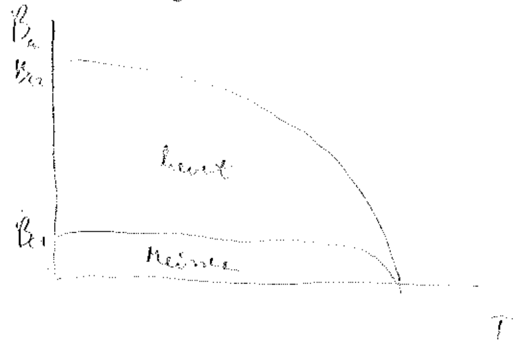
* vortexek megjelenése kis mágneses térben, H_{c1}.

II.-fajú SzV.



$B_{c1} = ?$

vortexrács rácsállandója $a^2 \sim \frac{\Phi_0}{B}$ $\quad B \to 0 \rightsquigarrow a \to \infty$

ha $a \gg \lambda \rightsquigarrow$ vortexek függetlenek egymástól, kölcsönhatásuk elhanyagolható

Mikor jelenik meg az első vortex?

$$\tilde{F}_S \big|_{\text{vortex nélkül}} = \tilde{F}_S \big|_{\text{első vortexnél}}$$ ($\underline{H}$ = const. melletti feltétel)

($\tilde{F}_S = F_S - \frac{1}{\mu_0} \underline{H} \cdot \int \underline{B}\, d^3r$, mint előző órán)

↑ azért ezt használjuk, mert $\underline{H}$ = const., de $\underline{B}$ inhomogén)

vortex nélkül $F_S = \tilde{F}_S$, mert $\underline{B} = 0$

vortex vonal hossza

$$F_S = F_S - \frac{1}{\mu_0} H_{c1} \underbrace{\int B\, d^3r}_{\Phi_0 L} + \underbrace{\epsilon_1 L}_{\text{vortex energiája}}$$

ϵ_1 vortexvonal hosszmenti energiája [J/m]

$$F_S = F_S + \epsilon_1 L - \frac{1}{\mu_0} H_{c1} \Phi_0 L \qquad \int B\, d^3r = L \int B\, d^2r = L \Phi_0$$

$$F_S = F_S - \frac{1}{\mu_0} H_{c1} \Phi_0 L + \epsilon_1 L$$

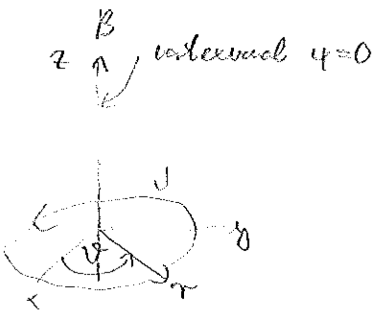
$$\boxed{H_{c1} = \mu_0 \frac{\epsilon_1}{\Phi_0}}$$

Egyetlen vortex szerkezete

vortexrács rácsállandója: $a^2 \sim \frac{\Phi_0}{B}$ $\quad B \to 0 \rightsquigarrow a \to \infty$

ha $a \gg \lambda_L \rightsquigarrow$ a vortexek nem hatnak kölcsön

$\rightsquigarrow$ tekinthetünk egyetlen vortexet az anyagban



$$\psi = \Psi_\infty f(r) e^{i\vartheta} \qquad \left(\oint \nabla\varphi \, dl = 2\pi\right)$$

$$\underline{A} = A(r)\,\hat{\vartheta} \qquad (\text{"minimális modell"})$$

$$(\nabla \times A)_z = \frac{1}{r}\left[\frac{\partial}{\partial r}(r A_\theta) - \frac{\partial A_r}{\partial \vartheta}\right] = \frac{1}{r}\frac{\partial}{\partial r}\left[r A(r)\right]$$

$$(\nabla \times A)_r = (\nabla \times A)_\vartheta = 0$$

$$\underline{B} = B(r)\,\hat{\underline{z}} \qquad \frac{1}{r}\frac{\partial}{\partial r}\left[r A(r)\right] = B(r)$$

$$A(r) = \frac{1}{r}\int_0^r r' B(r')\, dr'$$

$$r \to 0 \quad B(r) \approx B(0) \quad \rightsquigarrow \quad A(r) = \frac{B(0)\, r}{2}$$

$$r \to \infty \quad \oint \underline{A}\, d\underline{l} = 2\pi r A_\infty = \Phi_0 \quad \rightsquigarrow \quad A_\infty = \frac{\Phi_0}{2\pi r}$$ (nem [illegible]!)

