration shows that the approximation is justified for spot sizes larger than $(\kappa_1/\kappa)\, L$.

In the next section, we apply the above model to ZnSe interference filters.

III. Application to ZnSe Interference Filters

In a recent publication [8], we described thermally induced optical bistability in ZnSe interference filters. Here we present further experimental results and compare these to our theoretical model.

The structure of the filter was described in [8]. The theoretical determination of the absorbance of the filter is a rather difficult task due to its complex structure and uncertainties in the material parameters. In view of this fact, we determined experimentally the reflection and transmission coefficients of our sample as a function of temperature. For this measurement, we used very low laser power in order to avoid any heating by the beam.

We found that, at least near the transmission peak, both the transmission coefficient τ and the reflection coefficient R can be described by the standard Fabry–Perot relation for high-finesse cavities:

$$\tau = \frac{\tau_0}{1 + G(\theta - \theta_0)^2} \qquad 1 - R = \frac{1 - R_0}{1 + G(\theta - \theta_0)^2}$$

where θ is the angle of incidence of the laser beam, θ_0 is the angle of maximum transmission, and G is the half width. From the relation $A = 1 - \tau - R$, it follows that

$$A = \frac{A_0}{1 + G(\theta - \theta_0)^2}. \tag{8}$$

In Fig. 2, we show some experimental curves measured at different temperatures. The temperature dependences of A_0, τ_0, and θ_0 are shown in Fig. 3; these can be regarded as linear functions of T in the temperature range investigated. Although we do not make any attempt here to relate the temperature variations of these parameters to material properties, we note that while the temperature dependences of A_0, G, and τ_0 are connected first of all with $d\alpha/dT$, the temperature shift of the peak θ_0 is a consequence of the increase of the refractive index. Knowing the temperature dependence of A, it is straightforward to calculate the spot temperature as a function of the input power P_{in} on the basis of the approximation given in Section II [see (7)]. The transmitted power (P_T) can be calculated by integrating the transmitted intensity over the spot

$$P_T = \int_{\boldsymbol{\rho}} I_{in}(\rho)\, \tau(T(\rho))\, d^2\boldsymbol{\rho}$$

where, for $T(\rho)$, the temperature profile of Fig. 1 can be used.

The results for the ZnSe filter are shown in Fig. 4. The milliwatt scale on the x axis of the calculated curves correspond to $\rho_0 = 60\ \mu$m and $\kappa = 9 \times 10^{-3}$ W/°C · cm (heat conductivity of glass). Experimental curves are presented for comparison. The similarity between the shapes of the calculated and experimental curves is obvious; furthermore, the present model provides the correct order of magnitude of the critical powers of bistability.

As mentioned in Section II, the model predicts a linear dependence of the switching powers on the spot size. In Fig. 5, the experimental verification of this relation is presented. This linear dependence is different from what is expected in the case of diffusive electronic nonlinearities where at large spot sizes $P_{cr} \sim \rho_0^2$ [14]. The linear relationship in the present case can be taken as an evidence of the thermal origin of the nonlinearity.

In a further experiment, we investigated the profile of the outcoming beam. The layer was imaged on the detec-

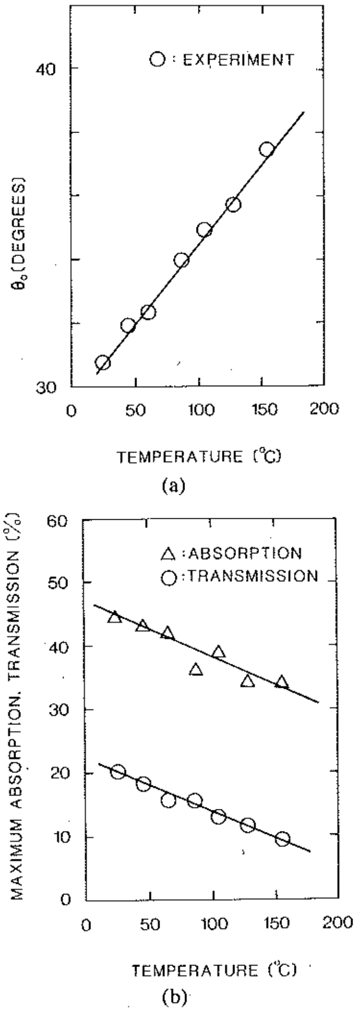

Fig. 3. (a) Maximum transmission angle (θ_0) as a function of temperature. (b) Maximum absorption and (A_0) transmission (τ_0) as a function of temperature.

tor so that the recorded beam profiles represented the intensity distribution at the layer–substrate interface. A first approximation for this distribution from our model can be obtained by multiplying the Gaussian input beam by the $\tau(T(r))$ transmission coefficient using the temperature profile presented in Fig. 1.

