THEORY OF OPTICAL BISTABILITY IN METAL MIRRORED FABRY-PEROT CAVITIES CONTAINING THERMO-OPTIC MATERIALS

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We present and analyse a new method for achieving all-optical bistability, in metal mirrored Fabry-Perot cavities containing thermo-optic material. It is shown how "butterfly" bistability is achieved in metal/metal mirrored cavities. Optimisation for low switching powers is considered for a dielectric/metal mirrored cavity. The use of nematic liquid crystals with such a system makes submilliwatt switching powers easily achievable.

1. Introduction

In a recent paper [1] we discussed the optimisation of nonlinear narrow-pass band interference filters for low power optical bistability. One of the conclusions of this work was that the source of heating of the central spacer of any nonlinear Fabry-Perot does not need to be absorption in the spacer material itself. It is necessary only that the absorbing region experiences the optical feedback of the cavity. It is perhaps intuitively obvious, and we have confirmed by numerical calculations for nonlinear multilayer dielectric filters, that the optimum position for the absorber is at the back reflective surface of the cavity. This gives a direct-contact thermal source whilst avoiding the loss of finesse that occurs for high absorption internal to the cavity.

Very simple nonlinear cavities can therefore be constructed with thermo-optic materials as the spacers, in good thermal contact with thin metallic mirrors. We analyse here the optical responses of such cavities and of alternative structures with dielectric reflectors at the front face of the spacer and metallic back-face reflectors, fig. 1.

In the former case we obtain and explain a novel "butterfly"-bistability. Experimental observations of the butterfly are reported in the accompanying power

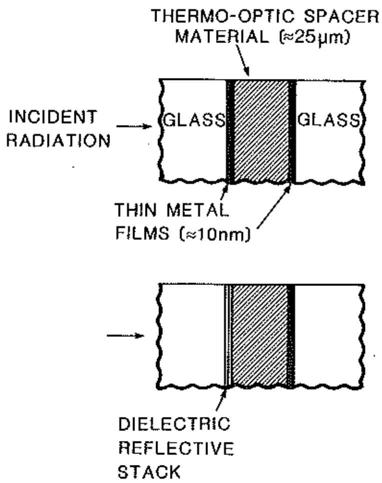

Fig. 1. Schematic of the nonlinear Fabry-perot schemes analysed in this paper.

[2], for simple, liquid spacer-materials between metal mirrors. For dielectric/metal cavities we show that lower power bistability than observed in dielectric-/dielectric systems is to be expected for optimised cases. Ref. [2] reports on submilliwatt bistability (of the conventional anti-clockwise hysteresis-loop variety) in such a dielectric material cavity, containing a nematic liquid crystal. The analyses given here apply equally to semiconductor or other solid spacers and to liquid media.

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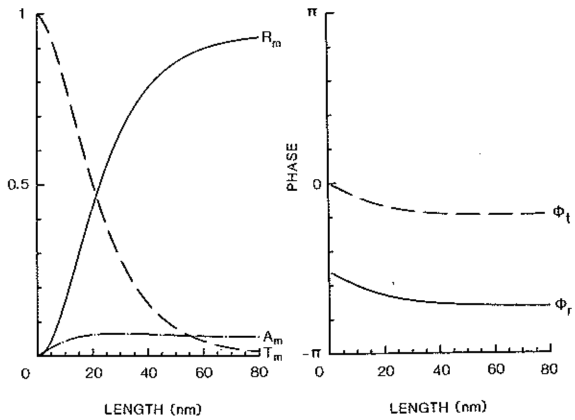

Fig. 2. Theoretical optical properties of a gold film; ($n=0.12$, $\kappa=3.29$). (a) Transmissivity T_M, reflectivity R_M, and absorptance A_M. (b) The phase changes for transmission ϕ_t and reflection ϕ_r. Plots are functions of the film thickness, the film is surrounded by two media both of refractive index 1.5.

