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Scattering of light by an electron in the presence of a magnetic field and a strong microwave field

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Abstract. Scattering of light by an electron embedded simultaneously in a uniform magnetic field and in a microwave field is first analysed within the framework of classical electrodynamics. It is shown that the intensities of the different components of the scattered light can be increased by the presence of the magnetic field. Then coherent quantum states of the electron in the same field configuration are constructed and are applied to the description of the scattering process using semiclassical methods. The results of the classical and semiclassical calculations are compared and discussed and various limiting cases are considered to show the connection with previously published work.

1. Introduction

Laser-induced and laser-assisted phenomena have recently been very popular topics of investigation (Ehlotzky 1981). Induced and inverse bremsstrahlung, in particular, have been considered to be of importance for fusion research (Seely 1974a), but interest has also been directed towards more general aspects to this problem which are related to the low-frequency limit and classical correspondences (Mittleman 1982). Introducing a constant magnetic field adds new features to the scattering process considered, of which the appearance of resonances between different Landau states is the most prominent one (Seely 1974b).

One of the problems in the scattering of electrons in the simultaneous presence of a strong radiation field and a magnetic field is the fact that in the limit $B\to 0$ we do not recover the formulae derived in the magnetic-field-free case, as long as we consider the transitions to occur between discrete Landau states during the scattering process (Bergou *et al* 1982). This difficulty has its origin in the different boundary conditions which are usually implied and therefore the imposition of the boundary conditions requires more detailed investigations (Faisal 1982).

In order to get some insight into the boundary value problem involved we have chosen to investigate first an apparently simpler problem which also permits a treatment in entirely classical terms. We shall investigate in the following the scattering of laser radiation by a free electron which is simultaneously embedded in a homogeneous magnetic field and a strong microwave field propagating in the direction of the B field.

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This problem is of some practical interest for laser diagnostics of a magnetised plasma (Sheffield 1975) where the microwave field adds some new features (Guccione-Gush and Gush 1975) and enhancements to the predicted scattering spectra. Moreover we shall see that the classical and quantum mechanical formulations of the problem most easily permit one to investigate different limiting cases if the electron is described by a properly chosen coherent superposition of Landau states. This approach will then also be the starting point for a similar investigation of potential scattering of electrons in a microwave and a magnetic field (Varró *et al* 1984).

In the first section we shall give a classical treatment of the problem by solving the Lorentz equation of motion and evaluating the spectrum emitted by the electron. Then in § 2 we shall treat the Schrödinger equation for a non-relativistic electron in a microwave and a magnetic field and shall find the exact solutions by the algebraic techniques developed earlier (Bergou *et al* 1982). From these states we shall construct coherent electron wavefunctions which in the limit $\hbar \to 0$ follow the classical electron orbits. These coherent states will then be used in § 3 to evaluate semiclassically the emitted radiation field. A short summary will complete our investigations which are to be considered as a first step towards a better understanding of the boundary value problem for particle scattering in a radiation and a magnetic field.

2. Classical description of the scattering process

In the present section we shall analyse the scattering of light by an electron in the presence of a constant homogeneous magnetic field and a low-frequency microwave radiation field within the framework of classical electrodynamics.

The non-relativistic equation of motion of the electron of charge $-e$ and mass m reads

$$\dot{\boldsymbol{v}} = -\omega_c(\boldsymbol{v} \times \boldsymbol{\mathscr{E}}_z) - \frac{e}{m}(\boldsymbol{E}_\omega + \boldsymbol{E}_\Omega) \tag{1}$$

where $\omega_c \equiv eB/mc$ is the cyclotron frequency. The magnetic field $\boldsymbol{B} = B\boldsymbol{\mathscr{E}}_z$ is chosen to point in the positive z direction. The low-frequency wave is assumed to propagate parallel to the magnetic field and it is taken to be right circularly polarised with Cartesian components

$$\boldsymbol{E}_\omega = F(\sin \omega t, -\cos \omega t, 0). \tag{2}$$

The incident light of frequency Ω propagates along the direction $\boldsymbol{n}$ and it is assumed to be linearly polarised with the polarisation vector given by $\boldsymbol{\mathscr{E}} = \mathscr{E}_1\boldsymbol{\mathscr{E}}_x + \mathscr{E}_2\boldsymbol{\mathscr{E}}_y + \mathscr{E}_3\boldsymbol{\mathscr{E}}_z$. The corresponding electric field is then taken to have the form

$$\boldsymbol{E}_\Omega = \boldsymbol{\mathscr{E}} f \sin[\Omega(t - c^{-1}\boldsymbol{n} \cdot \boldsymbol{r})]. \tag{3}$$