In Fig. 6, experimental (upper traces) and theoretical results (lower traces) are presented. The calculation reproduces the main features of the experimental beam profiles. At lower powers, the beam is nearly Gaussian; after switching to the "on" state, the edges become very steep and a local minimum appears at the center. The reason for this shape is the following. The transmission coefficient at a fixed θ is a Lorentzian-like function of T with a maximum at a certain T_0. In the "on" state, the temperature at the center is higher than T_0. Moving outwards from the center, the temperature drops; therefore, both τ and the transmitted intensity (τI) increase. Towards the edge of the beam, T becomes smaller than T_0. As both the input intensity and τ are decreasing in this region, the transmitted intensity drops very sharply.

Finally, we note that, as explained in Section II, the

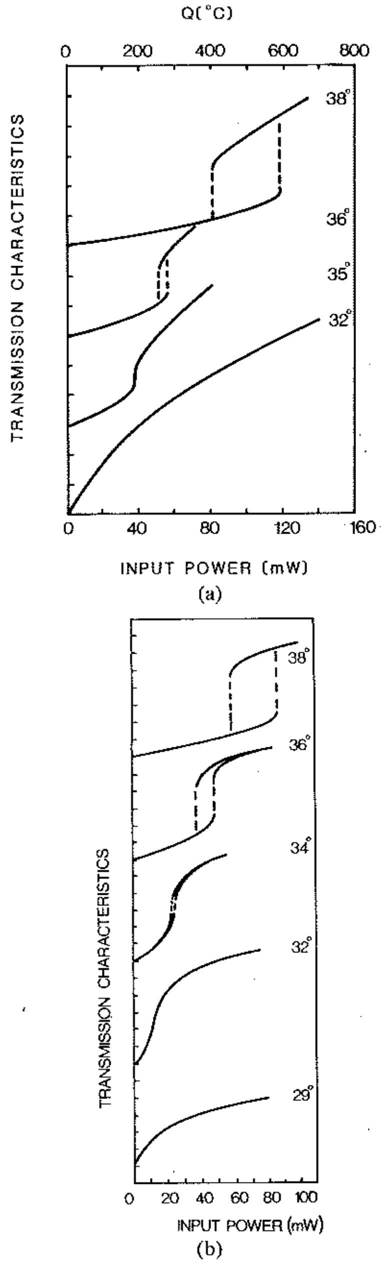

Fig. 4. (a) Theoretical characteristics of ZnSe filter for different detuning ($Q = P_{in}/2\sqrt{\pi}\, k\rho_0$). (b) Corresponding experimental results.

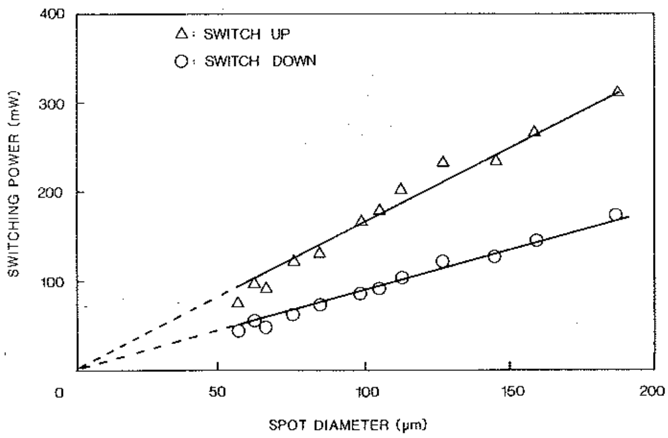

Fig. 5. Spot diameter dependence of switching powers.