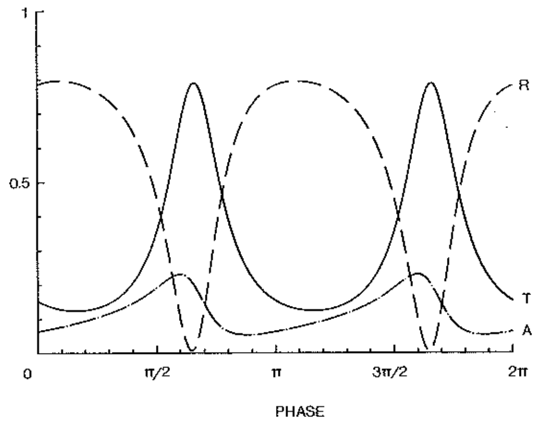

Fig. 3. Transmissivity (full line), reflectivity (dashed line) and absorptance (chain line) of a Fabry-Perot etalon consisting of a spacer of refractive index 1.5, sandwiched between the 20 nm gold coating on glass substrates (see fig. 1). Plots are functions of the spacer, single-path phase change.

2. Optical bistability in metal mirrored cavities

The reflectivity at a dielectric/metal interface is influenced by the refractive indices of the two media and by the metal extinction coefficient. For a thin metal film one obtains partial reflection, partial transmission, and of course absorption; these parameters, and the phase change on reflection and transmission, all depend on the film thickness. They are plotted in fig. 2, for the optical constants appropriate to gold at 633 nm [3] (assuming perfect quality films). The calculations are for a liquid/metal/glass-substrate partial mirror.

For a Fabry-Perot etalon constructed from two such mirrors we plot in fig. 3 the cavity-phase dependence of the optical transmissivity, reflectivity and absorptance, as given by the formulae

$$T=T_M^2(1-R_M)^{-2} \times[1+F\sin^2(\phi-\phi_r)]^{-1}\,, \tag{1}$$

$$R=R_M\{(1+T_M-R_M)^2-4[T_M\sin^2(\phi+\phi_t) -R_M\sin^2(\phi+\phi_r)-R_M T_M\sin^2(\phi_r-\phi_t)]\} \times\{(1-R_M)^2[1+F\sin^2(\phi+\phi_r)]\}^{-1}\,, \tag{2}$$

$$A=1-T-R\,. \tag{3}$$

R_M, T_M refer to the reflectivity and transmissivity of the metallic films, ϕ_r and ϕ_t are the corresponding phase changes; $F=4R_M(1-R_M)^{-2}$. ϕ is the single-pass phase-change across the cavity. At frequency ω and spacer refractive index n:

$$\phi=\omega nD/c\,. \tag{4}$$

Note from fig. 3 that the peak transmissivity, minimum reflectivity and peak absorptance occur at different values of ϕ; this is in contrast to the case of a dielectric/dielectric Fabry-Perot. This is because when the *transmission* is on-resonance there is a phase shift of $2(\phi_t-\phi_r)$ between the directly reflected incident field and the field propagating in the reflected direction after multiple cavity transits. This phase shift is no longer either zero or π as in the case of non-absorbing mirrors, so the resultant reflected field is not on-resonance.

The absorption within the metal films causes a temperature rise across the thin cavity. Heat loss is primarily into the substrate and the temperature rise inside the cavity is essentially [4],

$$\Delta T\simeq(\partial T/\partial P_A)\,AP_0\,, \tag{5}$$

where P_0 is the incident power and $\partial T/\partial P_A$ is the temperature rise per unit power absorbed which depends on the cavity construction and spot size. For a cavity medium of thermo-optic coefficient $\partial n/\partial T$ the cavity phase change is

$$\Delta\phi \simeq \frac{\phi_0}{n}\frac{\partial n}{\partial T}\frac{\partial T}{\partial P_A} A(\phi_0+\Delta\phi)\, P_0 \,. \qquad (6)$$

The following results require only that $\Delta\phi/\phi_0$ be proportional to AP_0, and not on the details of the thermal modelling.

