

High-precision measurement of azimuthal rotation of liquid crystals on solid substrates

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A high-accuracy optical method for the determination of the rotation of the liquid-crystal director on solid substrates is reported. The measurements involve the use of a photoelastic modulator in conjunction with a white light source. The detection of the first and second harmonics of the light signal allows for the determination both of the director position and the twist deformation at the exit face of the cell. The method is illustrated by experiments on the liquid crystal pentylcyanobiphenyl aligned by polyimide layers. The importance of director gliding is pointed out as well. © 2005 American Institute of Physics. [DOI: 10.1063/1.2031927]

I. INTRODUCTION

The study of alignment properties of liquid crystals on solid substrates, in particular, on polymer surfaces, is not only a challenging task for fundamental research, but it is also very important for device applications. Reliable technologies were developed to ensure proper orientation of nematic liquid crystals on the cell substrates with sufficient stability under the influence of applied fields or with time. The quantitative characterization of the anchoring properties is, however, sometimes a difficult task. One of the problems is to find consistent procedures to determine small rotations of the liquid crystal on the cell boundaries.

Several methods were proposed and used to determine the surface reorientation of the nematic liquid-crystal director. These methods, however, often give markedly different results, even when the same system is investigated. As an example, the reported values of the anchoring strength for azimuthal reorientation in planar nematic cells deviate by more than an order of magnitude. The anchoring strength is usually characterized by the extrapolation length, which is the ratio of surface rotation of the director to the director gradient at the interface.¹ Vilfan and Copic² investigated thermal fluctuations using light scattering and found $\xi = 450$ nm for the azimuthal extrapolation length of pentylcyanobiphenyl (5CB) on polyimide (PI). Zhang *et al.*³ analyzed the optical transmission curves in an electric field applied parallel to the substrates (in-plane switching). From the measurements they estimated $\xi \approx 100$ nm. Another method is to measure the deviation of the actual twist angle from the applied twist in planar cells using a polarizing microscope.^{4,5} This method, however, is not suitable to measure strong anchoring energies, i.e., extrapolation length well below 100 nm. Faetti and Nobili⁶ and Faetti⁷ developed procedures to measure such small extrapolation lengths. In their methods a magnetic field was used to control the twist deformation in the sample. The surface director rotation was measured by detecting either the change of the intensity of the reflected beam, or the polarization direction of the transmitted beam in

a wedge-shaped cell. With the latter method they found for 5CB on PI $\xi = 34$ nm. Clearly, further precise methods should be useful in clarifying the real anchoring properties of liquid crystals on solid substrates. The determination of the azimuthal anchoring strength is especially important for in-plane switching, which became recently a significant display technology.

In the present paper, we describe an optical method, which allows for measuring surface director rotations as small as a fraction of an arcminute. Our method is similar to that of Faetti and Nobili insofar as the twist deformation is induced by a magnetic field. To determine the director orientation at the substrate, a photoelastic modulator (PEM) is used in conjunction with a white light source. PEM is a very efficient device to analyze the polarization state of a light beam;⁸ some details are given in Sec. II. The purpose of applying white light instead of a laser beam is to avoid interference effects in the liquid-crystal cell (and also in the PEM itself), which can give spurious results.^{6,7} We show that the method is suitable not only for the precise measurement of the surface director orientation but also for the direct determination of the twist deformation at the interface. The extrapolation length can be therefore deduced from the data without additional measurements. We also show that the measured director orientation is determined by the alignment at the exit face of the cell alone; it is insensitive to the alignment at the entrance face. As a consequence, it is possible to compare the alignment properties of the two substrates of the cell independently. Finally, the method can be used for dynamic investigations on a time scale shorter than the bulk reorientation time of the director. This circumstance is useful in the study of the so-called gliding phenomenon, i.e., the slow drift of the surface orientation after the bulk equilibrium is attained.

