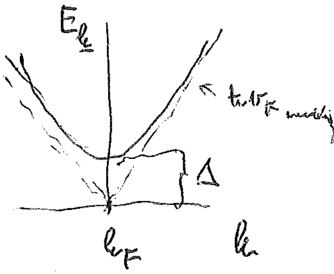
BCS állapot véges hőmérsékleten

$$H^{-\mu N} = \Delta E + \sum_{k} E_k \left(\gamma^+_{k0}\gamma_{k0} + \gamma^+_{k1}\gamma_{k1}\right)$$

kondenzációs energia $= -\frac{1}{2} N(0)\Delta^2$ ~~BCS-ből~~ gyenge k.c.   ha Δ_k k-független $= \Delta$

$$E_k = \sqrt{\xi_k^2 + |\Delta_k|^2} \qquad \xi_k = \epsilon_k - \mu$$



~~Önkonzisztens: $\gamma_{k0} = u_k c_{k\uparrow}$~~

$\gamma^+_{k0} = u_k^* c^+_{k\uparrow} - v_k^* c_{-k\downarrow}$

$\gamma^+_{k1} = u_k^* c^+_{-k\downarrow} + v_k^* c_{k\uparrow}$

$$\left.\begin{aligned} c_{k\uparrow} &= u_k^*\gamma_{k0} + v_k\gamma^+_{k1} \\ c_{-k\downarrow} &= -v_k^*\gamma_{k0} + u_k\gamma^+_{k1} \end{aligned}\right\}$$ „Bogoljubovok" eltüntető operátorok

$$\left.\begin{aligned} |u_k|^2 + |v_k|^2 &= 1 \\ |v_k|^2 - |u_k|^2 &= -\frac{\xi_k}{E_k} \end{aligned}\right\} \begin{Bmatrix} |u_k|^2 \\ |v_k|^2 \end{Bmatrix} = \frac{1}{2}\left(1 \mp \frac{\xi_k}{E_k}\right)$$

$$\cancel{2}u_k^* v_k = \langle c_{-k\downarrow} c_{k\uparrow}\rangle = \frac{\Delta_k}{\cancel{2}E_k}$$ ← rendparaméter

normál fázisban $\Delta_k \equiv 0$

u_k és v_k rel. fázisa $= \Delta_k$ fázisa

~~Gap~~ önkonzisztencia-egyenlet Δ_k-ra (gap-egyenlet)

$$\Delta_k = -\sum_{k'} V_{kk'} \langle c_{-k'\downarrow} c_{k'\uparrow}\rangle = -\sum_{k'} V_{kk'} u_{k'}^* v_{k'} \langle 1 - \gamma^+_{k'0}\gamma_{k'0} - \gamma^+_{k'1}\gamma_{k'1}\rangle$$

$T = 0 \quad \langle\gamma^+_{ki}\gamma_{ki}\rangle = 0 \quad i = 0, 1$

Δ_k ~~$= -\sum_{k'} V_{kk'} u_{k'}^* v_{k'} = -\sum_{k'}$~~ $\qquad T > 0 \quad \langle\gamma^+_{ki}\gamma_{ki}\rangle = f(E_k) \equiv$

$$\equiv \frac{1}{e^{\beta E_k} + 1} \qquad \beta = \frac{1}{k_B T}$$

$$\Delta_{\underline{k}} = -\sum_{\underline{k}'} V_{\underline{k}\underline{k}'} u_{k'}^{*} v_{k'} \left[1-2f(E_{k'})\right]$$

$$= -\sum_{k'} V_{kk'} \frac{\Delta_{k'}}{2E_{k'}} \operatorname{th}\left(\frac{1}{2}\beta E_{k'}\right)$$

$$1-\frac{2}{e^x+1} = \frac{e^x-1}{e^x+1} = \operatorname{th}\frac{x}{2}$$

Cooper modell: $V_{kk'} = -V$ ha $|\xi_k| < \hbar\omega_c$

$$\left[\frac{1}{\mathcal{N}(0)V} = \frac{1}{2}\int_{-\hbar\omega_c}^{\hbar\omega_c} \frac{\operatorname{th}\left(\frac{1}{2}\beta E\right)}{E} d\xi = \int_0^{\hbar\omega_c} \frac{\operatorname{th}\left(\frac{1}{2}\beta E\right)}{E} d\xi\right]$$

