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LETTER TO THE EDITOR

Small angle scattering of slow electrons by helium atoms in a CO_2-laser field: a collective model

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Abstract. According to recent experiments by Wallbank and Holmes (1993 *Phys. Rev.* A **48** R2515; 1994 *J. Phys. B: At. Mol. Opt. Phys.* **27** 1221) on small angle, slow electron scattering by He atoms in a CO_2-laser field, the observed cross section data on free–free transitions are by many orders of magnitude larger than the predictions by the Kroll–Watson scattering formula (1973 *Phys. Rev.* A **8** 804), which are based on the consideration of scattering in a laser field by a single atom. Instead, we show here that a simple model of the collective and coherent scattering of electrons by a quasi-macroscopic laser-induced polarization potential reproduces the observed data surprisingly well.

The scattering of electrons by atoms in a laser field has been investigated over the years by many authors. Reviews on this subject on the more theoretical side can be found in the books by Mittleman [1] and Faisal [2] and in a very recent review, Mason [3] discusses the experimental situation.

In a series of recent experiments, Wallbank and Holmes [4–6] consider the scattering of slow electrons at small scattering angles by He and Ar atoms in a CO_2-laser field. These authors show that their cross section data of free–free transitions are by many orders of magnitude larger than the values predicted by the Kroll–Watson scattering formula [7], whereas for large scattering angles, the agreement between theory and experiment is satisfactory. Wallbank and Holmes suggest that the disagreement at small scattering angles may have its origin in laser-induced polarization effects, which should be dominant at small scattering angles [8]. However, in two short communications by Rabadán *et al* [9] and by Geltman [10], it has been demonstrated recently that under the above experimental conditions the polarization effects are negligibly small due to the small polarizability of He in its $1\,^1S$ ground state [11]. In a very recent publication we arrive at a similar conclusion [12]. The situation for argon is not much better.

All these considerations, presented so far, are based on the assumption that the theoretical cross sections, evaluated for a single-atom scattering event, can essentially be directly related to the experimental data. Apparently, this is not so under the above experimental conditions and we shall present in the following a collective model of electron scattering by a quasi-macroscopic laser-induced dipole potential being a coherent superposition of the contributions of the individual polarization potentials of the He atoms of the gas jet emanating from the nozzle of the atom gun (see figure 1 of [5]). As we shall show in the

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following, the cross section data of free–free transitions, evaluated from this model, are in surprisingly good agreement with the experimental results of Wallbank and Holmes [4, 5].

Our model is essentially based on the following considerations. In the experimental setup, used by Wallbank and Holmes [4–6] (see figure 2 of [5]), the interaction region of the beam of He atoms with the electron beam and laser beam is close to the nozzle of the atom gun. Near the nozzle, the density of atoms is of the order of magnitude of 10^{15}–10^{16} atoms per cm^3 [13] and this beam is very cold, which means that all atoms have mostly the same velocity $\boldsymbol{v}$, that has the magnitude 1 to 2×10^5 cm s^{-1} [13]. On the other hand, in the first experiment [4], to which we shall refer henceforth, the scattered electrons had an energy of 9.5 eV, corresponding to an electron velocity of 2×10^8 cm s^{-1}. Hence, during the passage time $t = 10^{-9}$ s of the electrons, which are passing perpendicularly through the atom beam of about 0.1 cm diameter, the atoms may be assumed to be at rest at fixed positions. At the same time, the CO_2-laser beam of about 0.15 cm diameter [13], polarized in the direction of the ingoing electron beam, induces a dipole-potential, the action of which has to be summed up coherently over the interaction volume of the three beams intersecting perpendicular to each other. Since the scattered electrons have been observed at the small scattering angle of 9° [4], we shall simplify our analysis by assuming exact forward scattering and we shall thus end up with an essentially one-dimensional problem.

The geometry, on which our calculation will be based, is depicted in figure 1. An atom beam, for simplicity of rectangular cross section, is propagating in the y direction. It is intersecting with an electron beam, oriented in the z direction and a laser beam along the x direction, the polarization vector of which is oriented along the z axis. The electrons interact with an effective atomic layer of thickness d in the interval $-d/2 \leqslant z \leqslant d/2$, where it is essential that this layer is simultaneously totally occupied by the laser field.