II. THE PRINCIPLE OF PHOTOELASTIC MODULATOR MEASUREMENTS

The main part of PEM is a quartz window, which is periodically compressed and stretched with the help of a piezoelectric transducer, typically in the frequency range of 20-100 kHz. The mechanical stress induces an optical aniso-

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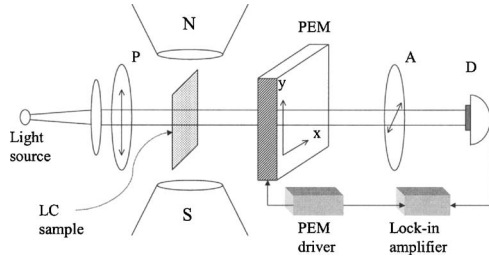


FIG. 1. Setup used in the experiments. P is the polarizer, A is the analyzer, D is the detector, and N and S are the poles of the electromagnet. The PEM and the analyzer are mounted on the same stage and can be rotated together.

trophy in the window, producing a periodical phase difference between the polarization components of a light beam passing through it. In the standard setup (Fig. 1), behind the PEM the light passes through an analyzer polarized at 45° with respect to the modulator axis. The first and second harmonic Fourier components of the detector signal are measured with a lock-in amplifier.

For a monochromatic light, the first harmonic signal (FHS) is proportional to $S_3 J_1(\lambda Ad)$, where S_3 is the Stokes parameter defined as

$$S_3 = i\langle E_x E_y^* - E_x^* E_y \rangle. \quad (1)$$

E_x and E_y are the electric-field strengths of the light beam along the modulator axis and in the perpendicular direction, respectively. J_1 is the first-order Bessel function, λ is the wavelength, A is the amplitude of the refractive index modulation, and d is the window thickness. We note that A can be regulated through the amplitude of the driving voltage of the PEM. The second harmonic signal (SHS) is proportional to $S_2 J_2(\lambda Ad)$, where S_2 is

$$S_2 = \langle E_x E_y^* + E_x^* E_y \rangle, \quad (2)$$

and J_2 is the second-order Bessel function. When the PEM and the analyzer are rotated together by an angle ψ , the Stokes parameters for the latter position of the setup, S'_2 and S'_3 , can be expressed with the help of S_2 and S_3 as

$$S'_2 = S_2 \cos 2\psi + S_1 \sin 2\psi, \quad (3)$$

$$S'_3 = S_3,$$

where S_1 is the Stokes parameter

$$S_1 = \langle E_x E_x^* - E_y E_y^* \rangle.$$

From the above relations it follows that for an elliptically polarized light $S_2=0$ when the principal axis of the polarization ellipsoid is along x . Therefore this direction can be found by rotating the PEM until the SHS becomes zero. We note that the SHS varies linearly with angle around the zero position [see Eq. (3)]. This fact makes possible very sensitive detection of changes in the direction of the principal axis. S_3 characterizes the ratio of the principal axes of the ellipsoid; for linear polarization it is zero.

For white light, the Stokes parameters for different wavelengths are added together. The FHS and SHS are proportional to the “effective” Stokes parameters, \hat{S}_3 and \hat{S}_2 , respectively, defined by the relations

$$\hat{S}_3 = \int_{\lambda} w(\lambda) S_3(\lambda) J_1(\lambda Ad) d\lambda \int_{\lambda} w(\lambda) J_1(\lambda Ad) d\lambda,$$

$$\hat{S}_2 = \int_{\lambda} w(\lambda) S_2(\lambda) J_2(\lambda Ad) d\lambda \int_{\lambda} w(\lambda) J_2(\lambda Ad) d\lambda,$$

where $w(\lambda)$ is the spectral density of the light source, multiplied by the detector sensitivity at the given wavelength. An effective Stokes parameter \hat{S}_1 can be defined as

$$\hat{S}_1 = \int_{\lambda} w(\lambda) S_1(\lambda) J_2(\lambda Ad) d\lambda \int_{\lambda} w(\lambda) J_2(\lambda Ad) d\lambda.$$

With this definition, the transformation equation [Eq. (3)] is valid also for the effective Stokes parameters.

III. DETERMINATION OF THE SURFACE DIRECTOR ORIENTATION AND TWIST DEFORMATION WITH PEM

The setup used for our method is depicted in Fig. 1. Similar to the experiments of Faetti and Nobili a uniformly twisted planar nematic cell is prepared, with the boundary conditions $\varphi(0)=\varphi_0$ and $\varphi(L)=\varphi_L$, where φ is the azimuthal angle of the director and L is the sample thickness. The sample is placed into an electromagnet, which can deform the initial director alignment. A polarized white light beam passes through the liquid-crystal cell and enters the PEM. The measurements consist of detecting the first and second harmonic signals as a function of the magnetic field.

