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Coherent states of an electron in a homogeneous constant magnetic field and the zero magnetic field limit

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Abstract. Coherent states of an electron embedded in a constant homogeneous magnetic field are constructed. The centres of the probability distributions belonging to these states gyrate along possible classical trajectories. Suitable packets of such coherent states are defined which reduce to properly normalised free electronic states in the zero magnetic field limit. A simple example is given to illustrate the dynamics of free electron localisation due to the presence of a magnetic field.

1. Introduction

The stationary solutions to the Schrödinger equation of an electron in a constant homogeneous magnetic field, the Landau states, have long been known and can be found in every textbook on quantum mechanics (Landau and Lifshitz 1975). Though the Landau states are very powerful and a commonly used mathematical tool for describing various physical processes which take place in uniform magnetic fields, they have a serious insufficiency. Namely, they do not reproduce any kind of free electronic states in the zero magnetic field limit.

Recently many physicists have for example worked on the theory of potential scattering in the presence of external electromagnetic fields. In particular, induced and inverse bremsstrahlung processes in the presence of a uniform magnetic field are being very extensively studied due to their importance in fusion research (Seely 1974, Ferrante *et al* 1979, Bergou *et al* 1982). The difficulty with these calculations is that the scattering cross sections do not take the field-free form if Landau states are used as initial and final states (see also Ventura 1973). The source of these inconsistencies is clearly the application of Landau states, so it is important to find suitable superpositions of the Landau states which behave properly in the zero magnetic field limit (see also Faisal 1982).

The aim of this paper is to present a possible solution to the problem of the zero magnetic field limit. In § 2 we give an elementary method to construct coherent states of a Schrödinger electron embedded in a constant homogeneous magnetic field. Our result coincides with that obtained by Malkin and Man'ko (1969) who used a different approach to the problem. In § 3 it is proved that a special class of coherent states is able to reproduce free electron states in the field-free limit. In § 4 a short summary completes our analysis.

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2. Coherent states of an electron in a homogeneous constant magnetic field

Coherent states of a charged particle in a magnetic field were first published by Malkin and Man'ko (1969) who have developed a general formal theory of such states based on suitably defined creation and annihilation operators of the quantum excitation of the system under discussion. The more general problem of coherent states of charged particles in a uniform electric field and a magnetic field has been thoroughly discussed by Johnson *et al* (1983). In the present paper we concentrate on the problem of the zero magnetic field limit and we do not need the complete theory developed by Malkin and Man'ko. The explicit form of the mentioned coherent states in coordinate representation will be used in § 3.

Due to this, we think that it is not completely useless to construct them by using an alternative and simpler (though less general) method which is based simply on the well known one-dimensional harmonic oscillator wavefunctions.

The coherent states to be presented here correspond to gaussian probability distributions of the electron's position on the x–y plane gyrating along circles with the usual cyclotron frequency $\omega_c \equiv (|e|B/Mc)$ (e and M are the electron's charge and mass, respectively, c is the velocity of light in vacuum and B is the magnetic field strength). So the centres of these distributions move along possible classical trajectories. Their widths are the same, namely the usual magnetic length $\gamma^{-1/2} \equiv (2\hbar c/|e|B)^{1/2}$ ($\hbar$ is the Planck constant divided by 2π). The mentioned coherent states are parametrised by two complex numbers representing the velocity, the initial phase and the position vector of the guiding centre of the gyration.

The Schrödinger equation for an electron in a uniform constant magnetic field directed along the z axis reads

$$\begin{aligned}\frac{1}{2M}\left[\left(p_x-\frac{|e|B}{2c}y\right)^2+\left(p_y+\frac{|e|B}{2c}x\right)^2\right]\phi &= \left(\frac{p_x^2}{2M}+\tfrac{1}{2}Mx^2\omega^2+\frac{p_y^2}{2M}+\tfrac{1}{2}My^2\omega^2+\omega L_z\right)\phi \\ &= \mathrm{i}\hbar\,\partial\phi/\partial t, \qquad (2.1)\end{aligned}$$

