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Free-free transitions in a bichromatic laser field

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We consider potential scattering of electrons in a laser field. The field has frequencies ω and 2ω, which are out of phase by an angle φ. We find considerable modifications of the laser-induced scattering cross sections as a function of the phase angle φ, which for currently available experimental facilities should be observable effects.

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The scattering of electrons by an atomic system in the presence of a laser field has been investigated by many authors since the pioneering work of Bunkin and Fedorov [1]. A summary of the earlier work can be found in a book by Mittleman [2], and more recent investigations are presented in a book by Faisal [3]. Finally, a good survey of very recent work is given in the review by Francken and Joachain [4].

In most of the theoretical work presented so far in this field, the laser radiation has been treated as a classical field with a single frequency ω, or some narrow-band multimode approximation has been employed, yielding better agreement with the experiments by Weingartshofer *et al.* [5]. If the laser frequency ω and intensity I are sufficiently low so that the excitations of atomic transitions can be neglected, the atomic target can be described by a short-range potential $V(\mathbf{x})$, and the scattering can be treated in the first Born approximation, as was done by Bunkin and Fedorov [1].

Stimulated by the recent experimental work of Muller *et al.* [6] and the theoretical investigations of Schafer and Kulander [7] on phase-dependent effects in multiphoton ionization induced by a laser field and its second harmonic, and by similar effects observed in classical Thomson scattering in a powerful bichromatic field [8], we have investigated, in the approximation of Bunkin and Fedorov, the potential scattering of electrons in a low-frequency bichromatic laser field having frequencies ω and 2ω, which are out of phase by an angle φ. Considering the setup and the corresponding parameter values of the experiments by Weingartshofer *et al.* [5], our numerical investigations of the laser-induced scattering cross sections predict considerable asymmetries of the spectra of the scattered electrons as a function of the phase φ, which should be easily observable effects.

The scattering of electrons by a short-range potential $V(\mathbf{x})$ in a bichromatic laser field in dipole approximation is described by the Schrödinger equation.

$$\left[\frac{1}{2m}\left[\hat{\mathbf{p}}+\frac{e}{c}\mathbf{A}(t)\right]^2+V(\mathbf{x})\right]|\Psi(t)\rangle=i\hbar\partial_t|\Psi(t)\rangle\ , \qquad (1)$$

where the vector potential $\mathbf{A}(t)$ of the field of frequency ω and its second harmonic 2ω will be chosen in the form

$$\mathbf{A}(t)=\boldsymbol{\varepsilon}(cF/\omega)\cos\omega t+\boldsymbol{\varepsilon}_2(cF_2/2\omega)\cos(2\omega t+\varphi)\ , \qquad (2)$$

with F and F_2 being the corresponding two field strengths. For a short-range scattering potential $V(\mathbf{x})$, we obtain as zeroth order of approximation of (1) the well-known Gordon-Volkov plane-wave solutions

$$|\Psi_{\mathbf{p}}(t)\rangle=|\mathbf{p}\rangle\exp\left[-\frac{i}{\hbar}\int^t d\tau\frac{1}{2m}\left[\mathbf{p}-\frac{e}{c}\mathbf{A}(\tau)\right]^2\right] \qquad (3)$$

for an electron of initial momentum $\mathbf{p}$. Hence, in the first Born approximation, the matrix element of transition from an initial state $|\Psi_{\mathbf{p}_i}(t)\rangle$ to a final state $|\Psi_{\mathbf{p}_f}(t)\rangle$ reads

$$T_{fi}=-\frac{i}{\hbar}\int_{-\infty}^{+\infty}dt\langle\Psi_{\mathbf{p}_f}(t)|V(\mathbf{x})|\Psi_{\mathbf{p}_i}(t)\rangle\ . \qquad (4)$$