Algebrai egyenlet $\Delta(T)$-re

1) T_c meghatározása:

$T \to T_c \rightsquigarrow \Delta \to 0 \qquad E_k \to \xi_k$

$(\ln x)' = \ln x$

$T=T_c$-re

$$\frac{1}{\mathcal{N}(0)V} = \cancel{\mathcal{N}(0)V}\int_0^{\hbar\omega_c} \frac{\operatorname{th}\frac{1}{2}\beta\xi}{\xi} d\xi \Rightarrow \int_0^{x_c = \frac{1}{2}\beta\hbar\omega_c} \frac{\operatorname{th} x}{x} dx$$

$$x = \frac{1}{2}\beta\xi$$

$$= \ln x \operatorname{th} x \Big|_0^{x_c} - \int_0^{x_c} dx \underbrace{\ln x \operatorname{sech}^2 x}_{g(x)}$$

~~$x \gg 1 \to f(x)$~~ $\quad x \to \infty \quad g \propto e^{-2x}$

$e^{-2x_c} = e^{-\beta\hbar\omega_c} \ll 1$

gyenge csatolás: $T_c \ll \hbar\omega_c$

$x_c \gg 1$

$$\cancel{\int_0^{x_c} f\,dx =} \int_0^{\infty} f\,dx + O\left(e^{-2x_c}\right)$$

$$\int_0^\infty \ln x \,\mathrm{sech}^2 x \, dx = -\ln\left(\frac{4e^\gamma}{\pi}\right)$$ $\gamma \approx 0{,}577$ Euler-áll.,

$x_c \gg 1$ (gyenge csat.) ahol $x_c \approx \frac{1}{2}$

$$\frac{1}{N(0)V} = \ln\left(\frac{1}{2}\beta\hbar\omega_c \cdot \frac{4e^\gamma}{\pi}\right)$$

$$k_B T_c = \frac{2e^\gamma}{\pi}\hbar\omega_c \, e^{-1/N(0)V}$$

$\Delta(T=0)$

(1.) $$\boxed{\frac{\Delta_0}{k_B T_c} = \frac{\pi}{e^\gamma} \approx 1{,}764}$$ $$\frac{2\Delta}{T_c} = 3{,}53$$

(2.) $T \to 0$ ~~$\ln\frac{\Delta(0)}{\Delta(T)} \approx 2K_0\left(\frac{\Delta}{T}\right) =$~~ Lásd: Abrikosov.

$$\Delta(T) \approx \Delta_0 - \sqrt{2\pi\Delta_0 T}\, e^{-\Delta_0/T}$$

(3) $T \to T_c$

$$\Delta(T) \approx \sqrt{\frac{8\pi^2}{7\zeta(3)}}\, T_c \sqrt{1-\frac{T}{T_c}} \approx 3{,}06\, T_c \sqrt{1-\frac{T}{T_c}}$$

$$\frac{\Delta(T)}{\Delta_0} \approx 1{,}74\sqrt{1-t}$$

Értelmezés (3). $\Delta \propto \sqrt{1-t}$ átlagtér elméletben ~~mindig ez~~
jól egyezik a kísérletekkel

(2) $T \to 0$ ~ gerjesztések ~~szám~~ $e^{-\Delta_0/T}$ miatt kifagynak

(1) Összemérhető a kísérletekkel

Entrópia „[illegible]"

kétféle [illegible]

$$\text{Entrópia}\quad S_{es} = -2k_B \sum_k \left[(1-f_k)\ln(1-f_k) + f_k \ln f_k\right]$$

elektron [illegible]

$$C_{es} = T\frac{dS_{es}}{dT} = -\beta\frac{dS_{es}}{d\beta}$$

$$C_{es} = 2\beta k_B \sum_k \frac{\partial f_k}{\partial\beta}\underbrace{\ln\frac{f_k}{1-f_k}}_{-\beta E_k} = -2\beta^2 k_B \sum_k E_k \frac{\partial f_k}{\partial \beta}$$