where $\omega \equiv (|e|B/2Mc) = (\omega_c/2)$ is the Larmor frequency, $L_z \equiv xp_y - yp_x$ is the z component of the angular momentum operator, and $p_x \equiv -\mathrm{i}\hbar(\partial/\partial x)$, $p_y \equiv -\mathrm{i}\hbar(\partial/\partial y)$ are the canonical momentum operators. In (2.1) and henceforth we adopt a symmetric gauge vector potential $(B/2)(-y, x, 0)$ and leave out of account the irrelevant z-dependence of the electronic motion. By introducing the polar coordinates ρ and φ with the definitions $x=\rho\cos\varphi$, $y=\rho\sin\varphi$, equation (2.1) can be brought to the form

$$-\frac{\hbar^2}{2M}\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\phi}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\phi}{\partial\varphi^2}\right]-\frac{\mathrm{i}\hbar\omega_c}{2}\frac{\partial\phi}{\partial\varphi}+\tfrac{1}{8}M\omega_c^2\rho^2\phi=\mathrm{i}\hbar\frac{\partial\phi}{\partial t}. \qquad (2.2)$$

The stationary solutions to (2.2) are the well known Landau states (Landau and Lifshitz 1975)

$$\phi_{lm}(\xi,\varphi;t)=\left(\frac{\gamma}{\pi}\right)^{1/2}\left(\frac{l!}{(|m|+l)!}\right)^{1/2}\mathrm{e}^{\mathrm{i}m\varphi}\,\mathrm{e}^{-\xi/2}\xi^{|m|/2}L_l^{|m|}(\xi)\exp\left(-\frac{\mathrm{i}}{\hbar}E_{lm}t\right) \qquad (2.3)$$

where

$$\xi \equiv \gamma\rho^2, \qquad \gamma \equiv \frac{|e|B}{2\hbar c} = \frac{M\omega_c}{2\hbar}, \tag{2.3a}$$

$$m = 0, \pm 1, \pm 2, \ldots, \qquad l = 0, 1, 2, \ldots, \qquad E_{lm} \equiv \hbar\omega_c[l + \tfrac{1}{2}(m + |m| + 1)] \tag{2.3b}$$

and $L_l^{|m|}$ denote associated Laguerre polynomials (Gradshteyn and Ryzhik 1980). These states form an orthonormalised set

$$\int_0^{2\pi} d\varphi \int_0^\infty \frac{d\xi}{2\gamma} \phi^*_{lm}(\xi, \varphi; t)\phi_{l'm'}(\xi, \varphi; t) = \delta_{ll'}\delta_{mm'}. \tag{2.3c}$$

From (2.3) it is clear that the stationary states ϕ_{lm} reduce to physically meaningless results in the zero magnetic field limit ($\gamma \to 0$).

Instead of using the states given by (2.3) for constructing coherent states we first turn back to the original equation (2.1) and look for another class of solutions to it.

The Hamiltonian in (2.1) represents two coupled linear oscillators with the energy-conserving coupling term ωL_z. This term can be easily eliminated by introducing the ansatz

$$\phi = \exp[-(i/\hbar)\omega L_z t]\psi = \exp(-\omega t\, \partial/\partial\varphi)\psi, \tag{2.4}$$

with ψ satisfying the Schrödinger equation of two independent oscillators of the same frequency ω. The stationary solutions to this reduced equation are simply products of the corresponding linear oscillator wavefunctions (Landau and Lifshitz 1975)

$$\psi_{nk}(x, y; t) = \phi_n(x)\phi_k(y)\, e^{-i\omega(n+k+1)t}, \qquad n, k = 0, 1, 2, \ldots, \tag{2.5}$$

where

$$\phi_n(x) = \left(\frac{M\omega}{\pi\hbar}\right)^{1/4} \frac{1}{(2^n n!)^{1/2}} \exp\left[-\left(\frac{M\omega}{2\hbar}\right)x^2\right] H_n\left[\left(\frac{M\omega}{\hbar}\right)^{1/2} x\right], \tag{2.5a}$$

$H_n(x)$ is a Hermite polynomial (Gradshteyn and Ryzhik 1980). To obtain $\phi_k(y)$ we should replace in (2.5*a*) n and x by k and y, respectively.