First we consider light propagation through the liquid-crystal cell in the so-called Mauguin approximation,⁹ valid for the limit $d\varphi/dz \ll 1/\lambda$, where z is the direction along the cell normal. In this limit the light polarization follows the director “adiabatically,” so a beam polarized parallel to the director at the entrance face emerges from the cell polarized parallel to the director at the exit face. From the relations presented in Sec. II it follows that \hat{S}_2 is proportional to $\sin 2\varphi_L$, where φ_L is the director angle at the exit face, i.e., the angle between the polarization direction of the output light beam and the modulator axis. The polarization direction can be obtained by rotating the PEM unit until the SHS disappears. When the magnetic field is switched on SHS reappears, indicating a rotation of the director at the exit surface of the cell. The new position of the director can be found either by rotating the PEM until the SHS disappears again, or by calibrating the proportionality factor between SHS and $\sin 2\varphi_L$ through rotating the PEM by a known angle.

In the Mauguin limit one does not expect to obtain FHS, as \hat{S}_3 is zero for linearly polarized light. As it will be presented in Sec. IV, in the experiments we do observe FHS. In addition, in the above considerations we did not take into account the effect of a possible reorientation of the director at the entrance face. In order to deal with these problems, we consider a next approximation, which we will call generalized adiabatic propagation (GAP). The details of GAP are given in the Appendix; here we summarize the results.

In the absence of magnetic field the cell is uniformly twisted. There is an exact solution for light propagation in this configuration, given first by De Vries in 1951.¹⁰ There are four elliptically polarized normal modes; in the Mauguin limit they correspond to forward and backward propagating extraordinary and ordinary beams, respectively. The principal axes of the ellipse are parallel and perpendicular to the director. In the GAP limit we assume that these modes propagate independently, even when a nonuniform twist deformation is created by the magnetic field. According to this approximation, if only one of the normal modes is excited at the entrance face, the exit beam also corresponds to the same normal mode. It should be noted that a *linearly* polarized input does not correspond exactly to a single normal mode. For example, if the input light is polarized parallel to the director at the entrance face, beside the forward propagating *e* mode, a forward propagating *o* mode is excited with a certain amplitude as well. For monochromatic light, due to the interference between these modes, the major axis of the output polarization ellipsoid can deviate from the director. This effect can appear as a spurious director rotation.⁷

As pointed out in the Appendix, the above problem is avoided if a wide distribution of wavelengths is used. In this case the \hat{S}_2 Stokes parameter becomes zero when the director at the exit face is parallel to the modulator axis, at arbitrary input polarization. Consequently, the same procedure can be used to determine the director orientation as described earlier for the Mauguin limit. Director rotation at the entrance face has no influence on the method.

The GAP approximation also accounts for the observation of FHS. The normal modes are elliptically polarized hence the \hat{S}_3 Stokes parameter has a small but finite value. As we show in the Appendix, in the limit of slow spatial director variations it is proportional to the twist deformation at the exit boundary, i.e., to $\varphi'(L)$. According to this description, when twisted cells are used in the experiment FHS should be observed even without magnetic field, corresponding to the uniform twist deformation in the undistorted cell. This signal can be used to calibrate the relation between FHS and the twist deformation at the exit face.

To sum up, the director orientation at the exit surface can be obtained by determining the PEM position at which the SHS disappears. In the actual experiments, the compensation is done at zero magnetic field and then SHS is detected as a function of the magnetic field. For calibration of the angular rotation the PEM is rotated by a known angle at fixed magnetic-field strength and detecting the corresponding SHS. The twist deformation at the exit surface can be deduced from the FHS; the calibration factor can be obtained from the zero-field value of FHS, which corresponds to $(\varphi_L - \varphi_0)/L$.

IV. RESULTS AND DISCUSSION

In the experiments a Hinds Instruments photoelastic modulator was used with a resonance frequency of 42 kHz. The light source was a tungsten lamp. A filter was placed in front of the detector to cut off infrared light. The cells were prepared using commercially available plates, covered with