(GL1) $$f - f^3 - \xi^2\left[\left(\frac{1}{r} - \frac{2\pi A}{\Phi_0}\right)^2 f - \frac{1}{r}\frac{d}{dr}\left(r\frac{df}{dr}\right)\right] = 0$$

$$(\mathrm{rot}\, B)_\vartheta = -\frac{\partial B_z}{\partial r} \qquad (\ )_z = (\ )_r = 0$$

$$\underline{J} = \frac{1}{\mu_0}\,\mathrm{rot}\,\underline{B} = -\frac{1}{\mu_0}\underbrace{\frac{dB}{dr}}_{<0}\,\hat{\vartheta} = -\frac{1}{\mu_0}\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rA)\right]$$

(GL2) $$-\frac{1}{\mu_0}\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(rA)\right] = \frac{e\hbar}{m}\Psi_\infty^2 f^2\left(\underbrace{\frac{1}{r}}_{\nabla\varphi} - \frac{2\pi}{\Phi_0}A\right)$$

Általában csak numerikusan tudjuk megoldani

$r \to 0$ aszimptotika, t.f. legyen $f = c r^n \quad n \geq 0$

$$A(r) = \frac{B(0)\, r}{2}$$

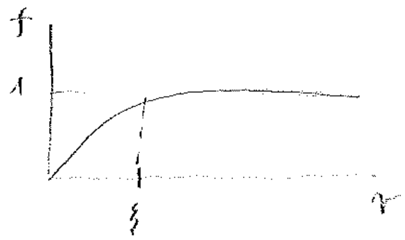
$$(GL1) \quad f - f^3 - \xi^2 \left[\left(\frac{1}{r} - \frac{\pi B(0) r}{\phi_0} \right)^2 f - \frac{1}{r} \frac{d}{dr}\left(r \frac{df}{dr} \right) \right] = 0$$

$$c r^n - c^3 r^{3n} - \xi^2 \left[\left(\frac{1}{r} - \frac{\pi B(0) r}{\phi_0} \right)^2 c r^n - n^2 c r^{n-2} \right] = 0$$

vezető tag $r \to 0$ határesetben: $r^{n-2}(1 - n^2) = 0 \quad \rightsquigarrow \boxed{n = 1}$

$$\boxed{f(r) \propto r, \quad r \to 0}$$

Következő tagig elmenve: $f \approx c r \left\{ 1 - \frac{r^2}{8 \xi^2} \left[1 + \frac{B(0)}{B_{c2}} \right] \right\}$



Nagy κ közelítés

$\lambda \gg \xi \qquad \kappa \gg 1$

ξ kicsi, majdnem mindenhol $f = 1$

London-egyenlet: $\mu_0 \lambda^2 \,\mathrm{rot}\, \underline{j} + \underline{B} = 0$ fluxoid minden hurokra 0!

$$\underline{j} = \underbrace{\frac{e\hbar}{m}\psi_\infty^2}\left(\underline{\nabla}\varphi - \frac{2\pi}{\Phi_0}\underline{A}\right)$$

$$\frac{1}{\mu_0\lambda_L^2} = \frac{ne^2}{m} \qquad \frac{e^2\frac{1}{2}n}{m}\,\frac{\hbar}{e} = \frac{e^2 n}{m}\,\frac{\hbar}{2e}$$

$$\underline{j} = \frac{1}{\mu_0\lambda_L^2}\,\frac{\Phi_0}{2\pi}\left(\underline{\nabla}\varphi - \frac{2\pi}{\Phi_0}\underline{A}\right)$$

$$\underline{A} + \mu_0\lambda_L^2\,\underline{j} = \frac{\Phi_0}{2\pi}\underline{\nabla}\varphi \rightsquigarrow \oint\left(\underline{A} + \lambda_L^2\,\mathrm{rot}\,\underline{B}\right) dl = \Phi_0$$

$$\int_F \underbrace{\left(\mathrm{rot}\,\underline{A} + \lambda_L^2\,\mathrm{rot}\,\mathrm{rot}\,\underline{B}\right)}_{=0 \text{ London-egyenlet}} \cdot d\underline{f} = \Phi_0$$

$$\underline{B} + \lambda_L^2\,\mathrm{rot}\,\mathrm{rot}\,\underline{B} = \Phi_0\,\hat{\underline{z}}\,\delta^2(\underline{r})$$

$$\mathrm{rot}\,\mathrm{rot}\,\underline{B} = \mathrm{grad}\,(\cancel{\mathrm{div}\,\underline{B}}) - \nabla^2\underline{B}$$

$$\boxed{\nabla^2\underline{B} - \frac{1}{\lambda_L^2}\underline{B} = -\frac{\Phi_0}{\lambda_L^2}\,\hat{\underline{z}}\,\delta^2(\underline{r})}$$