Remembering the expansion of coherent states of a one-dimensional harmonic oscillator in terms of stationary number states (see e.g. Schiff 1955 or Glauber 1963) and taking into account the ansatz (2.4), we define the following coherent states parametrised by two complex numbers α and β:

$$\begin{aligned}\phi_{\alpha\beta}(x, y; t) \equiv{}& \exp\left(-\omega t \frac{\partial}{\partial\varphi}\right)\left(\sum_{n=0}^{\infty} \frac{[(M\omega/2\hbar)^{1/2}\alpha]^n}{\sqrt{n!}} \phi_n(x)\, e^{-in\omega t}\right) \\ &\times \left(\sum_{k=0}^{\infty} \frac{[(M\omega/2\hbar)^{1/2}\beta]^k}{\sqrt{k!}} \phi_k(y)\, e^{-ik\omega t}\right) \\ &\times \exp\left[-\frac{1}{2}\left(\frac{M\omega}{2\hbar}\right)(|\alpha|^2 + |\beta|^2)\right] e^{-i\omega t}.\end{aligned} \tag{2.6}$$

Introducing polar coordinates ρ and φ into (2.6) and using the generating formula for Hermite polynomials (Gradshteyn and Ryzhik 1980)

$$\sum_{n=0}^{\infty} \frac{s^n}{n!} H_n(x) = \exp(-s^2 + 2sx),$$

after applying the rotation operator $\exp(-\omega t\, \partial/\partial\varphi)$ we get the following explicit form for $\phi_{\alpha\beta}$:

$$\begin{aligned}\phi_{\alpha\beta}(\rho,\varphi;t) = {} & (M\omega_c/2\pi\hbar)^{1/2}\exp(-\tfrac{1}{2}\mathrm{i}\omega_c t)\exp\{-(M\omega_c/4\hbar)\\ & \times[\rho^2+|\alpha_-|^2+|\alpha_+|^2+2\alpha_-\alpha_+\exp(-\mathrm{i}\omega_c t)\\ & -2\alpha_-\rho\,\mathrm{e}^{\mathrm{i}\varphi}\exp(-\mathrm{i}\omega_c t)-2\alpha_+\rho\,\mathrm{e}^{-\mathrm{i}\varphi}]\},\end{aligned} \tag{2.7}$$

where

$$\alpha_\pm \equiv \tfrac{1}{2}(\alpha \pm \mathrm{i}\beta). \tag{2.7a}$$

Of course, the coherent state $\phi_{\alpha\beta}$ given by (2.7) satisfies the Schrödinger equation (2.2) by construction, but this can also be proved by direct substitution.

Apart from notational differences (2.7) coincides with the coherent state given by Malkin and Man'ko (see equation (39) in their paper).

To see the physical meaning of the coherent state $\phi_{\alpha\beta}$ first let us cite the classical description of cyclotronic motion. The Newtonian equation of an electron in the uniform magnetic field under discussion reads $\dot{\boldsymbol{v}} = -\omega_c(\boldsymbol{v}\times\boldsymbol{n})$, where $\boldsymbol{n}$ is a unit vector along the z axis and $\boldsymbol{v}$ is the velocity of the electron. The solution to this classical equation is as follows:

$$\begin{aligned}x_c(t) &= (v/\omega_c)[\cos(\omega_c t+\chi-\pi/2)-\cos(\chi-\pi/2)]+x(0),\\ y_c(t) &= (v/\omega_c)[\sin(\omega_c t+\chi-\pi/2)-\sin(\chi-\pi/2)]+y(0),\end{aligned} \tag{2.8}$$

where $(x(0), y(0))$ is the initial position vector and $v(\cos\chi, \sin\chi) \equiv (v_x(0), v_y(0))$ is the initial velocity on the x–y plane at $t=0$.