In order to obtain the 'effective' laser-induced dipole-potential, acting on a single scattered electron in the interaction region, we proceed as follows. We write down for this region the total potential energy of the ensemble of electrons (composed of all the electrons of the atomic shells with the coordinates $\boldsymbol{\xi}_j$ and of the scattered electron of coordinate $\boldsymbol{x}$) and of the nuclei with coördinates $\boldsymbol{x}_j$. Then we go over to the Kramers–Henneberger frame of reference [14] using the substitutions $\boldsymbol{\xi}_j \to \boldsymbol{\xi}_j + \boldsymbol{\alpha}$ and $\boldsymbol{x} \to \boldsymbol{x} + \boldsymbol{\alpha}$. Thus we find that the only two terms which will yield a laser-induced dipole-potential are given by

$$V = -Ze^2 \sum_j \frac{1}{|\boldsymbol{x} + \boldsymbol{\alpha} - \boldsymbol{x}_j|} - Ze^2 \sum_{i,j} \frac{1}{|\boldsymbol{\xi}_i + \boldsymbol{\alpha} - \boldsymbol{x}_j|} + \cdots \qquad (1)$$

while all the rest remain static contributions, which do not give rise to laser-induced nonlinear processes. In (1), however, only the first term is of interest to us, since the second term refers to the electrons of the atomic shells. For a single nucleus at the position $\boldsymbol{x}_j$, the first term of (1) yields the potential

$$V(\boldsymbol{x} - \boldsymbol{x}_j + \boldsymbol{\alpha}) = -\frac{Ze^2}{|\boldsymbol{x} - \boldsymbol{x}_j + \boldsymbol{\alpha}|} = -\frac{Ze^2}{|\boldsymbol{x} - \boldsymbol{x}_j|} + \frac{Ze^2 \boldsymbol{\alpha}(\boldsymbol{x} - \boldsymbol{x}_j)}{|\boldsymbol{x} - \boldsymbol{x}_j|^3} + \mathrm{O}\left(\frac{Ze^2\alpha^2}{|\boldsymbol{x} - \boldsymbol{x}_j|^3}\right) \qquad (1a)$$

where

$$\boldsymbol{\alpha} = \alpha_0 \boldsymbol{\varepsilon} \sin(\omega t - k x_j) \qquad \alpha_0 = \frac{\mu_0 c}{\omega} \qquad \mu_0 = \frac{eF_0}{m\omega c}. \qquad (2)$$

In (1a), the second term in the power expansion in terms of α furnishes our effective laser-induced polarization-potential originating in the nuclear Coulomb field

$$V_p(\boldsymbol{x} - \boldsymbol{x}_j; t) = \frac{\alpha_p e F_0 \boldsymbol{\varepsilon} \cdot (\boldsymbol{x} - \boldsymbol{x}_j)}{|\boldsymbol{x} - \boldsymbol{x}_j|^3} \sin(\omega t - k x_j) \qquad (3)$$

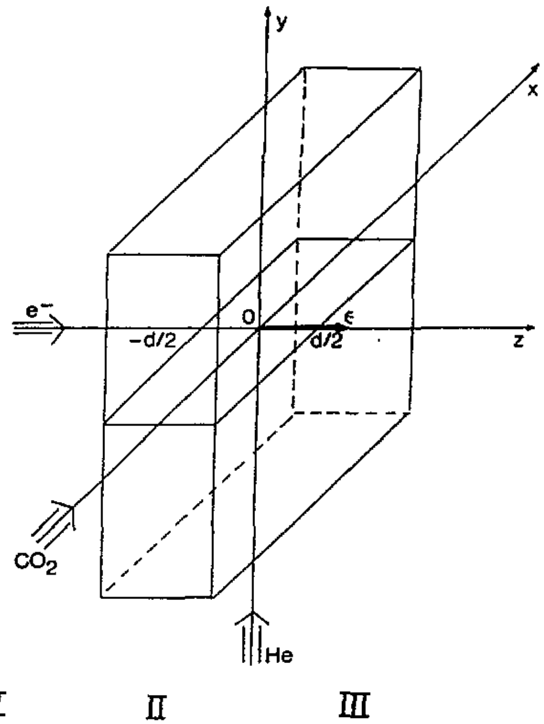

Figure 1. This figure shows the geometry of the intersection of the atomic beam of rectangular cross section, propagating in the y direction, with the electron beam, propagating in the z direction, and the laser beam, propagating in the x direction, having its vector of linear polarization parallel to the z axis.

and the effective polarizability, $\alpha_p = Ze^2/m\omega^2$, turns out, in the present case, to be by a factor 10^3 larger than the polarizability of the electron shell of He in its $1\,^1\mathrm{S}$ ground state [11], which has turned out to yield a negligibly small effect [9, 10, 12]. Why we do not need to take into account the higher order terms in the power expansion (1a), we shall discuss later on. Since the electrons move in the direction of the laser polarization, there is no space-dependence of the radiation field in this direction and the effect of the remaining x dependence will turn out to be negligibly small.