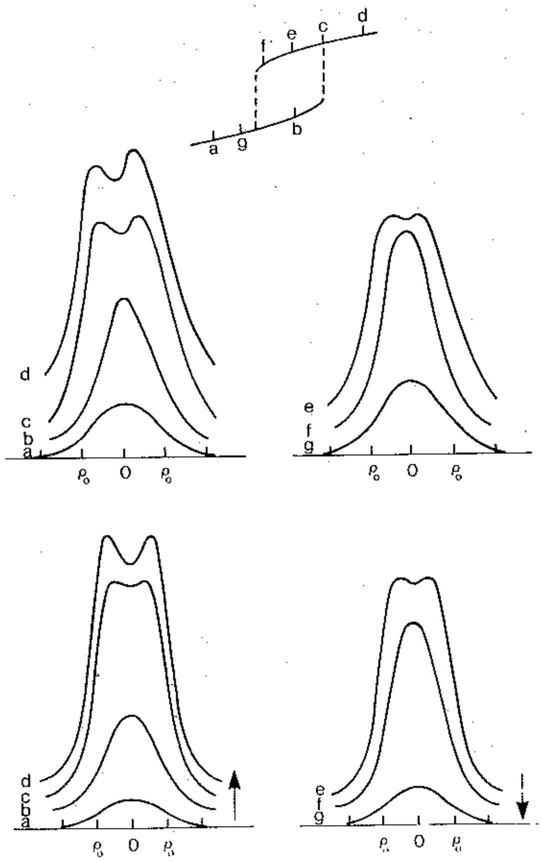

Fig. 6. Comparison between experimental observation and theoretical prediction of transmitted profile, $\theta = 38°$. Upper curves: experimental. Input powers: *a*: 17.5 mW, *b*: 31 mW, *c*: 39 mW, *d*: 51 mW, *e*: 31 mW, *f*: 23.5 mW, *g*: 21.5 mW. Lower curves: theory. Q (proportional to input power): *a*: 390°C, *b*: 782°C, *c*: 792°C, *d*: 942°C, *e*: 633°C, *f*: 511°C, *g*: 507°C.

model is only valid for spot sizes not smaller than $\kappa_1/\kappa \cdot L$. In the present case, $\kappa_{\text{ZnSe}}/\kappa_{\text{glass}} \approx 20$ and $L \approx 1\ \mu$m. This means that for spot sizes smaller than 20 μm, the present model is not applicable. In particular, we do not expect that the observed linear spot size dependence should hold for very small spots.

IV. Conclusions

The model presented in this paper provides a simple description of thermally induced optical bistability. It applies to thin absorbing layers, evaporated onto a thick substrate. Experimental results on ZnSe interference filters are in good agreement with the model.

The theoretical description given in Section II can be readily extended to include the interaction of two or more spots, e.g., for two spots, the equations corresponding to (7) of Section II are

$$T_S^{(1)} - T_F = \frac{P_{\text{in}}}{2\sqrt{\pi}\,\kappa\rho_0}\,(A(T_S^{(1)}) + \nu A(T_S^{(2)}))$$

$$T_S^{(2)} - T_F = \frac{P_{\text{in}}}{2\sqrt{\pi}\,\kappa\rho_0}\,(A(T_S^{(2)}) + \nu A(T_S^{(1)}))$$

where $T_S^{(1)}$ and $T_S^{(2)}$ denote the spot temperature at the center of the first and second spot, respectively; it is assumed that the spot size and input power are the same for the two spots. ν is a coupling constant which can be given as

$$\nu = \frac{1}{\sqrt{\pi}} \int_{\Omega'} \frac{e^{-\Omega'^2}}{|\boldsymbol{\Omega}' - \boldsymbol{R}|}\, d\boldsymbol{\Omega}'$$

where $\boldsymbol{R}$ is the vector between the centers of the spots. For large separations, ν is proportional to $1/R$.

A further possible generalization of the model is to include transient phenomena. In this case, the mathematical problem is much more complicated as the temperature at a given position and time depends not only on the spatial distribution of the temperature at that moment, but on the previous history of the spot also. We plan to discuss these further generalizations in a future paper.

Acknowledgment

We are grateful to Dr. B. S. Wherrett and Dr. E. Abraham for many useful discussions.