In addition, we assume that Ω is much larger than ω and ω_c, and that the field $\boldsymbol{E}_\Omega$ is much weaker than the low-frequency field $\boldsymbol{E}_\omega$, i.e. the corresponding intensity parameters satisfy the condition $\mu \equiv eF/m\omega c \gg ef/m\Omega c \equiv \mu_\Omega$. The equation of motion (1) can be easily solved if the space dependence of $\boldsymbol{E}_\Omega$ is neglected. It is well known that this (dipole) approximation is valid for a classical free electron if the amplitude of the quivering motion of the electron caused by the electric field is much smaller than the wavelength of the field, i.e. $\mu_\Omega \ll 1$. On the other hand, we know that μ is the ratio of the velocity amplitude of the quivering motion caused by the field $\boldsymbol{E}_\omega$ to the velocity of light, so we must demand that $\mu \ll 1$ for non-relativistic electrons. Using the above approximations we obtain the following solution to equation (1)

$$v_z = \frac{ef}{m\Omega} \cos \Omega t \qquad z = \frac{ef}{m\Omega^2} \sin \Omega t \tag{4}$$

$$\dot{u} = v \exp[-\mathrm{i}(\omega_c t + \chi)] + \frac{eF}{m(\omega - \omega_c)} \exp(-\mathrm{i}\omega t) + \frac{ef}{2m}(\mathscr{E}_1 - \mathrm{i}\mathscr{E}_2)\left(\frac{\exp(\mathrm{i}\Omega t)}{\Omega + \omega_c} + \frac{\exp(-\mathrm{i}\Omega t)}{\Omega - \omega_c}\right) \tag{5}$$

$$u = \mathrm{i}\frac{v}{\omega_c} \exp[-\mathrm{i}(\omega_c t + \chi)] + \mathrm{i}\frac{eF}{m\omega(\omega - \omega_c)} \exp(-\mathrm{i}\omega t) - \mathrm{i}\frac{ef}{2m\Omega}(\mathscr{E}_1 - \mathrm{i}\mathscr{E}_2)\left(\frac{\exp(\mathrm{i}\Omega t)}{\Omega + \omega_c} - \frac{\exp(-\mathrm{i}\Omega t)}{\Omega - \omega_c}\right). \tag{6}$$

In equations (5) and (6) we have introduced the complex trajectory defined by $u \equiv x(t) - \mathrm{i}y(t)$. The solutions given by equations (4), (5) and (6) correspond to an adiabatic switching-on of the fields, so they describe an electron having the initial velocity components $v(\cos \chi, \sin \chi, 0)$. From equation (5) it can be seen that for keeping the electronic motion in the non-relativistic regime we have to demand that $\mu(\omega/\Delta\omega) \ll 1$ holds which is a stronger condition than $\mu \ll 1$. This has to be so if the system is close to resonance, i.e. $|\omega - \omega_c| \equiv \Delta\omega \ll \omega$. In the case of exact resonance ($\omega = \omega_c$) the term $eF/m(\omega - \omega_c)$ in equation (5) has to be replaced by $-\mathrm{i}(eF/m)t$ which yields a divergent velocity as time goes on. Of course, under any real experimental conditions the electron is always subject to certain friction forces (due to collisions or simply due to the radiation reaction force) which results in finite velocity amplitudes even at exact resonance. This aspect of the problem shall not be considered here.