Now, if we calculate the modulus squared of $\phi_{\alpha\beta}$ given by (2.7) we can easily realise that if we identify α_- and α_+ with the following combination of classical quantities,

$$\alpha_- = (v/\omega_c)\,\mathrm{e}^{-\mathrm{i}(\chi-\pi/2)}, \qquad \alpha_+ = u(0)-\alpha_-^*, \qquad u(0)\equiv x(0)+\mathrm{i}y(0), \tag{2.9}$$

the result for $|\phi_{\alpha\beta}|^2$ can be brought to the form

$$|\phi_{\alpha\beta}|^2 = \left(\frac{M\omega_c}{2\pi\hbar}\right)\exp\left[-\left(\frac{M\omega_c}{2\hbar}\right)[(x-x_c(t))^2+(y-y_c(t))^2]\right]. \tag{2.10}$$

Equation (2.10) tells us that $|\phi_{\alpha\beta}|^2$ is a gaussian probability distribution of width $(2\hbar/M\omega_c)^{1/2} = \gamma^{-1/2}$ whose centre follows the classical trajectory (2.8) without changing shape, so $\phi_{\alpha\beta}$ is really a coherent state in the usual sense. By taking the classical limit $\hbar\to 0$ we get from (2.10) a two-dimensional Dirac delta distribution describing the classical motion of a point-like electron.

The connection between the coherent states $\phi_{\alpha\beta}$ and the usual Landau states given by (2.7) and (2.3), respectively, can be established by expanding the exponential

$$\exp\left[-\left(\frac{M\omega_c}{4\hbar}\right)[2\alpha_-\alpha_+\exp(-\mathrm{i}\omega_c t)-2\alpha_-\rho\,\mathrm{e}^{\mathrm{i}\varphi}\exp(-\mathrm{i}\omega_c t)-2\alpha_+\rho\,\mathrm{e}^{-\mathrm{i}\varphi}]\right]$$

into power series and rearranging the triple sum we get after that expansion. The result is

$$\phi_{\alpha\beta} \equiv \phi_{ab}(\xi,\varphi;t) = \sum_{m=-\infty}^{\infty}\sum_{l=0}^{\infty} C_{lm}\phi_{lm}(\xi,\varphi;t), \tag{2.11}$$

where we have introduced the notations

$$a \equiv (M\omega_c/2\hbar)^{1/2}\alpha_-, \qquad b \equiv (M\omega_c/2\hbar)^{1/2}\alpha_+, \tag{2.11a}$$

$$C_{lm} = \begin{cases} \dfrac{(-ab)^l a^m}{[l!(l+m)!]^{1/2}} \exp[-\frac{1}{2}(|a|^2+|b|^2)], & m \geq 0, \\ \dfrac{(-ab)^l b^{|m|}}{[l!(l+|m|)!]^{1/2}} \exp[-\frac{1}{2}(|a|^2+|b|^2)], & m < 0. \end{cases} \tag{2.11b}$$

The probability distribution of Landau states ϕ_{lm} is a two-dimensional Poisson distribution. It can be proved that

$$\sum_{m=-\infty}^{\infty} \sum_{l=0}^{\infty} |C_{lm}|^2 = \left(\sum_{m=-\infty}^{\infty} \left|\frac{a}{b}\right|^m I_m(2|ab|) \right) \exp(-|a|^2 - |b|^2) = 1. \tag{2.11c}$$

In (2.11c) we have used the power series representation of the modified Bessel function I_m

$$I_m(z) = \sum_{l=0}^{\infty} \frac{(z/2)^{m+2l}}{(m+l)!\, l!},$$

and the relation

$$\exp\left[\frac{z}{2}\left(s + \frac{1}{s}\right)\right] = \sum_{m=-\infty}^{\infty} s^m I_m(z)$$

(Gradshteyn and Ryzhik 1980). Of course, it is not very surprising that the distribution $|C_{lm}|^2$ is properly normalised as stated by (2.11c) since the coherent state $\phi_{\alpha\beta}$ is normalised to unity and the states ϕ_{lm} form an orthonormalised set (see (2.3c)).