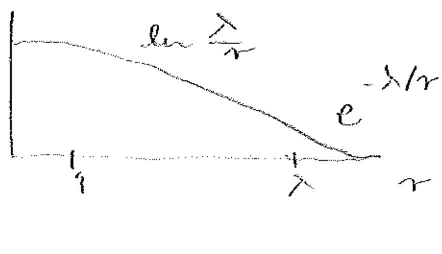
$\underline{B}$ a 2d Helmholtz-egyenlet Green-fv.-e

Szakirodalomból: $\boxed{B(r) = \frac{\Phi_0}{2\pi\lambda^2} K_0\left(\frac{r}{\lambda}\right)}$

Hankel-fv.

$r \gg \lambda \rightsquigarrow \quad B(r) \approx \frac{\Phi_0}{2\pi\lambda^2}\left(\frac{\pi}{2}\,\frac{\lambda}{r}\right)^{1/2} e^{-r/\lambda}$

$\xi \ll r \ll \lambda \quad B(r) \approx \frac{\Phi_0}{2\pi\lambda^2}\left(\ln\frac{\lambda}{r} + 0{,}12\right)$



Szupravezető áram:

$$J(r) = \frac{\Phi_0}{2\pi \lambda_L^3 \mu_0} K_1\left(\frac{r}{\lambda_L}\right) = \begin{cases} \propto \dfrac{1}{r} & \xi \ll r \ll \lambda \\ \propto e^{-r/\lambda_L} & r \gg \lambda \end{cases}$$

Vortexvonal energiája $\kappa \gg 1$ közelítésben

Vortexvonal hosszmenti energiája:

$\xi^2 \propto \left(\frac{\xi}{\lambda}\right)^2$ rendű, elhanyagolható

$$\mathcal{E}_1 = \int \left(\underbrace{\frac{B^2}{2\mu_0}}_{\text{mágneses}} + E_{kin} \right) \underbrace{dS}_{dx\,dy} + \cancel{E_{mag}}$$

$E_{kin} = ?$ $\quad n\frac{1}{2} m^* v^2 \quad v = \frac{J}{ne}$

$$E_{kin} = \frac{1}{2}\frac{m}{ne^2} J^2 \qquad \underline{J} = \frac{1}{\mu_0}\,\mathrm{rot}\,\underline{B} \qquad J^2 = \frac{1}{\mu_0^2}|\mathrm{rot}\,\underline{B}|^2$$

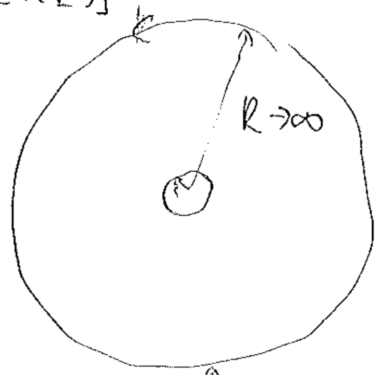
$$E_{kin} = \frac{1}{2\mu_0}\frac{m}{ne^2\mu_0}|\mathrm{rot}\,\underline{B}|^2 = \frac{1}{2\mu_0}\lambda^2|\mathrm{rot}\,\underline{B}|^2$$

$$\mathcal{E}_1 = \frac{1}{2\mu_0}\int \left(B^2 + \lambda^2 |\mathrm{rot}\,\underline{B}|^2 \right) \underbrace{dS}_{dx\,dy \text{ sík}}$$

$$(\underline{\nabla}\times\underline{B})(\underline{\nabla}\times\underline{B}) = \underline{B}\,(\underline{\nabla}\times\underline{\nabla}\times\underline{B}) + \underline{\nabla}[\underline{B}\times(\underline{\nabla}\times\underline{B})]$$

$$\mathcal{E}_1 = \frac{1}{2\mu_0}\int \underbrace{(\underline{B} + \lambda^2\,\mathrm{rot}\,\mathrm{rot}\,\underline{B})}_{\hat{z}\,\Phi_0\,\delta^2(\underline{r})}\cdot\underline{B}\,dS + \frac{\lambda^2}{2\mu_0}\oint \underline{B}\times\mathrm{rot}\,\underline{B}\,d\underline{l}$$

$$\mathcal{E}_1 = \lim_{R\to 0}\oint_{\text{Belső kör}} \underline{B}\times\mathrm{rot}\,\underline{B}\,d\underline{l}$$



erre 0 az $\oint$ mert nincs mágneses

$$\mathcal{E}_1 = \frac{\lambda^2}{2\mu_0}\left[B\frac{dB}{dr}2\pi r \right]_\xi$$

$$B(r) = \frac{\Phi_0}{2\pi\lambda^2}\ln\frac{r}{\lambda} \qquad \frac{dB}{dr} = \frac{\Phi_0}{2\pi\lambda^2}\frac{1}{r}$$