Now we calculate the radiation field emitted by the electron at the point of observation ($\boldsymbol{R} = R\boldsymbol{n}'$) which is far away from the bounded region of the electronic motion. To this end we use the general formula (Jackson 1962)

$$\boldsymbol{E}_{\mathrm{rad}}(\boldsymbol{R}, t) \simeq -\frac{e}{Rc^2}\boldsymbol{n}' \times \left[\boldsymbol{n}' \times \int \mathrm{d}t' \, \dot{\boldsymbol{v}}(t')\delta\left(t - \frac{R}{c} + \frac{1}{c}\boldsymbol{n}' \cdot \boldsymbol{r}(t') - t'\right)\right]. \tag{7}$$

Since in the present paper we study the scattering of the high-frequency wave the relevant source term in equation (7) is determined by

$$\dot{\boldsymbol{v}}_\Omega(t') \equiv -\frac{ef}{m}\boldsymbol{\mathscr{E}} \sin[\Omega(t' - c^{-1}\boldsymbol{n} \cdot \boldsymbol{r}(t))]. \tag{8}$$

In equation (8) we again take into account the space dependence of the incident light because otherwise we would lose the information about its propagation direction. According to our original assumptions we can obtain from equation (6) the relevant contribution to the trajectory

$$\boldsymbol{r}(t') \simeq \boldsymbol{\mathscr{E}}_x\left(\frac{v}{\omega_c}\sin(\omega_c t' + \chi) + \frac{eF}{m\omega(\omega - \omega_c)}\sin \omega t'\right) - \boldsymbol{\mathscr{E}}_y\left(\frac{v}{\omega_c}\cos(\omega_c t' + \chi) + \frac{eF}{m\omega(\omega - \omega_c)}\cos \omega t'\right). \tag{9}$$

After putting equations (9) and (8) into equation (7) and expressing the delta function by the usual Fourier integral the time integration can be easily carried out if we use the Jacobi–Anger formula (Erdélyi *et al* 1953)

$$\exp(\mathrm{i}a\sin\phi)=\sum_{n=-\infty}^{\infty}J_n(a)\exp(\mathrm{i}n\phi)$$

for the generation of ordinary Bessel functions J_n of order n. We obtain

$$\boldsymbol{E}_{\mathrm{rad}}(\boldsymbol{R},t)=\frac{e^2f}{Rmc^2}\boldsymbol{n}'\times(\boldsymbol{n}'\times\boldsymbol{\mathscr{E}})\sum_{nl}J_n\left(\frac{v}{\omega_{\mathrm{c}}}Q_{nl}^{\perp}\right)$$
$$\times J_l(\mu(\omega/\Delta\omega)Q_{nl}^{\perp}/k)\sin[\Omega_{nl}(t-R/c)+n\chi-(n+1)\phi_{nl}] \qquad (10)$$

where $\Omega_{nl}\equiv\Omega+n\omega_{\mathrm{c}}+l\omega$ and $Q_{nl}^{\perp}$ is the perpendicular component of the scattering vector

$$\boldsymbol{Q}_{nl}\equiv\frac{1}{c}(\Omega_{nl}\boldsymbol{n}'-\Omega\boldsymbol{n})\equiv\boldsymbol{K}'_{nl}-\boldsymbol{K}$$

with $Q^{\perp}\equiv((\boldsymbol{Q}\mathscr{E}_x)^2+(\boldsymbol{Q}\mathscr{E}_y)^2)^{1/2}$. Moreover, $\sin\phi_{nl}=\mathscr{E}_y\boldsymbol{Q}_{nl}/Q_{nl}^{\perp}$ and $k\equiv\omega/c$ is the wavenumber of the microwave. Equation (10) reveals that the scattered radiation will be decomposed into contributions of outgoing waves of frequencies Ω_{nl}. The differential scattering cross sections for these incoherent components can be obtained by standard procedures, to yield

$$\mathrm{d}\sigma_{nl}=\mathrm{d}\sigma_{\mathrm{Th}}J_n^2(\rho_{\mathrm{c}}Q_{nl}^{\perp})J_l^2(\mu(\omega/\Delta\omega)Q_{nl}^{\perp}/k) \qquad (11)$$

where $\rho_{\mathrm{c}}=v/\omega_{\mathrm{c}}$ and $\mathrm{d}\sigma_{\mathrm{Th}}\equiv(e^2/mc^2)^2|\boldsymbol{\mathscr{E}}\times\boldsymbol{n}'|^2\,\mathrm{d}\Omega$ is the usual differential cross section of ordinary Thomson scattering.