The device response is determined numerically by stepping the phase of the cavity $(\phi_0+\Delta\phi)$ from a chosen initial detuning ϕ_0. T and R are obtained from eqs. (1) and (2). Eq. (6) is used to determine that value of the incident power P_0 that is consistent with each $\Delta\phi$ [5]. Alternatively a graphical solution analogous to that used by Marburger and Felber [6] for the all-dielectric case can be employed. To demonstrate the latter we plot in fig. 4(a) the absorbance A, (i) in the modified Airy form given by eqs. (1)–(3), and (ii) from eq. (6), in the form

$$A=\frac{\Delta\phi}{\phi_0}\left(\frac{1}{n}\frac{\partial n}{\partial T}\frac{\partial T}{\partial P_A}\right)^{-1}\frac{1}{P_0}\,. \qquad (7)$$

We have considered the case for negative $\partial n/\partial T$ (appropriate to most liquids). As usual, bistability occurs for P_0 regions in which the straight lines (of gradient inversely proportional to P_0) intercepts the modified Airy curve at three ϕ-values. The *absorptance* hysteresis is of the conventional form, with switch-up to a position close to the absorptance peak and switch-down to the absorbance minimum, occurring at a lower power level. However because the transmissivity, reflectivity and absorptance extrema occur at different ϕ (different cavity temperatures) a switch to higher absorptance does not necessarily correspond to a switch to higher transmission, see fig. 4, and a "butterfly"-bistability in transmitted power can result. We note that the details of the nonlinear responses of metal/metal cavities will depend strongly on both the metal used and on the film thickness. In addition the absorptance peaks (fig. 3) are asymmetric and different critical switching powers occur for the same magnitudes but opposite signs of $\partial n/\partial T$.

Fig. 5 shows a comparison of calculation and experimental measurements for the case of an aluminium cell containing a 30 μm film of an alcohol (Glenfiddich malt whisky) [2]. The dotted portion of the steady-state theoretical response is unstable and one anticipates down-switching at both the power levels P_1 and P_2, as found experimentally. Note that in the transmission positive-gradient regions may be unstable, and negative-gradient regions stable in contrast to conventional wisdom. Also the central point at which the two bistable branches cross is degenerate only in so far as the transmitted power levels are equal on the two branches. That is, the absorptance levels and the system temperatures differ, one cannot jump from one branch to the other at this point.

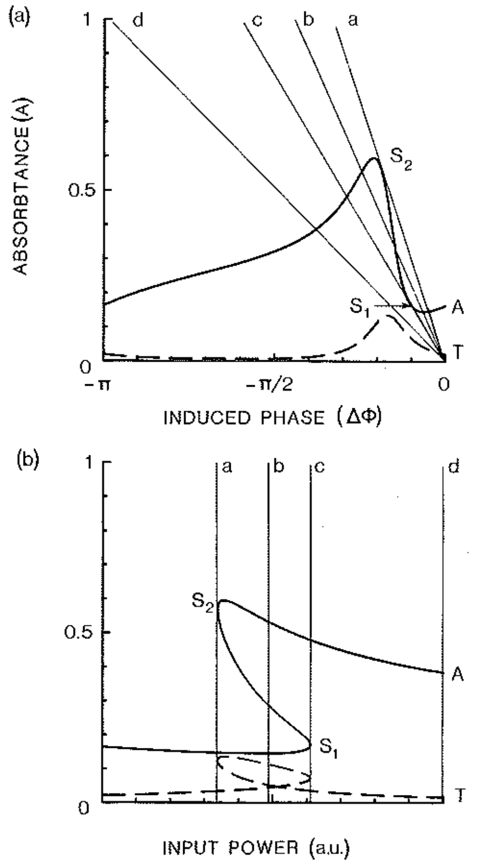

Fig. 4. Graphical analysis for a metal/metal nonlinear Fabry-Perot as used in the experiments of ref. [2]. (a) Phase dependences of the absorptance A, as given by eqs. (3) and (7). Spacer $n=1.5$, metal coatings consist of 10 nm of aluminium. The initial detuning ϕ_0 (modulo π) has been chosen as 0.05. The phase dependence of the transmissivity is shown as a dashed line, to indicate the shift in peak position with respect to the absorptance. (b) The points of intersection in (a) are the steady-state solutions for the absorptance A. These are plotted with emphasis on the four power levels a–d. The corresponding transmissivity is shown as the dashed line.