$$= \cancel{-2\beta^2 k_B \quad 2\beta k \sum_k \left(-\frac{\partial f_k}{\partial}\right.}$$

$$= -2\beta^2 k_B \sum_k E_k \frac{df_k}{d(\beta E_k)}\underbrace{\left(E_k + \beta\frac{dE_k}{d\beta}\right)}_{\frac{d(\beta E_k)}{d\beta}}$$

$$= 2\beta k_B \sum_k \left(-\frac{\partial f_k}{\partial E_k}\right)\left(E_k^2 + \frac{1}{2}\beta\frac{d\Delta^2}{d\beta}\right) = C_{s1}^{(1)} + C_{es}^{(2)}$$

T-független gyakran ~~számolható~~ járulék és gyakran T-függő

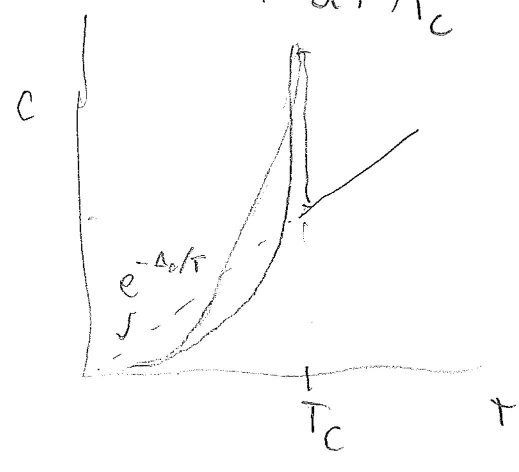
$$T \to T_C \quad \rightsquigarrow \quad E_k^2 \to \xi_k^2 \quad \rightsquigarrow \quad C_{es1}^{(1)} \to C_{en} = \frac{2\pi^2}{3}N(0)k_B^2 T$$

és $C_{es}^{(2)}$ még

$$\Delta C = (C_{es} - C_{en})\Big|_{T_C} = N(0)k_B\beta^2\frac{d\Delta^2}{d\beta}\underbrace{\int_{-\infty}^{\infty}\left(-\frac{\partial f}{\partial E}\right)d\xi}_{1}$$

$$= N(0)\left(-\frac{d\Delta^2}{dT}\right)_{T_C}$$

$$\frac{\Delta C}{C_{en}} = 1.43$$



~~Gerjesztett állapotok~~

~~$\gamma^{+}_{k0} =$~~

~~BCS alapállapot~~

$$\gamma_{k0}|\Psi_G\rangle = \gamma_{k1}|\Psi_G\rangle = 0$$

megegyezik a BCS alapállapottal?

célunk

$$\gamma_{k0}|\Psi_G\rangle = (\tilde{u}_k c_{k\uparrow} - \tilde{v}_k c_{-k\downarrow}) \prod_{\ell} (u_\ell + v_\ell c^{+}_{\ell\uparrow} c^{+}_{-\ell\downarrow})|0\rangle$$

k-t tartalmazó tényező:

$$\tilde{u}_k u_k \underbrace{\cancel{c_{k\uparrow}}}_{0} + \tilde{u}_k v_k \underbrace{c_{k\uparrow} c^{+}_{k\uparrow}}_{1} c^{+}_{-k\downarrow} - \tilde{v}_k u_k c^{+}_{-k\downarrow} - v_k^2 c_{-k\downarrow} c^{+}_{k\uparrow} \underbrace{c^{+}_{-k\downarrow}}_{0}$$