If no low-frequency radiation is present ($\mu=0$) equation (11) reduces to the well known formula describing the scattering by an electron in a magnetised plasma (Sheffield 1975). On the other hand, if we assume that the magnetic field is zero then in equations (5) and (6) there are no terms oscillating with the cyclotron frequency and the cross section reduces to the following form

$$\mathrm{d}\sigma_l=\mathrm{d}\sigma_{\mathrm{Th}}J_l^2(\mu Q_l^{\perp}/k) \qquad Q_l^{\perp}\equiv Q_{0l}^{\perp}. \qquad (12)$$

Formula (12) was first given by Guccione-Gush and Gush (1975) and rederived by one of the present authors in a paper concerned with x-ray scattering in the presence of a strong laser field (Ehlotzky 1978).

If the electron is initially at rest ($\rho_{\mathrm{c}}=0$), equation (11) reduces to the formula

$$\mathrm{d}\sigma_l=\mathrm{d}\sigma_{\mathrm{Th}}J_l^2(\mu(\omega/\Delta\omega)Q_l^{\perp}/k). \qquad (13)$$

If we compare equations (12) and (13) we can see that the argument of the Bessel function in equation (13) contains an extra factor $(\omega/\Delta\omega)$ which can be quite large when the system is close to resonance. This means that due to the presence of the magnetic field the scattering pattern determined by equation (13) could be more easily observed than the corresponding one in the magnetic-field-free case of equation (12).

3. Coherent states of an electron embedded in a magnetic field and in a radiation field

The Schrödinger equation of an electron embedded in the homogeneous magnetic field $\boldsymbol{B}=B(0,0,1)$ with vector potential $\boldsymbol{A}_B=(-\frac{1}{2}By,\frac{1}{2}Bx,0)$ and in the fields given in equations (2) and (3) reads

$$\frac{1}{2m}[(\hat{p}+A_1)^2+(m\omega_c\hat{q}+A_2)^2+(\hat{p}_z+A_3)^2]\Psi = i\hbar\frac{\partial\Psi}{\partial t} \tag{14}$$

where

$$\hat{p}\equiv\hat{p}_x-\tfrac{1}{2}m\omega_c\hat{y} \qquad \hat{q}\equiv(1/m\omega_c)(\hat{p}_y+\tfrac{1}{2}m\omega_c\hat{x}) \tag{14a}$$

satisfy the commutation rule $[\hat{p},\hat{q}]=-i\hbar$, and we abbreviate $(A_1, A_2, A_3)\equiv ec^{-1}(A_x, A_y, A_z)$. (A_x, A_y, A_z) is the vector potential of the sum of the two fields $\boldsymbol{E}_\omega$ and $\boldsymbol{E}_\Omega$.

By using the ansatz

$$\Psi=\exp\left(\frac{i}{\hbar}p_z z-\frac{i}{\hbar}\int^t\frac{1}{2m}[(p_z+A_3(\tau))^2+A_1^2(\tau)+A_2^2(\tau)]\,d\tau\right)\psi$$
$$=\psi_{p_z}(z,t)\psi\exp\left(-\frac{i}{\hbar}\int^t\frac{1}{2m}(A_1^2(\tau)+A_2^2(\tau))\,d\tau\right) \tag{15}$$

the z dependence and the unimportant time dependencies of Ψ can be eliminated. We note that in equation (15) $\psi_{p_z}(z,t)$ represents a non-relativistic Gordon–Volkov state with momentum p_z.