Using the Jacobi-Anger formula

$$\exp(iz\sin\alpha)=\sum_{n=-\infty}^{+\infty}J_n(z)\exp(in\alpha)$$

for the generation of Bessel functions $J_n(z)$ of the first kind of the order n, we can decompose the transition-matrix element (4) into an incoherent sum of multiphoton transition amplitudes

$$T_{fi}=-2\pi i\sum_n\langle\mathbf{p}_f|V(\mathbf{x})|\mathbf{p}_i\rangle B_n(a,b,\varphi)\times\delta(E_f-E_i-n\hbar\omega)\ , \qquad (5)$$

where $E_f=p_f^2/2m$ and $E_i=p_i^2/2m$ are the final and initial electron energies, respectively. Moreover, $\langle\mathbf{p}_f|V|\mathbf{p}_i\rangle$ is essentially the matrix element of elastic electron scattering by the potential V in the first Born approximation (at least for low frequencies ω), and the coefficients $B_n(a,b,\varphi)$ are generalized Bessel functions, which are defined by

$$B_n(a,b,\varphi)=\sum_{l=-\infty}^{+\infty}J_{n-2l}(a)J_l(b)\exp(-il\varphi) \qquad (5a)$$

with

$$a=[\mu(2mc^2E_i)^{1/2}/\hbar\omega]\boldsymbol{\varepsilon}\cdot[\mathbf{n}_i-(1+n\hbar\omega/E_i)^{1/2}\mathbf{n}_f] , \tag{5b}$$

$$b=[\mu_2(2mc^2E_i)^{1/2}/\hbar\omega]\boldsymbol{\varepsilon}_2\cdot[\mathbf{n}_i-(1+n\hbar\omega/E_i)^{1/2}\mathbf{n}_f] . \tag{5c}$$

In Eqs. (5b) and (5c) we have taken into account the energy conservation relation $E_f=E_i+n\hbar\omega$ represented by the δ function of (5) and we have introduced the unit vectors of initial and final electron propagation direction by $\mathbf{n}_i=\mathbf{p}_i/p_i$ and $\mathbf{n}_f=\mathbf{p}_f/p_f$, respectively. Furthermore, we have defined the usual dimensionless intensity parameters μ and μ_2 by

$$\mu=eF/mc\omega,\quad \mu_2=eF_2/2mc\omega , \tag{5d}$$

where the numerical value of μ can be expressed in the form $\mu=10^{-9}I^{1/2}/E_{\rm ph}$, with I being the intensity of the laser radiation of frequency ω measured in W cm^{-2} and $E_{\rm ph}=\hbar\omega$ is the energy of the laser photons measured in eV. We should also mention that in the evaluation of Eqs. (4) and (5) the contribution of the $\mathbf{A}^2$ part of the interaction in the integral of (3) drops out.

Finally, we obtain from (5), by standard procedures, the scattering cross sections of the laser-induced bremsstrahung processes of the order n,

$$\frac{d\sigma_n}{d\Omega}=\frac{p_f}{p_i}\frac{d\sigma_B}{d\Omega}|B_n(a,b,\varphi)|^2 , \tag{6}$$

where $d\sigma_B/d\Omega$ denotes the scattering cross section in the first Born approximation in the absence of the two laser fields. Formula (6) is our generalization of the Bunkin-Fedorov result for a bichromatic field.

For the following numerical examples we consider the scattering geometry and parameter values employed in the experiments by Weingartshofer *et al.* [5]. We take a CO_2 laser with $\hbar\omega\simeq0.1$ eV and its second harmonic and we choose as electron energy $E_i\simeq10$ eV. In this low-frequency case, $\hbar\omega/E_i=10^{-2}$ and hence the n dependence of the parameters a and b in (5b) and (5c) may be neglected to a very good approximation, at least for small n. For the same reason $p_i\simeq p_f$ in (6), and it will be sufficient to plot $d\sigma_n/d\sigma_B=|B_n(a,b,\varphi)|^2$ as a function of φ and n for fixed values of a and b. In order to simplify our analysis and concentrate mainly on the phase-dependent effects, we shall choose, in (5d), $F_2=F$, so that $\mu_2=\mu/2$, and we shall take $\boldsymbol{\varepsilon}_2=\boldsymbol{\varepsilon}$, corresponding to two laser fields of equal intensity and coinciding polarization vectors as in the work of Muller *et al.* [6]. In this case we find from (5b) and (5c) that $b=a/4$. For the intensity of the CO_2 laser we take $I=4\times10^7$ W cm^{-2}. Finally, in the scattering geometry of the experiments by Weingartshofer *et al.* [5], the scattered electrons are detected in the plane defined by the polarization vector $\boldsymbol{\varepsilon}$ and the momentum $\mathbf{p}_i$ of the incoming electrons. In this geometry the parameter a takes the form