3. The zero magnetic field limit

In the present section we shall consider only a special class of coherent states given by (2.7). We set $u(0) = 0$ in (2.9), so $\alpha_+ = -\alpha_-^*$ in this case. The special coherent state $\phi_{\alpha\beta}$ corresponding to these particular values of α_- and α_+ can now be characterised by a two-component real vector $\boldsymbol{K} \equiv [M\boldsymbol{v}(0)/\hbar] \equiv K(\cos\chi, \sin\chi)$, where $\boldsymbol{v}(0)$ is the initial velocity of the classical motion. The centre of the circle along which the electron gyrates is given by the vector $(1/2\gamma)\tilde{\boldsymbol{K}} \equiv (1/2\gamma)K(-\sin\chi, \cos\chi)$ which is perpendicular to the vector $\boldsymbol{K}$ as is shown in figure 1.

If the magnetic field goes to zero ($\gamma \to 0$) the radius of the trajectory goes to infinity and the circle is degenerated to a straight line representing a uniform motion of a free electron with the constant velocity $\boldsymbol{v}(0) = (\hbar\boldsymbol{K}/M)$. Since the centre of the probability distribution $|\phi_{\alpha\beta}|^2$ follows the classical trajectory and the width of the distribution is proportional to $B^{-1/2}$, one may expect that in the zero magnetic field limit the coherent state $\phi_{\alpha\beta}$ will be reduced to a free electronic plane wave with wavevector $\boldsymbol{K}$. To show this, first we write down the explicit form of such a coherent state:

$$\phi_{\alpha\beta} = \left(\frac{\gamma}{\pi}\right)^{1/2} \exp\left(-\frac{\mathrm{i}}{2}\omega_c t\right) \exp\left[-\frac{\gamma}{2}\rho^2 - \mathrm{i}\frac{\omega_c}{4\gamma}K^2\left(\frac{\exp(-\mathrm{i}\omega_c t) - 1}{-\mathrm{i}\omega_c t}\right)t + \tfrac{1}{2}\mathrm{i}K\rho\left(\mathrm{e}^{\mathrm{i}(\varphi-\chi)}\exp(-\mathrm{i}\omega_c t) + \mathrm{e}^{-\mathrm{i}(\varphi-\chi)}\right)\right]. \tag{3.1}$$

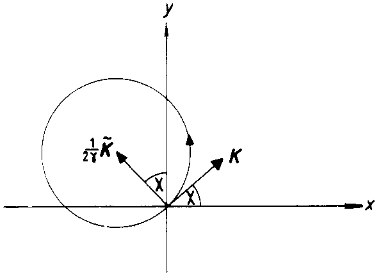

Figure 1. The classical trajectory of an electron passing through the origin of the x–y plane with initial velocity $v(0)=(\hbar K/M)$. The centre of the cyclotronic motion of the electron is represented by the vector $(\tilde{K}/2\gamma)$, where $\tilde{K}$ is perpendicular to K and has the same length as K, and $\gamma=(M\omega_c/2\hbar)=(|e|B/2\hbar c)$.

One can see that the exponential in (3.1) becomes a free electronic plane wave $\exp[-\mathrm{i}(\hbar K^2/2M)t+\mathrm{i}\boldsymbol{K}\boldsymbol{x}]$ in the zero field limit $\gamma\to 0$. Since $\phi_{\alpha\beta}$ contains the normalisation factor $(\gamma/\pi)^{1/2}$ the amplitude of the plane wave is zero. In order to get around this difficulty we introduce the wavefunction

$$\phi_{\boldsymbol{K}}(\boldsymbol{x};t\,|\,\gamma)\equiv(4\pi\gamma)^{-1/2}\phi_{\alpha\beta}(\rho,\varphi;t), \tag{3.2}$$

where $\phi_{\alpha\beta}$ is given by (3.1). We note that the scalar product of functions $\phi_{\boldsymbol{K}}$ reads

$$\int \mathrm{d}^2x\,\phi^*_{\boldsymbol{K}}(\boldsymbol{x};t\,|\,\gamma)\phi_{\boldsymbol{K}'}(\boldsymbol{x};t\,|\,\gamma)=\delta^{(2)}_{\gamma}(\boldsymbol{K}-\boldsymbol{K}'), \tag{3.2a}$$

where

$$\delta^{(2)}_{\gamma}(\boldsymbol{K}-\boldsymbol{K}')\equiv\frac{1}{4\pi\gamma}\exp\left(-\frac{1}{4\gamma}|\boldsymbol{K}-\boldsymbol{K}'|^2\right). \tag{3.2b}$$