Now we introduce creation and annihilation operators by the following definitions

$$\hat{a}\equiv\left(\frac{m\omega_c}{2\hbar}\right)^{1/2}\hat{q}+i\left(\frac{1}{2m\hbar\omega_c}\right)^{1/2}\hat{p}$$
$$\hat{a}^\dagger=\left(\frac{m\omega_c}{2\hbar}\right)^{1/2}\hat{q}-i\left(\frac{1}{2m\hbar\omega_c}\right)^{1/2}\hat{p}. \tag{16}$$

By means of these operators the reduced equation for ψ can then be put into the form

$$[\hbar\omega_c(\hat{a}^\dagger\hat{a}+\tfrac{1}{2})-i\hbar(g^*\hat{a}-g\hat{a}^\dagger)]\psi=i\hbar\frac{\partial\psi}{\partial t} \tag{17}$$

where

$$g\equiv g(t)\equiv(\omega_c/2m\hbar)^{1/2}(A_1-iA_2). \tag{18}$$

Equation (17) describes a displaced harmonic oscillator and its solution can be obtained with the help of the disentangling techniques which have been used earlier by the present authors (Bergou *et al* 1982). Using the ansatz

$$\psi=D_\sigma\Phi\exp\left(-i\int^t \mathrm{Im}(g^*\sigma)\,d\tau\right) \tag{19}$$

with

$$D_\sigma\equiv\exp(\sigma\hat{a}^\dagger-\sigma^*\hat{a}) \tag{20}$$

we see that the displacement operator D_σ eliminates the interaction terms from the left-hand side of equation (17) if the unknown complex function σ satisfies the following differential equation

$$\dot{\sigma}=-i\omega_c\sigma+g. \tag{20a}$$

A particular solution to equation (20a) is given by

$$\sigma(t)=\left(\frac{m\omega_c}{8\hbar}\right)^{1/2}\left(\frac{2}{m}\right)\exp(-\mathrm{i}\omega_c t)\int(A_1(\tau)-\mathrm{i}A_2(\tau))\exp(\mathrm{i}\omega_c\tau)\,\mathrm{d}\tau. \tag{20b}$$

Then the reduced equation for Φ is a Schrödinger equation for an electron being embedded in the magnetic field alone

$$\hbar\omega_c(\hat{a}^\dagger\hat{a}+\tfrac{1}{2})\Phi=\mathrm{i}\hbar\frac{\partial\Phi}{\partial t}. \tag{21}$$

The stationary solutions to equation (21) are

$$\Phi_n(t)=\Phi_n\exp[-\mathrm{i}\omega_c(n+\tfrac{1}{2})t] \qquad n=0,1,2,\ldots \tag{22}$$

where the Φ_n are normalised energy eigenstates which can be expressed in terms of cylindrical coordinates (Landau and Lifshitz 1958).

Instead of directly inserting these states $\Phi_n(t)$ into equation (19), we first construct a coherent superposition of these states by the following definition

$$\Phi_\beta\equiv\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!}}\phi_n\exp(-\tfrac{1}{2}|\beta|^2)$$
$$\beta\equiv\left(\frac{m\omega_c}{8\hbar}\right)^{1/2}\beta_1\equiv\left(\frac{m\omega_c}{8\hbar}\right)^{1/2}\beta_2\exp(-\mathrm{i}\omega_c t) \tag{23}$$

where β_2 is a time-independent but otherwise arbitrary complex number. $\Phi_\beta\exp(-\frac{1}{2}\mathrm{i}\omega_c t)$ is also a solution of equation (21). Moreover the Φ_β are eigenstates of the annihilation operator $\hat{a}$

$$\hat{a}\Phi_\beta=\beta\Phi_\beta \tag{24}$$

and they can be generated from the ground state by a displacement operation with parameter β (Glauber 1963)

$$\Phi_\beta=D_\beta\Phi_0. \tag{25}$$

In order to see the physical meaning of the coherent state Φ_β, we now determine its explicit form in coordinate representation by introducing cylindrical coordinates, which are related to the Cartesian coordinates x, y by $x=\rho\cos\phi$, $y=\rho\sin\phi$. Taking into account the definitions in equations (16), (14a) and (23), the eigenvalue equation (24) can be brought into the form

$$\left(\frac{\partial}{\partial\rho}-\frac{\mathrm{i}}{\rho}\frac{\partial}{\partial\phi}\right)\Phi_\beta=-\gamma[\rho-\beta_1\exp(\mathrm{i}\phi)]\Phi_\beta \tag{26}$$

with $\gamma\equiv m\omega_c/2\hbar$. The normalised solution to equation (26) is then

$$\Phi_\beta=(\gamma/\pi)^{1/2}\exp\{-\tfrac{1}{2}\gamma[\rho^2-\beta_1\rho\exp(\mathrm{i}\phi)+|\tfrac{1}{2}\beta_1|^2]\}. \tag{27}$$