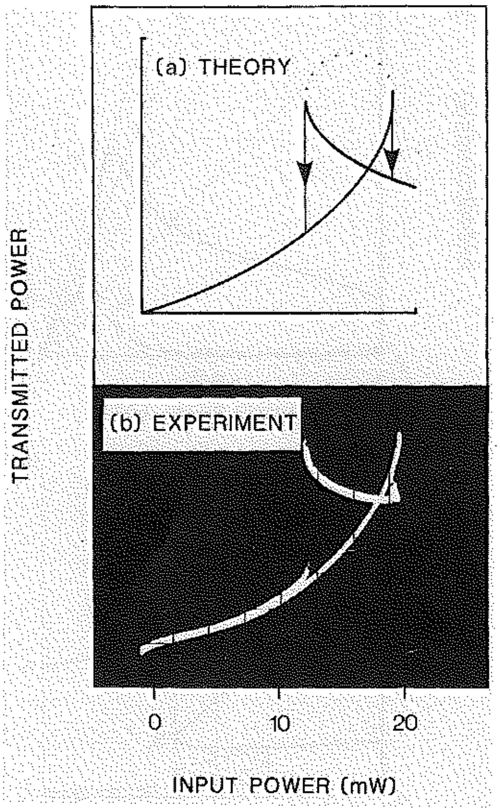

Fig. 5. (a) Theoretical "butterfly"-bistability in transmission (P_0T), obtained from the analysis shown in fig. 4. (b) Experimental "butterfly"-bistability observed for a cell containing an alcohol.

A similar shape of bistability can be achieved in a cavity containing a material that has both nonlinear absorption and refraction; e.g. dispersive bistability in the presence of increasing absorption [7–9]. In the present example only the refraction need be linear.

3. Optimisation of dielectric/metal cavities

One regains the conventional anti-clockwise hysteresis loop in transmission if the front partial mirror is replaced by a dielectric (non-absorbing) reflective stack. Having in mind arrays of bistable elements for optical 2-D information processing, it will be necessary to achieve 1–100 μW switching power levels in order to realise the parallelism that optics offers in principle, at moderate total power levels [10]. With the narrow-band pass filters presently used for cw thermally induced optical bistability, power levels are currently 1–100 mW [11]. Spot-size reduction towards diffraction limits and use of very high quality cavities may in principle produce the desired switching levels. There is however a restriction on the spacer absorption if the cavity is to be optimised, as there must be sufficient absorption to create the required temperature rise, but not too much absorption such that the cavity finesse is lost. This restriction places limitations on the spacer materials that can be used in conjunction with any given radiation wavelength.

In the following we compare optimisation of a dielectric/metal device, with that of the dielectric/dielectric case.

We will consider a form of the general dielectric-/metal case where we assume the metal is thick enough so that effectively the entire transmission of the Fabry-Perot is absorbed and contributes to the heating of the device. Such a device would obviously be used in reflection but the optimisation procedure will give a scaling for the more general case.

With the absorptance now given by $1-R$ the relevant cavity phase change is

$$\Delta\phi \simeq \frac{\phi_0}{n}\frac{\partial n}{\partial T}\frac{\partial T}{\partial P_A}[1-R(\phi_0+\Delta\phi)]P_0 , \tag{8}$$

$$1-R(\phi_0+\Delta\phi_0) = \frac{(1-R_F)(1-R_B\,e^{2\alpha D})}{(1-R_\alpha)^2[1+F\sin^2(\phi_0+\Delta\phi)]} \tag{9}$$

where

$$R_\alpha = (R_F R_B)^{1/2}\,e^{-\alpha D} , \quad F = 4R_\alpha/(1-R_\alpha)^2 .$$