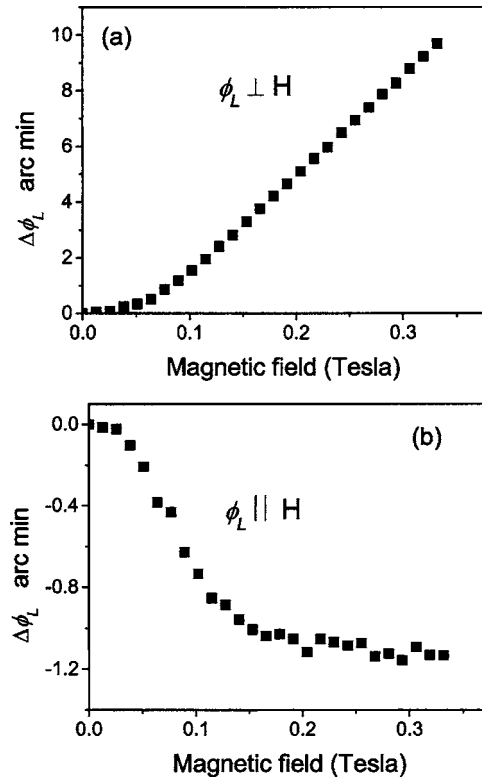


FIG. 2. Rotation of the director on the exit face as a function of the magnetic field. Director at exit face (a) perpendicular and (b) parallel to the magnetic field. $\Delta\phi_L = \phi_L(H) - \phi_L(0)$.

rubbed PI layers (E.H.C. Co., Japan). The nematic liquid crystal was 5CB, supplied by Merck Co. In the cell we applied a twist angle slightly smaller than 90° in order to avoid formation of domains with opposite twist directions. All measurements were carried out at 25°C .

As a first step of the measurements the PEM was rotated to a position where the SHS disappeared. In good agreement with the theoretical prediction, this position was insensitive to the direction of the input polarization; rotation of the input polarization by 10° caused about a $0.5'$ change in the zero position of the PEM. For calibration of the SHS the PEM was rotated by $20'$, with a precision of $1'$. The calibration was carried out without and with magnetic field; no significant difference was found between the two calibrations.

In Figs. 2 and 3, we present an example of the experimental results. The cell was $72\text{-}\mu\text{m}$ thick; the twist angle was 86° . In Fig. 2 the change of director orientation is displayed as a function of the magnetic field, as deduced from SHS measurements. The data in Fig. 2(a) were obtained with the director on the exit face approximately perpendicular to the magnetic field, while Fig. 2(b) shows the result when these directions were approximately parallel. The corresponding results for FHS are shown in Fig. 3 (squares and triangles, respectively).

In order to analyze the results, we note that according to the theoretical considerations, the FHS is proportional to the twist deformation at the exit face, $\varphi'(L)$. Using the Frank elastic theory of liquid crystals,¹¹ φ' can be readily obtained by solving the differential equation

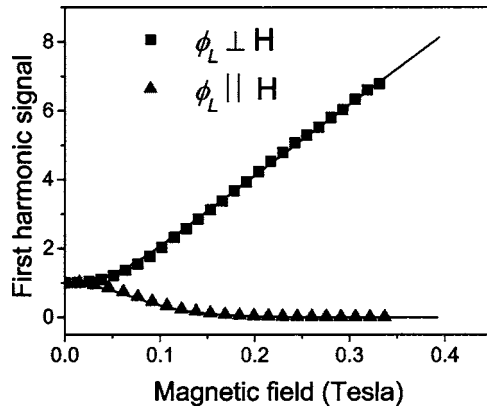


FIG. 3. Normalized FHS signal as a function of magnetic field with the director at the exit face perpendicular (squares) and parallel (triangles) to the field. Solid lines: theoretical fit with $\sqrt{K_2/\chi_a}=2.3 \times 10^{-6}$ T m.

$$K_2\varphi'' = \frac{1}{2}\chi_a H^2 \sin 2\varphi,$$

where K_2 is the twist elastic constant and χ_a is the anisotropy of the diamagnetic susceptibility. We define the magnetic-field direction as $\varphi=0$. The boundary conditions are $\varphi(0)=\varphi_0$ and $\varphi(L)=\varphi_L$. In Fig. 3, the solid lines corresponds to theoretical fits with $\varphi_0=4^\circ$ and $\varphi_L=90^\circ$ and $\varphi_0=-86^\circ$ and $\varphi_L=0^\circ$ for the fit of squares and triangles, respectively. [In the calculation we neglected the variation of φ_0 and φ_L with magnetic field, as it is less than $10'$ (see Fig. 2).] Both curves were fitted with $(K_2/\chi_a)^{1/2}=2.3 \times 10^{-6}$ T m. The fits are satisfactory, although the value of $(K_2/\chi_a)^{1/2}$ is about 15% higher than that reported in the literature.⁷ The deviation may be due to a small deviation between the assumed and actual director orientations at the substrates, relative to the magnetic field.