If we now choose $\beta_1=2(v/\omega_c)\exp[-\mathrm{i}(\omega_c t+\chi)]$, where v is the velocity and χ the phase of the classical gyration of the electron respectively, then the probability distribution for that coherent state is

$$|\Phi_\beta|^2=(\gamma/\pi)\exp\left\{-\gamma\left[\left(x-\frac{v}{\omega_c}\sin(\omega_c t+\chi)\right)^2+\left(y+\frac{v}{\omega_c}\cos(\omega_c t+\chi)\right)^2\right]\right\}. \tag{28}$$

Thus the centre of the Gaussian packet equation (28) moves along the classical trajectory, namely on a circle of radius $(v/\omega_c) \equiv \rho_c$ around the origin. $\gamma^{-1/2} \equiv \lambda_B$ is the characteristic length of the packet and it is inversely proportional to the square root of the magnetic field strength.

It can be shown that more general coherent states of a similar kind can be constructed by using a slightly different method. These generalised coherent states are characterised by two arbitrary complex numbers, determining on the one hand the velocity (radius) and the phase of the electron gyration, and on the other hand the position of the gyration centre in the xy plane. A detailed analysis of the generalised coherent states of this kind will be published in a forthcoming paper (Varró 1984).

By collecting all the information contained in the equations (25), (19) and (15) and by using the multiplication theorem $D_\sigma D_\beta = D_{\sigma+\beta} \exp[\mathrm{i}\,\mathrm{Im}(\sigma\beta^*)]$ of the displacement operators (Bergou *et al* 1982), the packet solution of the original Schrödinger equation (14) can be brought into the form

$$\Psi = \psi_{p_z} \Phi_\alpha \exp(-\mathrm{i}\delta) \tag{29}$$

where $\alpha \equiv \beta(t) + \sigma(t)$ and

$$\delta \equiv \int^t \left(\frac{1}{2m\hbar}(A_1^2 + A_2^2) + \mathrm{Im}(g^*\sigma) - \mathrm{Im}(\beta^*\sigma) \right) \mathrm{d}\tau + \tfrac{1}{2}\omega_c t. \tag{29a}$$

δ is real and a function of time. Φ_α is a coherent state of the form given in equation (27). Taking into account the expression for σ, (equation (20*b*)), we can easily show that $\alpha \equiv \beta + \sigma = \gamma^{1/2} u(t)$, where $u(t) = x(t) - \mathrm{i}y(t)$ is just the classical complex trajectory which is given in equation (6). Consequently in equation (29) Φ_α has the following explicit form

$$\Phi_\alpha(\rho, \phi) = (\gamma/\pi)^{1/2} \exp[-\tfrac{1}{2}\gamma(\rho^2 - 2u(t)\rho \exp(\mathrm{i}\phi) + |u(t)|^2)]. \tag{30}$$

If here we take the limit $\hbar \to 0$ $(\gamma \to \infty)$ the probability density $|\Phi_\alpha|^2$ becomes a Dirac delta function representing a point-like particle moving on the classical trajectory $u(t)$.

4. Semiclassical description of the scattering process

The probability density (P) and the probability current density ($\boldsymbol{j}$) of the electron being in the state Ψ (equation (29)) can be calculated from the usual definitions $P \equiv |\Psi|^2$, $\boldsymbol{j} \equiv \frac{1}{2}(\Psi^* \hat{\boldsymbol{v}} \Psi + \Psi \hat{\boldsymbol{v}}^* \Psi^*)$, where $\hat{\boldsymbol{v}}$ is the velocity operator in coordinate representation. We obtain

$$P = |\Phi_\alpha|^2 / L \tag{31a}$$

$$j_x = [v_x - \tfrac{1}{2}\omega_c(y - y(t))]P \tag{31b}$$

$$j_y = [v_y + \tfrac{1}{2}\omega_c(x - x(t))]P \tag{31c}$$

$$j_z = v_z P \tag{31d}$$

where (v_x, v_y, v_z) and $(x(t), y(t), 0)$ characterise the classical trajectory as given by equations (4), (5) and (6). In equation (31*a*) L is the normalisation length along the z direction, and from equation (30) we can easily determine $|\Phi_\alpha|^2$.