Once again by rewriting (8) as a function of $1-R$ we can form a graphical solution for the input/output characteristics where a straight line (eq. (8)) intersects an Airy function (eq. (9)). Using a similar analysis to Miller [12] we equate the first derivatives for switching and the second derivative for critical switching, to obtain the following expression for the critical switching power P_c (the lowest power at which bistability is possible).

$$P_c \simeq \frac{\lambda}{D}\frac{1}{|\partial n/\partial T|\;\partial T/\partial P_A} g(R_F, R_B, e^{-\alpha D}) , \tag{10}$$

where

$$g=\frac{\sqrt{2}}{16}\frac{(1-R_\alpha)^2}{(1-R_F)(1-R_B\,e^{-2\alpha D})}H(F)\ ,$$

$$H(F)=\{3(F+2)-[(3F+2)^2-8F]^{1/2}\}^2$$

$$\times\{(F+2)[(3F+2)^2-8F]^{1/2}$$

$$-(F+2)^2-2F^2\}^{-1/2}\ .$$

This expression is of a similar form for the case where the absorption is confined to the spacer layer only [1],

$$P_c\simeq\frac{\lambda}{D}\frac{1}{|\partial n/\partial T|\ \partial T/\partial P_A}f(R_F,R_B,e^{-\alpha D})\ , \qquad (11)$$

where

$$f=\frac{\sqrt{2}}{16}\frac{(1-R_\alpha)^2}{(1-R_F)(1+R_B\,e^{-\alpha D})(1-e^{-\alpha D})}H(F)\ .$$

For comparison we plot the cavity factors f and g in fig. 6, for various pairs of reflectivity, as a function of the cavity absorption, αD. It can be seen in the case of the cavity factor f there exists a minimum for each set of reflectivities which occurs at a specific αD which, in the high finesse approximation, is given by

$$\alpha D=(2-R_F-R_B)/4\ . \qquad (12)$$

Fore equal reflectivities, R_s, the minimum value of f is approximately $3\sqrt{3}(1-R_s)/2$. In contrast the minimum value of the cavity factor g occurs for zero absorption in the spacer. For equal reflectivities and in the high finesse approximation this minimum value is $4(1-R_s)/3\sqrt{3}$. Hence, providing all other factors are equal, we reduce the switching power by a factor of 8/27 by having all the absorption after the back mirror than in the spacer. It has already been established that for a dielectric/dielectric cavity the critical power is lower for a thick cavity of low absorption than for the converse, given the αD is set by condition (12) [1] and that this can be partly relaxed by compensating for lack of absorption by using metallic layer. That is, one requires only that $\alpha D<(2-R_F-R_B)/4$. For high 2-D uniformity one presently uses interference filters constructed by the vacuum thermal technique. This method restricts the spacer thickness to a maximum of a few microns, to avoid mechanical instability. For 95% reflectivity stacks the required αD is 0.025 so that α values in excess of 500 cm^{-1} are typically needed. Thus one must work with a semiconductor of absorption edge close to the operating frequency, for a dielectric/dielectric cavity. Equally if thick samples are to be used (given the spatial uniformity) materials of specific α values must be matched to the thickness. In the dielectric/metal case effectively non-absorbing spacer materials may be used. The operational wavelength/s of the device are then determined entirely by the cavity resonance positions.

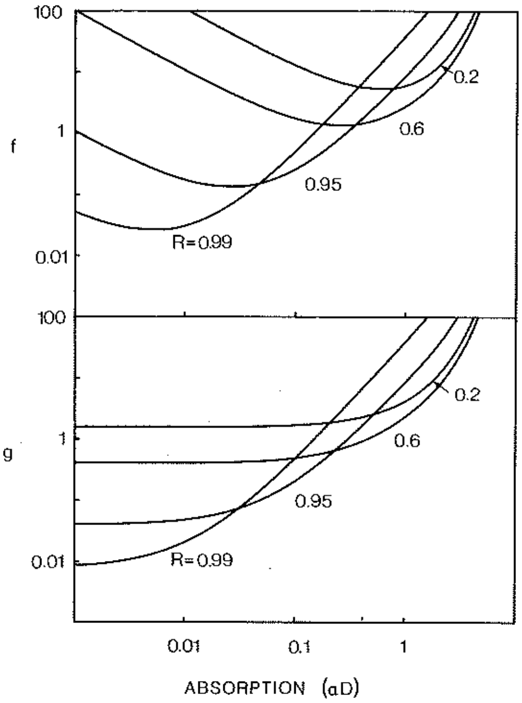

Fig. 6. The αD-dependence of the cavity factors f and g (see text) for the reflectivities indicated.