According to the accepted models of anchoring, the balance of two different surface torques determines the director orientation at the interface. The first torque is experienced by the liquid-crystal layer, it is equal to $K_2\varphi'(L)$. The second one is the due to the orienting action of the substrate; it can be given in the form $W(\varphi_e-\varphi_L)$, where W is an energy parameter characterizing the interface and φ_e is the azimuthal angle of the “easy axis,” i.e., the direction along which the liquid-crystal-substrate interaction energy is minimal. The director orientation is therefore

$$\varphi_L = \varphi_e - \xi\varphi'(L),$$

where $\xi=K_2/W$ is the extrapolation length. In Fig. 4 a direct proof of the above relation is presented, based on the data of Figs. 2(a) and 3. The absolute scale of the twist deformation was obtained by matching the zero-field value of the FHS with the initial twist of the cell, $(\varphi_L-\varphi_0)/L$. The extrapolation length is in this case 23 nm, which is a much lower value than those reported in Refs. 2 and 3, but it is compatible with the results of Faetti.⁷ We found that ξ changes to some extent across the substrate; it is the smallest at the center and increases towards the cell boundaries.

In Fig. 4 a small but systematic deviation from the predicted linear behavior can be observed. We believe that this

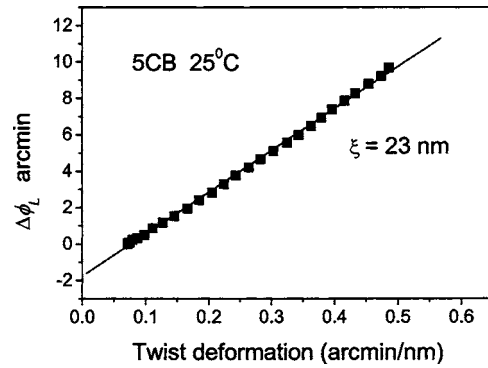


FIG. 4. Director rotation as a function of the twist deformation at the exit surface, field and director perpendicular. The curve is deduced from the data in Figs. 2(a) and 3.

divergence is due to director gliding,^{12–16} i.e., the drift of the director under constant applied field on time scales much longer than the bulk reorientation time.

The anchoring properties can be analyzed also for the geometry when the director at the exit face is parallel to the magnetic field. In this case we expect that the twist is removed by the field at the interface and as a consequence φ_L decreases from its initial value to φ_e . The experimental results are in good qualitative agreement with this expectation; the FHS decreases to zero (Fig. 3, triangles) while the variation of φ_L is significantly smaller and of opposite sign as in the other geometry. Quantitatively, however, these measurements are less accurate than the previous ones, as the measured signals are reduced by a factor of about 7. A similar evaluation of the extrapolation length as presented before yielded 15 nm. The deviation may be due to the fact that the measurements were not carried out on exactly the same spot, or to the influence of gliding on the experiment.

Finally, we discuss the limitations of the method. The measurement of the zero position of the SHS is limited by the thermal fluctuations of the liquid crystal, which introduces noise into the recorded signal. In steady-state experiments the noise can be reduced by time averaging; we found that with an integrating constant of a few seconds the fluctuations are in the order of a few tenths of an arcminute. As shown in the Appendix, the theoretical precision of the optical method for the present experimental circumstances is of similar magnitude. Therefore the accuracy of determining φ_L is less than an arcminute, which corresponds in our experiments to an error in the extrapolation length below 2 nm. In addition, the calibration procedures (magnetic field, rotation angle) and the imperfect determination of the alignment of the sample relative to the magnetic field may introduce systematic errors, which we estimate to cause a 10%–15% error in the evaluation of the correlation length for our experimental circumstances. In the case of thin samples or very strong magnetic fields, the proportionality between the FHS and $\varphi'(L)$ breaks down which has to be taken into account when applying the present method.

V. CONCLUSIONS

We presented a procedure for the precise determination of the azimuthal director orientation at a liquid-crystal-solid

substrate interface. Using this method changes of less than an arcminute in the director orientation can be reliably detected. In addition, the method allows for direct determination of the twist deformation at the interface, thus the extrapolation length can be calculated without any supplementary measurement. The procedure makes possible also dynamic measurements on time scales shorter than the bulk reorientation of the liquid-crystal layer. This fact is especially useful in the study of gliding, which in our opinion plays a much more significant role in anchoring properties than usually anticipated, even in the case of strong anchoring.