$\gamma_{k0}|\Psi_G\rangle = 0$ teljesül akkor és csak akkor, ha

$$\tilde{u}_k v_k = \tilde{v}_k u_k$$

$$\frac{v_k}{u_k} = \frac{\tilde{v}_k}{u_k} \qquad |u_k|^2 + |v_k|^2 = 1$$

$$|\tilde{u}_k|^2 + |\tilde{v}_k|^2 = 1$$

~~$c^{+}_{k}c_k$~~

$c_k c^{+}_k|0\rangle = 1|0\rangle$

$\rightsquigarrow$ $|u_k| = |\tilde{u}_k|$

$|v_k| = |\tilde{v}_k|$ + rel. fázisok megegyeznek

Gerjesztett állapot:

$$\gamma^{+}_{k0}|\Psi_G\rangle$$

k-t tartalmazó tényező

$$|u_k|^2 c^{+}_{k\uparrow} + u^{*}_k v_k \underbrace{\cancel{c^{+}_{k\uparrow} c^{+}_{k\uparrow}}}_{0} c^{+}_{-k\downarrow} - v^{*}_k u_k c_{-k\downarrow} - |v_k|^2 c_{-k\downarrow} c^{+}_{k\uparrow} c^{+}_{-k\downarrow}$$

$$= (|u_k|^2 + |v_k|^2)\, c^{+}_{k\uparrow}$$

$$\gamma^{+}_{k0}|\Psi_G\rangle = c^{+}_{k\uparrow} \prod_{\ell \neq k} (u_\ell + v_\ell c^{+}_{\ell\uparrow} c_{-\ell\downarrow})|0\rangle$$

Hasonlóan: $\gamma^{+}_{k1}|\Psi_G\rangle = c^{+}_{-k\downarrow} \prod_{\ell \neq k} (\qquad)|0\rangle$

Állapotsűrűség

$$\mathcal{N}(E') = \frac{1}{(2\pi)^3} \int_{S_{E'}} \frac{dS}{|\nabla_k E_k|} = \left.\frac{d\xi}{dE}\right|_{E'} \frac{1}{(2\pi)^3} \int_{S_{E'}} \frac{dS}{|\nabla_k \xi_k|} = \frac{d\xi}{dE} \mathcal{N}_n(E') \approx \frac{d\xi}{dE} \mathcal{N}(0)$$

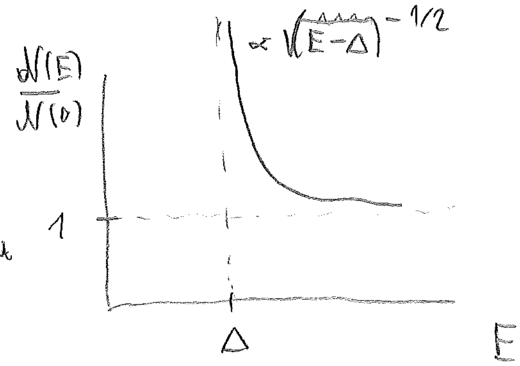
$$\left.\frac{\partial E_k}{\partial k}\right|_{E'} = \frac{\partial}{\partial k} \sqrt{|\xi_k|^2 + \Delta^2} = \frac{dE}{d\xi} \nabla_k \xi_k = \frac{1}{\left.\frac{d\xi}{dE}\right|_{E'}} \nabla_k \xi$$

k-tól független gap

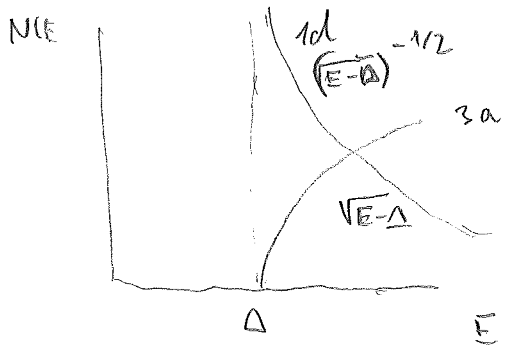
$$\xi(E) = \cancel{\sqrt{\Delta^2 - E^2}} \quad \sqrt{E^2 - \Delta^2}$$

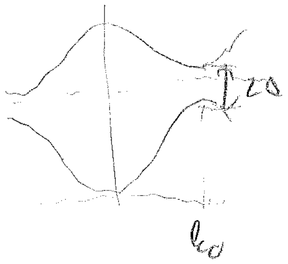
$$\frac{d\xi}{dE} = \frac{E}{\sqrt{\Delta^2 - E^2}}$$

$$\frac{\mathcal{N}(E')}{\mathcal{N}(0)} = \frac{E}{\sqrt{\Delta^2 - E^2}} \quad \begin{cases} E > 0 \\ 0 \text{ egyébként} \end{cases}$$