The function $\delta^{(2)}_{\gamma}(\boldsymbol{K}-\boldsymbol{K}')$ introduced in (3.2b) has a remarkable property that in the $\gamma\to 0$ limit it becomes a two-dimensional Dirac delta function $\delta^{(2)}(\boldsymbol{K}-\boldsymbol{K}')$:

$$\lim_{\gamma\to 0}\delta^{(2)}_{\gamma}(\boldsymbol{K}-\boldsymbol{K}')=\delta^{(2)}(\boldsymbol{K}-\boldsymbol{K}'). \tag{3.2c}$$

Taking into account the arguments mentioned before we can easily prove the relation

$$\lim_{\gamma\to 0}\phi_{\boldsymbol{K}}(\boldsymbol{x};t\,|\,\gamma)=\frac{1}{2\pi}\exp\left(-\mathrm{i}\frac{\hbar K^2}{2M}t+\mathrm{i}\boldsymbol{K}\boldsymbol{x}\right). \tag{3.3}$$

We must mention that the coherent states whose guiding centres are fixed during the limiting procedure (see Varró *et al* 1984a) are not able to reproduce free electron states.

The result expressed by (3.3) is still not satisfactory because $\phi_{\boldsymbol{K}}(\boldsymbol{x};t\,|\,\gamma)$ has a finite norm $(4\pi\gamma)^{-1/2}$, but after the limiting procedure it becomes a non-normalisable plane wave. In order to get rid of this difficulty we construct a superposition of functions $\phi_{\boldsymbol{K}}$ with the definition

$$\phi_g(\boldsymbol{x};t\,|\,\gamma)\equiv N_g^{-1}(\gamma)\int \mathrm{d}^2K\,g(\boldsymbol{K})\phi_{\boldsymbol{K}}(\boldsymbol{x};t\,|\,\gamma), \tag{3.4}$$

where the weight factor $g(\boldsymbol{K})$ is assumed to be normalised to unity but is otherwise an arbitrary function of $\boldsymbol{K}$,

$$\int \mathrm{d}^2K\,|g(\boldsymbol{K})|^2=1. \tag{3.5a}$$

In (3.4) we have introduced the real normalisation factor $N_g(\gamma)$ with the definition $N_g^2(\gamma)\equiv\|\int \mathrm{d}^2K\,g(\boldsymbol{K})\phi_{\boldsymbol{K}}\|^2$. Taking into account (3.2*a*) we get

$$N_g^2(\gamma)=\int \mathrm{d}^2K\int \mathrm{d}^2K'\,g^*(\boldsymbol{K})g(\boldsymbol{K}')\delta_\gamma^{(2)}(\boldsymbol{K}-\boldsymbol{K}'). \tag{3.5b}$$

We note that due to (3.5*b*), (3.2*c*) and (3.5*a*), $N_g(\gamma)$ tends to unity if γ tends to zero.

Now if we take the limit γ goes to zero from (3.4) we have

$$\lim_{\gamma\to0}\phi_g(\boldsymbol{x};t\,|\,\gamma)=\int \mathrm{d}^2K\,g(\boldsymbol{K})\frac{1}{2\pi}\exp\left(-\mathrm{i}\frac{\hbar K^2}{2M}t+\boldsymbol{K}\cdot\boldsymbol{x}\right)\equiv\psi_g(\boldsymbol{x};t). \tag{3.6}$$

Equation (3.6) represents the main result of the present paper. $\phi_g(\boldsymbol{x};t\,|\,\gamma)$ is a solution to the Schrödinger equation of the electron in the presence of a constant homogeneous magnetic field and $\psi_g(\boldsymbol{x};t)$ is a free electron wavepacket. Both of them are normalised to unity and $\phi_g(\boldsymbol{x};t\,|\,\gamma)$ reduces to $\psi_g(\boldsymbol{x};t)$ in the zero magnetic field limit. We note that though coherent states form a complete set, they are not orthogonal to each other. Thus, one may expect that the applicability of such states in scattering theory as initial and final states is questionable. This problem is the subject of our forthcoming paper (Varró *et al* 1984b).