Finally, the reader may ask why the screening of the nuclear Coulomb fields by the atomic electron shells does not influence the laser-induced dipole-potentials. As regards the static contributions in (1) and (1a), there is of course a strong screening effect, but the relevant laser-induced dipole-potential in (1a) has no counterpart with opposite sign to yield a partial cancellation. This is due to the fact that in the laser field the nuclei and electrons oscillate out of phase by 180° and thus form a dipole moment, whereas the electrons of the atomic shells and the scattered electron oscillate in phase and hence no dipole moment can build up.

Summarizing, we obtain the total effective polarization potential by taking the sum of (3) over the positions $\boldsymbol{x}_j$ of all the atomic nuclei in the interaction region. This yields

$$\sum_j V_p(\boldsymbol{x} - \boldsymbol{x}_j; t) \to 2\pi n_a \alpha_p e F_0 \sin(\omega t - kx) \int_{-d/2}^{d/2} \mathrm{d}z' \int_0^\infty \mathrm{d}\rho\, \rho \frac{(z - z')J_0(k\rho)}{[(z - z')^2 + \rho^2]^{3/2}} \qquad (4)$$

where the summation has been replaced by an integration because of the rather high density of atoms n_a in the gas jet. Moreover, choosing the z axis as polar axis, we introduced plane polar coordinates such that $x - x' = \rho\cos\theta$ and $y - y' = \rho\sin\theta$, which permits one to perform the θ-integration, yielding in (4) the ordinary Bessel function J_0. Since the wavelength for a CO_2-laser field is about 10^{-4} cm, the main contribution to the ρ-integration in (4) comes from small values of ρ. Hence, we may use the approximation $J_0(k\rho) \cong J_0(0) = 1$ and extend the interval of integration to infinity. Having performed the remaining elementary ρ-integration, we are left with

$$\int \mathrm{d}z' \int \mathrm{d}\rho[./.] = \int_{-d/2}^{d/2} \mathrm{d}z' \frac{z - z'}{|z - z'|} = \begin{cases} -d & z < -d/2 \quad \text{(I)} \\ 2z & -d/2 < z < d/2 \quad \text{(II)} \\ d & z > d/2 \quad \text{(III)}. \end{cases} \tag{5}$$

Thus we obtain from (4) and (5) as the approximate effective, quasi-macroscopic, collective and coherent oscillating polarization potential acting on the electrons

$$V_p(x, z, t) = \begin{cases} -2\pi n_a \alpha_p e F_0 d \sin(\omega t - kx) & z < -d/2 \quad \text{(I)} \\ 2\pi n_a \alpha_p e F_0 (2z) \sin(\omega t - kx) & -d/2 < z < d/2 \quad \text{(II)} \\ 2\pi n_a \alpha_p e F_0 d \sin(\omega t - kx) & z > d/2 \quad \text{(III)}. \end{cases} \tag{6}$$

Here we should observe that the contribution of the next higher term in the multipole expansion in (1) will be, contrary to (6), proportional to $n_a \alpha_p e F_0 d \mu_0 \sin^2(\omega t - kx)$. Since, however, Wallbank and Holmes [4,5] used a CO_2-laser power of some 10^8 W cm^{-2}, the corresponding intensity parameter (2) has the value $\mu_0 = 10^{-4}$ and, consequently, the higher order terms in the expansion in (1) yield negligible effects.

In the following, it is convenient to introduce the potential amplitude

$$\frac{V_p^0}{2} = 2\pi^2 n_a \alpha_p \mu_0 (2mc^2)(d/\lambda) \tag{7}$$

where λ is the wavelength of the laser field. Introducing here the relevant parameters by choosing $n_a = 1.25 \times 10^{15}$ cm^{-3}, $\alpha_p = 160 \times 10^{-24}$ cm^{-3}, $\mu_0 = 10^{-4}$, $2mc^2 = 10^6$ eV, $d = 0.1$ cm and $\lambda = 10^{-3}$ cm, we find for the total potential jump V_p^0 and the ratio $V_p^0/\hbar\omega$

$$V_p^0 = 7.89 \times 10^{-2} \text{ eV} \qquad V_p^0/\hbar\omega = 0.675 = b_0. \tag{8}$$