Különbség a 3d félvezetőtől

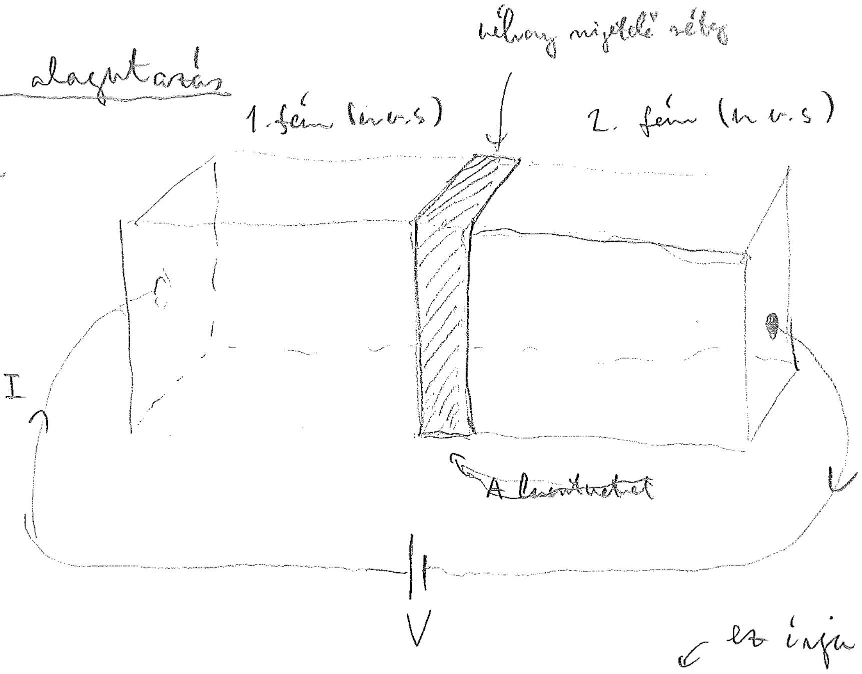




Szupra: az egész Fermi-felületen minimum van

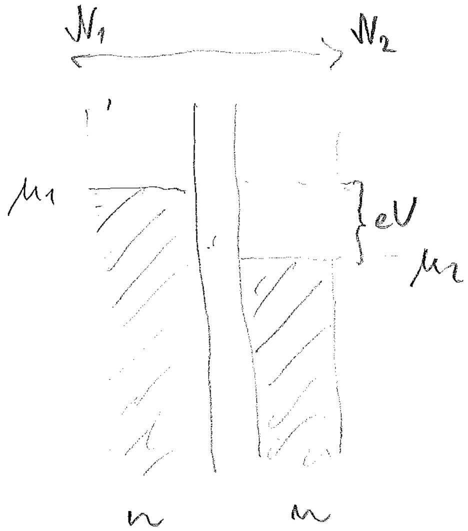
Elektron alagutazás

Kísérlet



~~alagutazást leíró~~ $H = H_1 + H_2 + H_T$ ← ez írja le az alagutazást

$$H_T = \sum_{\sigma k q} \Big(T_{kq} \underbrace{c^{\dagger}_{k\sigma} \tilde{c}_{q\sigma}}_{2 \to 1} + \underbrace{h.c}_{1 \to 2} \Big)$$



$$I_{1\to 2} = \overset{\text{kont.}}{A} \int_{-\infty}^{\infty} dE\, |T|^2 \overbrace{N_1(E) f(E)}^{\text{betöltött kezdeti állapot}}\, \overbrace{N_2(E+eV)[1-f(E+eV)]}^{\text{üres végállapot}}$$

normál - normál alagutazás

$$\cancel{I_{nn} = A|T|^2 \underbrace{N_1(0) N_2(0)}_{\text{kont. állapot.}} \int_{-\infty}^{\infty} [f}$$

$$I = I_{1\to 2} - I_{2\to 1}$$

$$N_1 f_1 N_2 (1-f_2) - N_2 f_2 N_1 (1-f_1) = N_1 N_2 (f_1 - f_2)$$

$$\boxed{I = A|T|^2 \int_{-\infty}^{\infty} N_1(E)\, N_2(E+eV)\, [f(E) - f(E+eV)]\, dE}$$