Before concluding this paper we give a simple example for the packet solutions of the type given in (3.4). Let us take the weight factor $g(\boldsymbol{K})$ in the form of a gaussian distribution of parameter Γ

$$g_0(\boldsymbol{K})\equiv\frac{1}{(2\pi\Gamma)^{1/2}}\exp\left(-\frac{1}{4\Gamma}|\boldsymbol{K}-\boldsymbol{K}_0|^2\right). \tag{3.7}$$

The probability distribution $|\phi_{g_0}(\boldsymbol{x};t\,|\,\gamma)|^2$ can be easily calculated, yielding

$$|\phi_{g_0}(\boldsymbol{x};t\,|\,\gamma)|^2=[\pi\sigma_\gamma^2(t)]^{-1}\exp\{-(\sigma_\gamma^2(t))^{-1}[(x-x_c(t))^2+(y-y_c(t))^2]\}, \tag{3.8}$$

where we have introduced the time-dependent width $\sigma_\gamma(t)$ by the definition

$$\sigma_\gamma^2(t)\equiv\frac{\Gamma}{2}\left[\frac{1}{\Gamma^2}+4\left(\frac{1}{\gamma\Gamma}+\frac{1}{\gamma^2}\right)\sin^2\frac{\omega_c}{2}t\right]\left(1+\frac{\gamma}{\Gamma}\right)^{-1}. \tag{3.8a}$$

In (3.8), $(x_c(t), y_c(t))$ represents a classical trajectory of the type given in (2.8) with $x(0)=y(0)=0$ and initial velocity $\boldsymbol{v}_0(0)=(\hbar\boldsymbol{K}_0/M)$. From (3.8) and (3.8*a*) one can see that the centre of the packet solution ϕ_{g_0} follows the classical trajectory as the simple coherent states $\phi_{\alpha\beta}$ do (see also (2.10)), but the distribution has a time-dependent width $\sigma_\gamma(t)$ which oscillates with the Larmor frequency $(\omega_c/2)$. That means that the maximum value of $|\phi_{g_0}|^2$ and the width of it vary periodically in an opposite manner. We can interpret this oscillation as a result of the competition between the natural spreading-out tendency of a free wavepacket and the contraction effect due to the presence of the magnetic field which results in the localisation of the electron.

4. Summary

The aim of the present paper was to show certain superpositions of Landau states which reduce to free electron plane waves or normalisable free electron wavepackets in the zero magnetic field limit.

The limiting procedure can be easily carried out for a classical electron if we solve the initial value problem of the Newtonian equation of motion. The classical trajectory of the well known cyclotronic motion is a circle whose radius is equal to the ratio of the modulus of the initial velocity and the cyclotron frequency. In the zero magnetic field limit the circle becomes a straight line describing a uniform motion of the electron with the same initial velocity and position as we had for the gyration along the circle when the magnetic field was on.

We have first constructed coherent states of the electron embedded in the magnetic field in the hope that they behave as properly as their classical counterparts in the zero magnetic field limit. It has been shown in the second part of our paper (see (3.3) and (3.6)) that this expectation was correct, so the states $\phi_{\boldsymbol{K}}$ introduced in (3.2) reduce to free electronic plane waves of wavevector $\boldsymbol{K}$ as the magnetic field goes to zero. We have also proved that suitable normalised superpositions of $\phi_{\boldsymbol{K}}$ reproduce normalised free electron wavepackets with the same norm in the zero field limit, thus, for coherent states the limiting procedure does not affect the normalisation.

Finally, we gave an example for gaussian packets of coherent states. The centres of the probability distributions belonging to these states again follow the classical trajectories, but the width of these distributions oscillates with the Larmor frequency. The latter result can be considered as a possible description of the dynamics of electron localisation due to the presence of the magnetic field.

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