$$a=[\mu(mc^2E_i)^{1/2}/\hbar\omega][\cos\psi_0-\cos(\psi_0+\theta)] , \tag{7}$$

where ψ_0 is the angle between $\boldsymbol{\varepsilon}$ and $\mathbf{p}_i$ and θ is the scattering angle, i.e., the angle between $\mathbf{p}_f$ and $\mathbf{p}_i$. In (7) we choose $\psi_0=38°$ and $\theta=155°$. Thus the parameter a will have the same order of magnitude as in the experiments by Weingartshofer *et al.* [5].

The results of our numerical evaluations of $d\sigma_n/d\sigma_B=|B_n(a,a/4,\varphi)|^2$ as a function of n and φ are depicted in Figs. 1–3. For brevity we shall write in the following $|B_n(\varphi)|^2$ or simply $|B_n|^2$.

Figure 1(a) shows the variation of $|B_0(\varphi)|^2$ as a function of φ between 0 and 2π. From this figure we expect maxima of the electron spectra at $n=0$ for $\varphi=\pi/2$ and $3\pi/2$ and minima for $\varphi=0$, π, and 2π. Evidently,

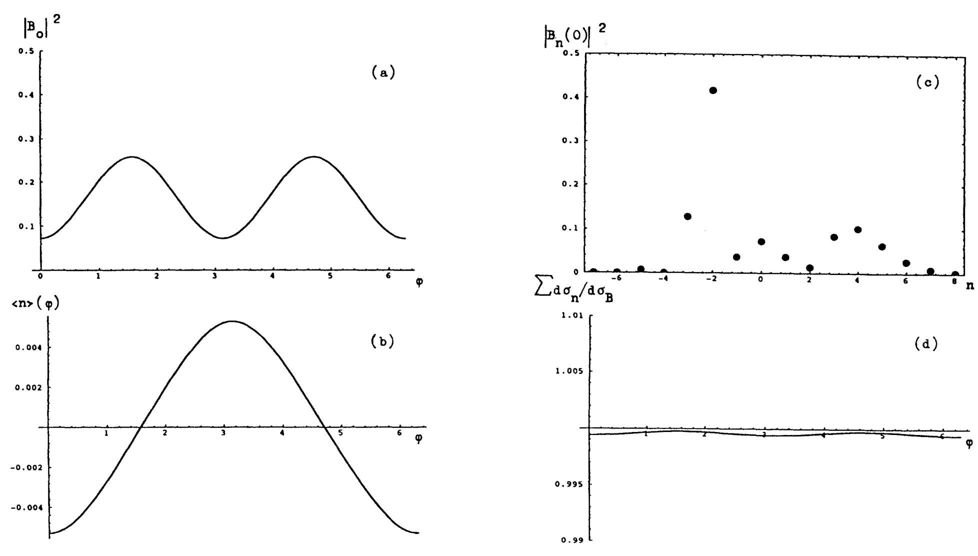

FIG. 1. In (a) the variation of $|B_0|^2$ is shown in the range $0\le\varphi\le2\pi$. (b) shows the average number $\langle n\rangle(\varphi)$ of emitted or absorbed laser quanta during the scattering process as a function of φ from 0 to 2π. (c) represents the electron spectrum $|B_n|^2$ for $\varphi=0$ in the range $-7\le n\le+7$, and (d) demonstrates the validity of the sum rule $\sum_n d\sigma_n=d\sigma_B$ for $O\le\varphi\le2\pi$.