1. Normál - normál alagutazás

$$I = A|T|^2 N_1(0) N_2(0) \int_{-\infty}^{\infty} \underbrace{[f(E) - f(E+eV)]}_{\simeq -\frac{\partial f}{\partial E} eV} dE \simeq A|T|^2 N_1(0) N_2(0)\, eV$$

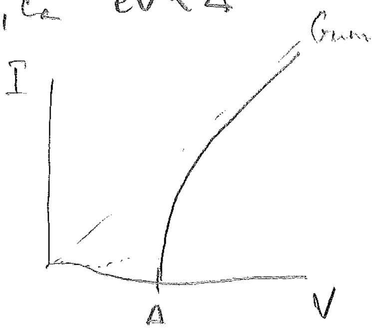
$$\equiv G_{nn} V$$

lineáris (Ohm-tv.)

T - független

2 Normál – szupravezető alagutazás

$T=0$: $I=0$, ha $eV<\Delta$



$$I_{ns} = A\,|T|^2\,\mathcal{N}_1(0)\int_{-\infty}^{\infty}\mathcal{N}_{2s}(E)\,[f(E)-f(E+eV)]\,dE$$

$$= \frac{G_{nn}}{e}\int_{-\infty}^{\infty}\frac{\mathcal{N}_{2s}(E)}{\mathcal{N}_2(0)}\,[f(E)-f(E+eV)]\,dE$$

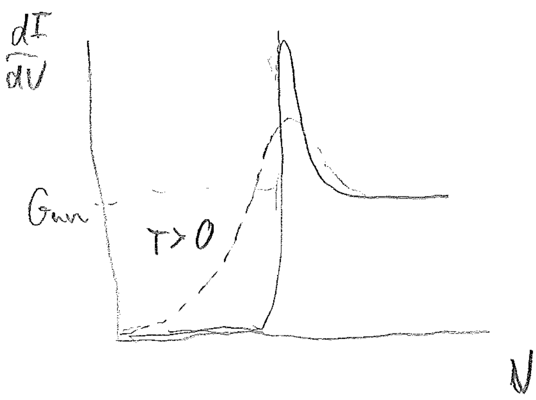
Differenciális vezetőképesség:

$$G_{ns} \equiv \cancel{G_{nn}}\,\frac{dI_{ns}}{dV} = G_{nn}\int_{-\infty}^{\infty}\frac{\mathcal{N}_{2s}(E)}{\mathcal{N}_2(0)}\left[-\frac{\partial f(E+eV)}{\partial(eV)}\right]dE$$

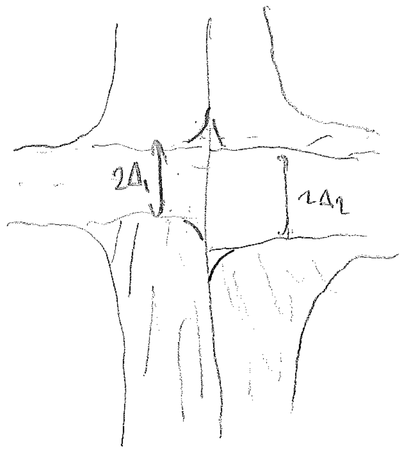
$$\downarrow T\to 0$$

$$\delta(E+eV)$$

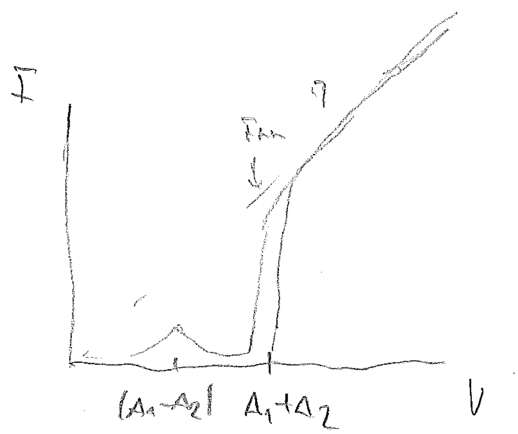
$$\boxed{G_{ns}\xrightarrow[T\to 0]{} G_{nn}\,\frac{\mathcal{N}_{2s}(e|V|)}{\mathcal{N}_2(0)}}$$