Because of the potentially high thermo-optic coefficient of nematic liquid crystals [13] we take as an example a non-absorbing nematic in a cell of 95% front-face (dielectric) reflectivity and 95% back-face (metal) reflectivity. The latter is achieved at a liquid-to-gold interface.

This design has a cavity factor $g\simeq 0.04$. For $r_0\simeq D$ and $\kappa_{sp}\approx\kappa_s/5$, where κ_{sp} and κ_s are the thermal conductivities of the spacer and substrate materials respectively, one obtains from a thermal analysis,

$$\partial T/\partial P_A \approx 0.15/D\kappa_s \ .$$

Using $\partial n/\partial T \approx -2\times10^{-3}$ K^{-1}, $\lambda_v = 633$ nm and $\kappa_s \approx 0.01$ W K^{-1} cm^{-1} one obtains

$$P_c \approx 80\ \mu\text{W} \ .$$

The results reported in the accompanying experimental paper, with $R_F \approx 95\%$, $R_B \approx 85\%$, lead to an observed critical power of 200 μW (220 μW predicted).

Use of reflected rather than transmitted signals from a nonlinear Fabry-Perot has an advantage, pointed out previously [5], that higher signal differences between switch-ON and switch-OFF are achievable. The value of signal difference depends on the width of the hysteresis loop used, and is limited by the ON and OFF resonance cavity response:

$$R_D = 4R_\alpha(1-R_F)(1-R_B\,e^{-2\alpha D})/(1-R_\alpha^2) \ . \qquad (13)$$

For given R_B, R_D is a maximum under the condition

$$R_F = R_B\,e^{-2\alpha D} \ . \qquad (14)$$

This condition is satisfied for any non-absorbing cavity of equal front and back reflectivities. In the designed cavity with $R_F = R_B = 0.95$ a reflected signal difference very close to 100% is achievable. This is particularly appropriate for those logic or bistable devices in which a large fan-out is required.

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References

[1] B.S. Wherrett, D. Hutchings and D. Russell, J. Opt. Soc. Am. 3 (1986) 351.
[2] A.D. Lloyd, I. Janossy, H.A. MacKenzie and B.S. Wherrett, Optics Comm. 61 (1987)
[3] Handbook of Optical Constants of Solids (Academic Press, 1985).
[4] I. Janossy, M.R. Taghizadeh, J.G.H. Mathew and S.D. Smith, IEEE J. Quantum Electron. QE-21 (1985) 1447.
[5] B.S. Wherrett, J. Quantum Electron. QE-20 (1984) 646.
[6] J.H. Marburger and F.S. Felber, Phys. Rev. A17 (1978) 335.
[7] B.S. Wherrett, F.A.P. Tooley and S.D. Smith, Optics Comm. 52 (1984) 301.
[8] A.C. Walker, J.S. Aitchison, S. Ritchie and P.M. Rodgers, Electron. Lett. 22 (1986) 366.
[9] H. Haug, private communication.
[10] B.S. Wherrett, Proc. Erice Summer School on Nonlinear optics: materials and devices, Proc. Phys., vol. 7 (Springer-Verlag, 1986) p. 180.
[11] Y.T. Chow, B.S. Wherrett, E.W. Van Stryland, B.T. McGuckin, D. Hutchings, J.G.H. Mathew, A. Miller and K. Lewis, J. Opt. Soc. Am. B, in press.
[12] D.A.B. Miller, IEEE J. Quantum. Electron. QE-17 (1981) 306.
[13] K.C. Chu, C.K. Chen and Y.R. Shen, Mol. Cryst. Liq. Cryst. 59 (1980) 97.

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