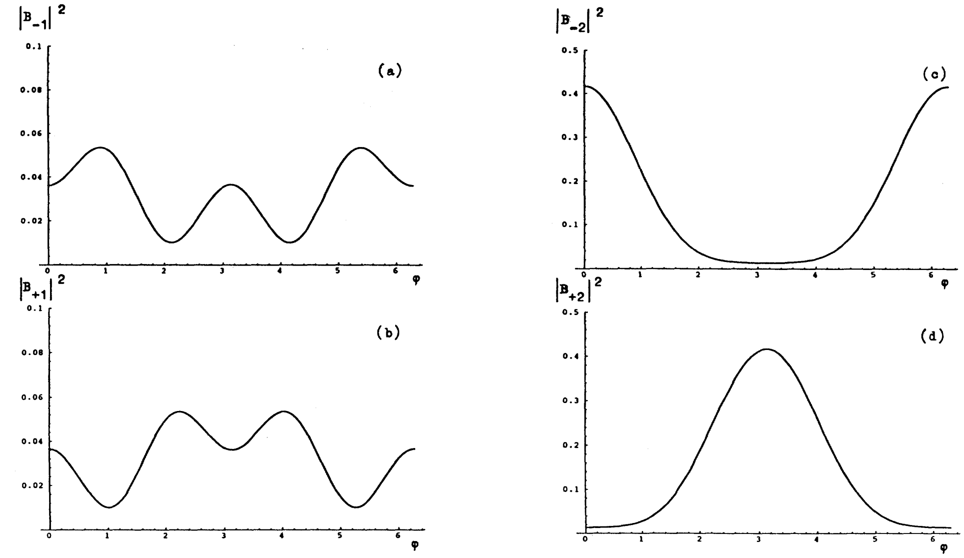

FIG. 2. This figure shows the variation of $|B_n|^2$ with $0 \le \varphi \le 2\pi$ for the following values of n: (a) $n=-1$, (b) $n=+1$, (c) $n=-2$, (d) $n=+2$.

$|B_0(\varphi)|^2$ is π periodic. These predictions nicely compare with Fig. 1 (b), in which the average number of photons, $\langle n \rangle(\varphi)=\sum_n n|B_n(\varphi)|^2$, which are emitted or absorbed are shown as a function of φ from 0 to 2π. Here we see that the minima of Fig. 1 (a) at $\varphi=0$ and 2π correspond to an average number of photons emitted, $\langle n \rangle < 0$, and the minimum at $\varphi=\pi$ corresponds to an average number of photons absorbed, $\langle n \rangle > 0$. Moreover, the maxima of Fig. 1(a) at $\varphi=\pi/2$ and $3\pi/2$ corresponds according to Fig. 1(b) to the average $\langle n \rangle=0$. The predictions of Figs. 1(a) and 1(b) for $\varphi=0$ are confirmed by the strongly asymmetric electron spectrum of Fig. 1(c), according to which $|B_n(\varphi=0)|^2$ has its maximum for $n=-2$, so that most of the electrons scattered emit two photons $\hbar\omega$ or one photon $2\hbar\omega$ into the bichromatic field. Finally, in Fig. 1(d) we have checked numerically the validity of the sum rule $\sum_n d\sigma_n = d\sigma_B$, or $\sum_n |B_n(\varphi)|^2=1$, which, as one can see, is very nicely fulfilled as a function of φ.