Szupravezető - szupravezető



T>0



Giaever (Nobel-díj, 1973)

Esaki, Josephson

Table 3.1 Properties of the Superconducting Elements[a]

Z	Element[b]	N_e	Crystal Structure[c]	T_c (K)	Θ_D (K)	B_c (mT)	$2B_c/T_c$ (mT/K)	γ $\left(\frac{mJ}{mole\,K^2}\right)$	$\chi \times 10^6$ (cm^3/mole)	λ	dT_c/dP (K/GPa)	P (GPa)	α	WF (eV)	$E_g = 2\Delta$ (meV)	E_g/kT_c	$D(E_F)$ (states, atom eV)	Z
4	Be	2	hcp	0.026	940			0.21						5.0				4
13	Al	3	fcc	1.18	420	10.5	18	1.4						4.3	0.35	3.4		13
21	Sc	3	hcc	0.01	470			10.9						5.9				21
22	Ti	4	hcp	0.40	415	5.6	28	3.3	155	0.38	0.6	0–1.4		4.33			≈ 1.4	22
23	V	5	bcc	5.40	383	141.0	52	9.82	300	0.60	6.3	0–2.5		4.3	1.6	3.4	≈ 2.1	23
30	Zn	12	hcp	0.85	316	5.4	12	0.66					0.45	4.3	0.23	3.2		30
31	Ga	3	orthr	1.08	325	5.83	11	0.60							0.33	3.5		31
40	Zr	4	hcp	0.61	290	4.7	15	2.77	129	0.41	15.0	0–2.0	0	4.05			≈ 0.8	40
41	Nb	5	bcc	9.25	276	206.0	45	7.80	212	0.82	−2.0	0–2.5		4.3	3.0	3.8	≈ 2.1	41
42	Mo	6	bcc	0.92	460	9.6	21	1.83	89	0.41	−1.4	0–2.5	0.37	4.6	0.26	3.4	0.65	42
43	Tc	7	hcp	7.8	411	141.0	36	6.28	270		−12.5	0–1.5		5.0	2.4	3.6		43
44	Ru	8	hcp	0.49	580	6.9	28	2.8	39	0.38	−2.3	0–1.8	0	4.7	0.15	3.5	0.91	44
48	Cd	12	hcp	0.517	210	2.8	11	0.69					0.5	4.2	0.14	3.2		48
49	In	3	tetrg	3.41	108	28.2	17	1.67						3.8	1.05	3.6		49
50	Sn(w)	4	tetrg	3.72	195	30.5	16	1.78					0.47	4.38	1.4	4.4		50
57	La(α)	3	hcp	4.88	152	80.0	33	9.8			190	0–2.3			1.5	3.5		57
57	La(β)	3	fcc	6.3	140	110.0	37	11.3			110							57
71	Lu	3	hcp	0.1		< 40.0									0.028	3.3		71
72	Hf	4	hcp	0.13	252	1.27	20	2.2	70	0.14	−2.6	0–1.0			0.044	3.9	0.83	72
73	Ta	5	bcc	4.47	258	82.9	37	6.15	162	0.75	−2.6				≈ 1.7	≈ 3.5	≈ 1.7	73
74	W	6	bcc	0.015	383	0.12	16	0.90	53	0.25				4.5	≈ 0.006	≈ 4.5	≈ 0.5	74
75	Re	7	hcp	1.70	415	20.0	24	2.35	68	0.37	−2.3	0–1.8	0.23		0.78	3.4	0.76	75
76	Os	8	hcp	0.66	500	7.0	21	2.35	13	0.44	−1.8		0.20		0.29	4.8	0.70	76
77	Ir	9	fcc	0.11	425	1.6	29	3.2	24	0.35					0.048	5.6		77
80	Hg(α)	12	trig	4.15	88	41.1	20	1.81					0.50	4.52	1.7	4.6		80
80	Hg(β)	12	tetrg	3.9	93	33.9	17	1.37										80
81	Tl	3	hcp	2.38	79	17.8	15	1.47		0.80			0.50	3.7	0.79	3.8		81
82	Pb	4	fcc	7.20	96	80.3	22	3.1		1.55			0.48	4.3	2.7	4.3		82
90	Th	4	fcc	1.38	165	16.0	23	4.32							0.41	3.4		90
91	Pa	5		1.4														91
95	Am	9	fcc	1.0														95