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APPENDIX

We consider a planar nematic liquid-crystal layer, in which the azimuthal director angle (φ) changes along the direction normal to the cell boundaries (z direction) while it is constant in the xy plane. We discuss the properties of a light beam propagating along the cell normal.

A general method to calculate the propagation of a monochromatic light beam in stratified liquid-crystal layers were given by Berremann and Scheffer¹⁷ and by Allia *et*

*al.*¹⁸ Following their approach, we introduce a four-component field vector $\mathbf{F}=[E_\xi, E_\eta, H_\eta, H_\xi]$, where \mathbf{E} and \mathbf{H} are the electric- and magnetic-field vectors; ξ and η are the axes in the xy plane parallel and perpendicular to the director, respectively. The field vector obeys the linear differential equation

$$\mathbf{F}' = ik_0 \mathbf{V} \mathbf{F}, \quad (\text{A1})$$

where the prime stands for derivation with respect to z , and $k_0 = 2\pi/\lambda$ (λ is the wavelength). In our specific case

$$\mathbf{V} = \begin{bmatrix} 0 & -ip & 1 & 0 \\ ip & 0 & 0 & -1 \\ n_e^2 & 0 & 0 & ip \\ 0 & -n_o^2 & -ip & 0 \end{bmatrix}.$$

Here $p = \varphi'/k_0$, and n_e and n_o are the extraordinary and ordinary refractive indices, respectively.

The solutions of Eq. (A1) for $p = \text{const}$ (uniform twist) were first given by De Vries.¹⁰ There are four independent normal modes of the form

$$\mathbf{F} = \mathbf{F}_l \exp(ik_0 m_l z),$$

from which the matrix $\mathbf{M} = [\mathbf{F}_1, \mathbf{F}_2, \mathbf{F}_3, \mathbf{F}_4]$ can be constructed,

$$\mathbf{M} = \begin{bmatrix} u_e & -iu_o r_o & u_e & iu_o r_o \\ iu_e r_e & u_o & -iu_e r_e & u_o \\ u_e(m_e - pr_e) & -iu_o(m_o r_o - p) & -u_e(m_e - pr_e) & -iu_o(m_o r_o - p_o) \\ -iu_e(m_e r_e - p) & -u_o(m_o - pr_o) & -iu_e(m_e r_e - p) & u_o(m_o - pr_o) \end{bmatrix}. \quad (\text{A2})$$

The normal modes are elliptically polarized, and the polarization ellipsoid rotates together with the director, the principal axes being along ξ and η . They can be specified into forward propagating extraordinary mode ($m_l = m_e, E_\eta/E_\xi = ir_e$), forward propagating ordinary mode ($m_l = m_o, E_\xi/E_\eta = ir_o$), backward propagating extraordinary mode ($m_l = -m_e, E_\eta/E_\xi = -ir_e$), and backward propagating ordinary mode ($m_l = -m_o, E_\xi/E_\eta = -ir_o$). The parameters u_e and u_o are chosen in a way that the z component of the vector $\mathbf{E} \times \mathbf{H}$ (which can be considered as the Poynting vector of the given mode) should be unity,

$$1/u_e^2 = m_e(1 + r_e^2) - 2pr_e \quad (\text{extraordinary modes}),$$

$$1/u_o^2 = m_o(1 + r_o^2) - 2pr_o \quad (\text{ordinary modes}).$$

The exact expressions for m_e , m_o , r_e , and r_o can be found in standard textbooks of liquid crystals (e.g., Ref. 9); here we only give their values in the limit $p \ll 1$,

$$m_e = n_e + O(p^2), \quad r_e = \frac{2pn_e}{n_e^2 - n_o^2},$$

$$m_o = n_o + O(p^2), \quad r_o = \frac{2pn_o}{n_e^2 - n_o^2}.$$

For nonuniformly twisted layers we can write the field vector as a superposition of the normal modes,

$$\mathbf{F} = \sum_{l=1}^4 C_l(z) \exp\left(ik_0 \int m_l dz\right) \mathbf{F}_l,$$

or

$$\mathbf{F} = \mathbf{M} \hat{\mathbf{C}}, \quad \hat{\mathbf{C}}_l = C_l \exp\left(ik_0 \int m_l dz\right). \quad (\text{A3})$$