References

[1] E. M. Epshtein, "Optical thermal breakdown of a semiconductor plate," *Sov. Phys. Tech. Phys.*, vol. 23, pp. 983–985, 1978.

[2] J. Hajto and I. Janossy, "Optical bistability observed in amorphous semiconductor films," *Phil. Mag. B*, vol. 47, pp. 347–366, 1983.

[3] D. A. B. Miller, A. C. Gossard, and W. Weigmann, "Optical bistability due to increasing absorption," *Opt. Lett.*, vol. 9, pp. 162–164, 1984.
[4] B. S. Wherrett, F. A. P. Tooley, and S. D. Smith, "Absorption switching and bistability in InSb," *Opt. Commun.*, vol. 52, pp. 301–306, 1984.
[5] M. R. Taghizadeh, I. Janossy, and S. D. Smith, "Optical bistability in bulk ZnSe due to increasing absorption and self-focusing," *Appl. Phys. Lett.*, vol. 46, pp. 331–333, 1985.
[6] M. Dagenais and W. F. Sharfin, "Cavityless optical bistability due to light induced absorption in cadmium sulfide," *Appl. Phys. Lett.*, vol. 45, pp. 210–212, 1984.
[7] F. V. Karpushko and G. V. Sinitsyn, "The anomalous nonlinearity and optical bistability in thin-film interference structures," *Appl. Phys. B*, vol. 28, p. 137, 1982.
[8] S. D. Smith, J. G. H. Mathew, M. R. Taghizadeh, A. C. Walker, B. S. Wherrett, and A. Hendry, "Room temperature, visible wavelength optical bistability in ZnSe interference filters," *Opt. Commun.*, vol. 51, pp. 357–362, 1984.
[9] G. R. Olbright, N. Peyghambarian, H. M. Gibbs, H. A. Macleod, and F. Van Milligen, "Microsecond room-temperature optical bistability and crosstalk studies in ZnS and ZnSe interference filters with visible light and milliwatt powers," *Appl. Phys. Lett.*, vol. 45, pp. 1031–1033, 1984.
[10] cf. E. Abraham and S. D. Smith, "Optical bistability and related devices," *Rep. Progr. Phys.*, vol. 45, pp. 815–885, 1982.
[11] N. N. Rozanov, "Hysterisis phenomena in distributed optical systems," *Sov. Phys. JETP*, vol. 53, pp. 47–53, 1981.
[12] The analogous electrostatic problem is treated in, e.g., B. I. Bleaney and B. Bleaney, *Electricity and Magnetism*. Oxford, England: Clarendon, 1957, p. 53.
[13] D. A. B. Miller, "Theory of optical bistability and differential gain due to absorption increasing with excitation," *J. Opt. Soc. Amer. B*, vol. 1, pp. 857–864, 1984.
[14] W. J. Firth, I. Galbraith, and E. M. Wright, "Diffusion and diffraction in dispersive optical bistability," *J. Opt. Soc. Amer. B*, vol. 2, pp. 1005–1009, 1985.

Istvan Janossy graduated in 1968 from the Eotvos Lonard University in Budapest, Hungary.

Since 1972 he has worked at the Central Research Institute for Physics of the Hungarian Academy of Sciences. He has studied electro-optical properties and instabilities in liquid crystals. In the recent years he has been investigating laser-induced effects in liquid crystals and amorphous semiconductors. Presently he is a visitor at the Physics Department, Heriot-Watt University, where he is participating in the optical bistability research.

Mohammad R. Taghizadeh received the B.Sc. degree in physics from Mashad University, Iran, in 1977, the M.Sc. degree in optoelectronics from Essex University in 1978, and the Ph.D. degree from Heriot-Watt University, Edinburgh, Scotland, in 1982, where he was working on high power tunable mid-infrared lasers.

He is currently a Research Associate at the Physics Department, Heriot-Watt University, where he is studying nonlinear optical properties of semiconductors, optical bistability, digital optical circuits, and optical computing.

J. Gordon H. Mathew, for a photograph and biography, see p. 99 of the January 1985 issue of this JOURNAL.

S. Desmond Smith, for a photograph and biography, see p. 783 of the July 1985 issue of this JOURNAL.