Table 4.1 Debye temperature Θ_D, Density of States $D(E_F)$, and Specific Heat Data[a]

Material	T_c (K)	Θ_D (K)	γ_n^*	γ_n $\left(\frac{mJ}{mole\,K^2}\right)$	$(C_s - C_n)/T_c$ (mJ/mole K^2)	$(C_s - C_n)/\gamma T_c$	A (mJ/mole K^4)	$D(E_F)$ (states/eV)	Reference
Cd	0.55	252		0.67	0.91	1.36			
Al	1.2	423		1.36	1.97	1.45			
Sn, white	3.72	196		1.78	2.85	1.60			
Pb	7.19	102		3.14	8.51	2.71			
Nb	9.26	277		7.66	14.8	1.93		1.55	
$Zr_{0.7}Ni_{0.3}$	2.3	203		4.04	≈ 6.7	≈ 1.65	0.23	2.0	
[illegible]	0.76			4					Sürgers *et al.* (1989)
									Vonsovsky *et al.* (1982, pp. 269ff.)
									Vonsovsky *et al.* (1982, pp. 269ff.)

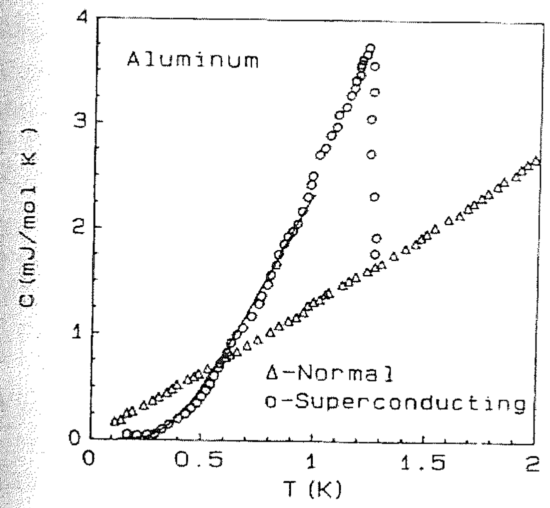


Figure 4.5 Specific heat jump in superconducting Al compared with the normal-state specific heat (Phillips, 1959; see Crow and Ong, 1990, p. 225).

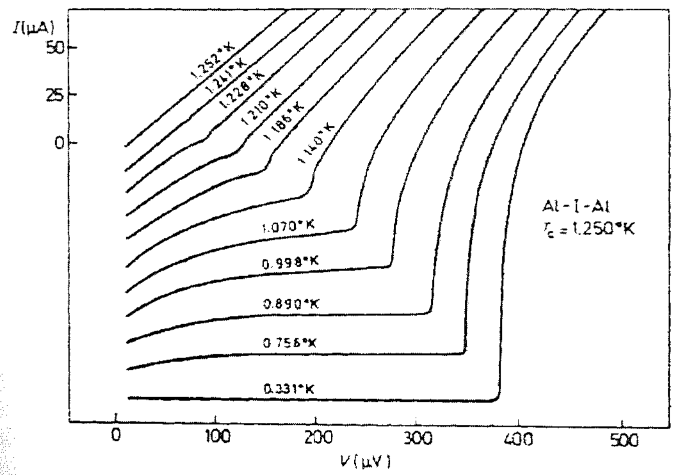


Figure 29.4 The I–V characteristics of an Al–AlO$_x$–Al junction. Successive curves are displaced for clarity. (After Blackford and March (1968).)