As a detailed consideration shows, in the limit $\varphi''/k_0^2(n_e - n_o) \ll 1$, the z dependence of the C_l coefficients becomes negligible, i.e., the modes propagate independently through the liquid-crystal layer (GAP). Let us consider a light beam entering the sample with $C_1(0) = c_1$ and $C_2(0) = c_2$. The backward propagating modes are not excited, hence $C_3(0) = C_4(0) = 0$. For simplicity, we assume that c_1 and c_2 are real quantities, although this is not essential for our considerations. In the GAP approximation, at the exit face,

$$C_1(L) = c_1 \exp(i\psi_e), \quad C_2(L) = c_2 \exp(i\psi_o),$$

with

$$\psi_e = k_0 \int m_e dz \approx k_0 n_e L \quad \text{and} \quad \psi_o = k_0 \int m_o dz \approx k_0 n_o L.$$

To calculate the Stokes parameters at the exit face with reference to the ξ, η system, we note that from Eqs. (A2) and (A3),

$$E_\xi = M_{11}C_1 + M_{12}C_2, \tag{A4}$$

$$E_\eta = M_{21}C_1 + M_{22}C_2.$$

Using the definition of the Stokes parameters and applying Eq. (A4) for $z=L$, we find that for monochromatic light,

$$S_1 = (1 - r_e^2)u_e^2c_1^2 - (1 - r_o^2)u_o^2c_2^2 + u_e u_o (r_e - r_o)c_1c_2 \sin(\psi_e - \psi_o),$$

$$S_2 = u_e u_o (1 - r_e r_o)c_1c_2 \cos(\psi_e - \psi_o),$$

$$S_3 = 2(r_e u_e^2 c_1^2 + r_o u_o^2 c_2^2) + u_e u_o (1 + r_e r_o)c_1c_2 \sin(\psi_e - \psi_o),$$

where the parameters u and r have to be taken at $z=L$.

For white light the Stokes parameters have to be averaged over the wavelength, as discussed in Sec. II. The terms containing the phase difference $\psi_e - \psi_o \approx k_0(n_e - n_o)L$ oscillate rapidly with the wavelength; for example, with $n_e - n_o = 0.2$, $L = 70 \mu\text{m}$, and $\lambda = 500 \text{ nm}$, the phase difference changes by 2π over a wavelength range of 20 nm. If the spectral width of the light beam is much larger than this interval, one can assume that the average of the terms containing $\psi_e - \psi_o$ is zero. Under this circumstance \hat{S}_2 is zero at arbitrary c_1 and c_2 . This fact was used in the measurements to determine the director orientation at the exit face.

To see the accuracy of the above assumption, we solved the light propagation equation (A1) numerically. In the calculation the sample thickness and refractive indices corresponded to the experimental conditions. It was found that for monochromatic light the angle between the zero position of S_2 and the director at the exit face deviates about 0.5° and the deviation oscillates with the wavelength, as expected from the previous considerations. For white light, the light spectrum has to be specified. Assuming a Gaussian distribution

for $w(\lambda)J_2(\lambda Ad)$ with a half-width of the order of 100 nm, we found that the deviation was reduced to less than 1 arc min.

In order to analyze the first harmonic signal, we consider \hat{S}_3 , which becomes for white light

$$\hat{S}_3 = 2(r_e u_e^2 c_1^2 + r_o u_o^2 c_2^2) \approx \frac{4p(L)}{n_e^2 - n_o^2} (c_1^2 + c_2^2). \tag{A5}$$

In the experiments a linearly polarized beam enters the sample, where the polarization may have a small angle α with the director at the entrance face. At $z=0$ the electric-field components are $E_\xi = E_0 \cos \alpha$ and $E_\eta = E_0 \sin \alpha$. Inverting Eq. (A4), and applying for $z=0$ one finds that

$$c_1 = \sqrt{n_e} E_0 \cos \alpha + O(p); \quad c_2 = \sqrt{n_o} E_0 \sin \alpha + O(p).$$

From these relations, combined with Eq. (A5), it follows that the \hat{S}_3 Stokes parameter is, in linear approximation (with respect to $k_0\varphi'$), proportional to $p(L)$. In the measurements we took advantage of this fact to determine $\varphi'(L)$.

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