$\ln K$

$$\mathcal{E}_1 = \frac{\Phi_0}{2\mu_0}B(\xi) \approx \frac{\Phi_0}{2\mu_0}B(0) = \frac{\Phi_0^2}{4\pi\mu_0\lambda^2}\ln\frac{\xi}{\lambda}$$

~~GL bevezet~~ Előző órából: $\xi = \frac{\Phi_0}{2\sqrt{2}\pi B_c \lambda}$

$$E_1 = \underbrace{\frac{B_c^2}{2\mu_0} 4\pi \xi^2}_{\text{kond. energia a magban}} \ln \kappa$$ elhanyagolt mag diszkussziója

$$B_{c1} = \mu_0 \frac{E_1}{\Phi_0} = \frac{\Phi_0}{4\pi\lambda^2} \ln \kappa$$

$$\frac{\Phi_0}{2\pi\xi^2} \cdot \frac{\xi^2}{\lambda^2} = B_{c2} \frac{1}{\kappa^2} = B_c \frac{1}{\kappa}$$, mert $B_{c2} = B_c \kappa$

$$B_{c1} \approx \frac{1}{2} \frac{B_c}{\kappa} \ln \kappa \qquad B_{c1} \approx \frac{B_c}{\kappa} \qquad \boxed{B_c^2 \approx B_{c1} B_{c2}}$$

$\int \nabla \times A \, dS = \oint A \cdot dl$ $\quad (-i\nabla)^2\psi \Rightarrow |-i\nabla\psi|^2 = +|\nabla\psi|^2$ $\quad$ 3

$\cancel{\alpha\psi} + \beta|\psi|^2\psi = \beta\psi_\infty^3 e^{i\varphi(r)}$ lineáris

Vortexekre ható erő

$\psi = \psi_\infty e^{i\varphi(r)}$

$\kappa \gg 1 \quad |\psi| = \psi_\infty$ ~ lineáris egyenletek ~ szuperpozíció

2 vortex $\quad B(r) = B_1(r) + B_2(r) = \hat{z}\left[B(|r - r_1|) + B(|r - r_2|)\right]$

$j(r) = j_1(r) + j_2(r)$

$H_{c1} \ll H \ll H_{c2}$

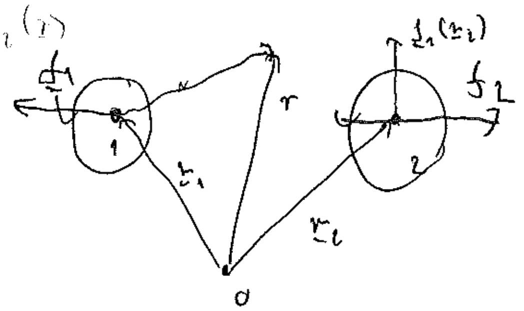
2 vortex $\quad \xi \ll r \ll \lambda$

$f \propto \ln \frac{\lambda}{r}$

$r \gg \lambda$

$f \propto e^{-\lambda/r}$

A vortexek taszítják egymást



~~B(r)~~

Kin. energia $\propto |\nabla\varphi|^2 \propto |\mathrm{rot}\, B|^2$

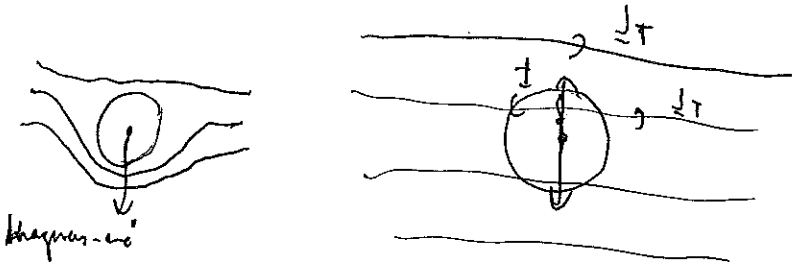
mágneses energia $\frac{B^2}{2\mu_0}$

$E = E_1 + E_2 + E_{12}$

$f = -\frac{\partial E_{12}}{\partial r}$

$$f_2 = j_1(r_2) \times \Phi_0 \hat{z}$$

Ugyanez a helyzet, ha külső transzportáramot kapcsolunk a mintára:



$$f = j_T \times \hat{z}\,\Phi_0$$

~~$B \gg B_{c1}$~~

$B < B_{c1}$ $\qquad$ $B > B_{c1}$