The current $(v_x, v_y, v_z)P$ represents a rigid, density distribution following the classical trajectory of the electron. If we eliminate this overall motion by transforming to the centre of the packet, we only see the transverse current

$$j_\perp = \tfrac{1}{2}\omega_c \rho e_\phi (\pi L \lambda_B^2)^{-1/2} \exp(-\rho^2/\lambda_B^2). \tag{32}$$

e_ϕ is the usual polar vector in the moving frame with $e_\phi = (-\sin\phi, \cos\phi, 0)$. Equation (32) shows that the packet rotates around its centre and that the velocities of the local current elements are different from one point to another. It can be demonstrated that the maximum value of these velocities is of order of the zero-point velocity v_0 which is defined by the relation $\hbar\omega_c = mv_0^2$. Usually v_0 is very small, so that the packet practically behaves like a rigid distribution.

In order to determine the scattered radiation we consider $-eP$ and $-ej$ as the actual charge and current distributions respectively, and we introduce them as source terms into Maxwell's equations. By using the same approximations and mathematical tools as in the classical description of the process, we can derive the following differential cross sections for the components of the scattered field of frequencies Ω_{nl}

$$d\sigma_{nl} = d\sigma_{\mathrm{Th}} |F(Q_{nl})|^2 J_n^2(\rho_c Q_{nl}^\perp) J_l^2(\mu(\omega/\Delta\omega) Q_{nl}^\perp/k). \tag{33}$$

Here we have used the same notations as in equation (11). $F(Q_{nl})$ is the form factor of the packet solution equation (29) or, in other words, it is the Fourier transform of the distribution $(\pi L\lambda_B^2)^{-1} \exp[-(x^2+y^2)/\lambda_B^2]$. So the modifications of the classical formula equation (11) due to the quantum spreading of the electron is determined by the factor

$$|F(Q_{nl})|^2 = \exp[-\tfrac{1}{2}(\lambda_B Q_{nl}^\perp)^2]. \tag{34}$$

As we have already mentioned, the characteristic length of the electronic wavefunctions is $\lambda_B \sim 10^5$ Å$/\sqrt{B}$, where B is measured in Gauss. For light of wavelength $\lambda \sim 5\times10^3$ Å we get $\lambda_B/\lambda \sim 20/\sqrt{B}$. Clearly the dipole approximation is valid only if λ_B is much smaller than the wavelength of the light. Thus we have to require $20/\sqrt{B} \ll 1$ to hold. Since $Q_{nl}^\perp \leqslant 2\pi/\lambda$ the squared modulus of the form factor equation (34) is of order of unity in that case and we recover the classical formula equation (11).

5. Summary

In the preceding sections we have seen that a very close relation can be established between the classical treatment of radiation scattering by an electron in a microwave and in a magnetic field and the corresponding quantum mechanical calculations if quantum mechanically the electron is described by a coherent superposition of Landau states. This also permits us to relate the various limiting cases to the proper classical situations and to trace the boundary value problem of quantum mechanics to the corresponding initial value problem of classical electrodynamics, thus showing in particular how to proceed in the limit $B \to 0$.

Since the generalised Thomson scattering considered here can be related to electron scattering at least in the high-energy limit by the Weizsäcker–Williams method of equivalent photons (Heitler 1954) we may expect that similar formulae to equations (33) and (34) can also be derived for electron scattering and thus the difficulty with letting $B \to 0$ can also be overcome.

Moreover our calculations have some practical implications since they indicate how a microwave field close to resonance with ω_c may add to the observation of the Bernstein modes (Bernstein 1958) and thus represent an additional diagnostic tool.

This, of course, would also require some averaging over a given velocity distribution (Segrè 1975).

Classically we could have added the action of damping phenomena by some phenomenological friction term, but quantum mechanically this is a much more complicated problem which we have not touched upon here.

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