In Fig. 2 we explicitly show the variation of $|B_n(\varphi)|^2$ as a function of φ from 0 to 2π for $n=\pm 1$ and ± 2. As we can see, the variations are particularly pronounced for even values of n and they are out of phase by π, if we compare the curves for $n>0$ and for $n<0$ with each other. In particular, in Figs. 2(c) and 2(d) $|B_{-2}(\varphi)|^2$ has its maxima for $\varphi=0$ and 2π and its minimum for $\varphi=\pi$, whereas $|B_{+2}(\varphi)|^2$ has its minima at $\varphi=0$ and 2π and its

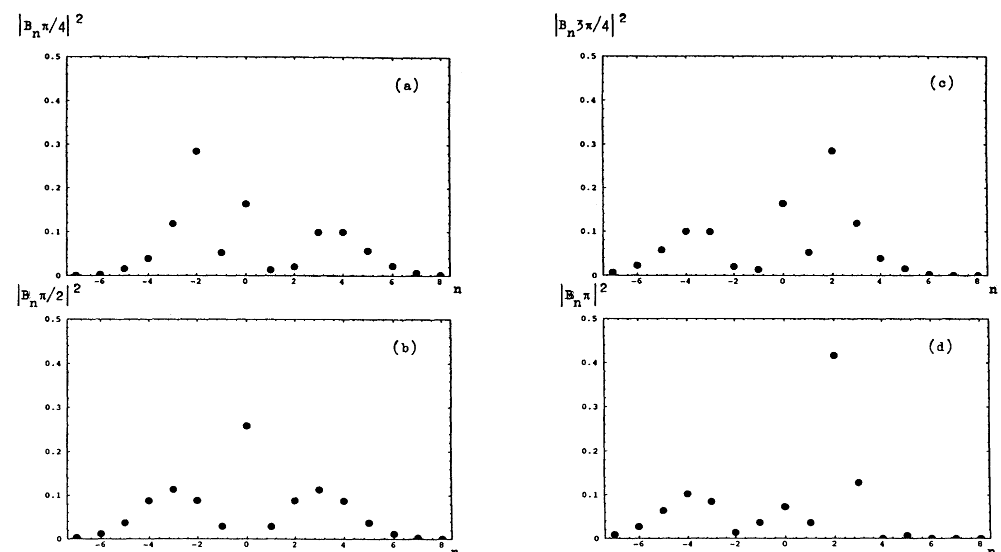

FIG. 3. In this figure four electron spectra $|B_n|^2$ are depicted as a function of n for the following values of the phase φ: (a) $\varphi=\pi/4$, (b) $\varphi=\pi/2$, (c) $\varphi=3\pi/2$, (d) $\varphi=\pi$. (a) and (c) as well as (d) and Fig. 1(c) are, respectively, mirror symmetric with respect to $n=0$, as discussed in the text.

maximum at $\varphi=\pi$. Apparently, the curves for $|B_n|^2$ and $|B_{-n}|^2$ are shifted in phase by π relative to each other. The result of this analysis for $\varphi=0$ and π is clearly demonstrated by the electron spectra of Figs. 1(a) and 3(d), respectively. Both figures are evidently mirror symmetric with respect to $n=0$.

Finally, in Fig. 3 we show a few electron spectra, which have been evaluated for particular values of the phase φ. As we can see, Figs. 3(a) and 3(b), corresponding to $\varphi=\pi/4$ and $3\pi/4$, respectively, are mirror symmetric with respect to $n=0$, as becomes evident from the foregoing figures. The same holds for Fig. 3(d) and Fig. 1(a) for $\varphi=\pi$ and 0, respectively, as discussed before. Figure 3(b) for $\varphi=\pi/2$ shows a symmetric electron spectrum, which has, however, not so strongly pronounced peaks, as in the other cases of asymmetric spectra. This becomes evident from our discussion of Fig. 2, in which, for particular values of φ, $|B_{\pm 2}|^2$ have especially large values, or vanish completely. This dominance of even values of n has certainly something to do with our particular choice of the bichromatic field.

Summarizing, we have found, in our simple analysis of potential scattering of electrons in a bichromatic laser field of frequency ω and 2ω being out of phase by an angle φ, similar phase-dependent effects as have been observed in multiphonon ionization by Muller *et al.* [6] and theoretically analyzed by Schafer and Kulander [7]. Also in our case, these effects should be accessible to experimental verification.

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