In order to investigate the scattering of electrons by the dipole potential (6), we consider the solution of the Schrödinger equation in the three regions (I), (II) and (III), which are also indicated in figure 1. For all three regions we may write the one-dimensional Schrödinger equation in z and t in the form, treating x as a parameter

$$\left[-\left(\frac{\hbar^2}{2m}\right)\frac{\partial^2}{\partial z^2} + V_p^0 f(z)\sin(\omega t - kx)\right]\psi = i\hbar\frac{\partial}{\partial t}\psi \tag{9}$$

which has the approximate parametric solution in x and z

$$\psi(x, z, t) = \exp(\mathrm{i}(pz - Et)/\hbar)\exp(\mathrm{i}(V_p^0/\hbar\omega) f(z)\cos(\omega t - kx)) \tag{10}$$

where

$$f(z) = \begin{cases} -1/2 & z < -d/2 \quad \text{(I)} \\ z/d & -d/2 < z < d/2 \quad \text{(II)} \\ 1/2 & z > d/2 \quad \text{(III)}. \end{cases} \tag{11}$$

As one can see, the solution (10) passes continuously from region (I) into (II) and (III). One can estimate the error which is made in region (II) by treating z parametrically. It turns out to be of the order of magnitude $V_p^0/\omega t$, where t is the interaction time. Inserting the above value for V_p^0, the CO_2-laser frequency and $t = 10^{-9}$ s, we find $V_p^0/\omega t = 10^{-7}$ eV which is negligibly small. Since we are interested in the total phase-change which a free electron plane wave suffers by passing through the oscillating dipole potential we make the simple phase transformation

$$\varphi(x,z,t) = \psi(x,z,t)\exp(\mathrm{i}(V_p^0/2\hbar\omega)\cos(\omega t - kx)) \tag{12}$$

and we then find from (10) and (11) for region (III)

$$\varphi_{\mathrm{III}}(x,z,t) = \exp(\mathrm{i}(pz - Et)/\hbar)\exp(\mathrm{i}(V_p^0/\hbar\omega)\cos(\omega t - kx)) \tag{13}$$

whereas for region (I) we simply get $\varphi_{\mathrm{I}}(z,t) = \exp(\mathrm{i}(pz - Et)/\hbar)$. Hence, after Fourier-decomposition of (13) we get

$$\varphi_{\mathrm{III}}(x,z,t) = \sum_n \varphi_n(x,z,t)$$
$$\varphi_n(x,z,t) = \exp(\mathrm{i}(pz - n\hbar kx - E_n t)/\hbar)(-\mathrm{i})^n J_n(V_p^0/\hbar\omega) \tag{14}$$

from which we obtain the desired transmission probabilities

$$P_n = J_n^2(b_0) \qquad b_0 = V_p^0/\hbar\omega \qquad E_n = E + n\hbar\omega \tag{15}$$

and these probabilities fulfil the sum rule $\sum_n P_n = 1$. Consequently, in this low frequency approximation there is no reflection of electrons by the oscillating dipole potential. The oscillating dipole potential layer apparently behaves like a phase-grating in time. We also observe in (14) that the momentum transfer $n\hbar k$ in the x direction is negligibly small for a CO_2-laser field. Moreover, (15) has the proper field-free limit, since for $b_0 = 0$, $P_n = J_n^2(0) = \delta_{n,0}$.

Since in the experiments of Wallbank and Holmes [4–6] a CO_2-laser source in multimode operation has been used, which is more appropriately described by a Gaussian chaotic field [15, 16], we average (15) over the corresponding Gaussian distribution function $F_0^{-2}\exp(-(F/F_0)^2)$. Thus we find

$$\langle P_n\rangle_{ch} = I_n(b_0^2/2)\exp(-b_0^2/2) \tag{16}$$

where we find from (8), $b_0^2/2 = 0.2277$.

For our numerical evaluation of the cross section data of free–free transitions, we assumed the following time-dependence of the intensity of the CO_2-laser pulse, roughly corresponding to the pulse-shape depicted in figure 2 of [4]

$$I(t) = (3.5\times10^8\ \mathrm{W\ cm^{-2}})\exp(-t/t_0) \tag{17}$$

where $t_0 = 5\times200$ ns, using the sampling time of 200 ns of Wallbank and Holmes [4] for determining the electron spectra at different parts of the laser pulse. We have discretized the pulse-shape (17) into 15 steps, $I(t_j) = (3.5\times10^8\ \mathrm{W\ cm^{-2}})\exp(-j/5)$, $j = 1,\ldots,15$. Hence, we calculated our cross section data from

$$\frac{\langle\langle \mathrm{d}\sigma_n\rangle\rangle}{\mathrm{d}\sigma_{el}} = \langle\langle P_n\rangle\rangle = \frac{1}{15}\sum_{j=1}^{15} I_n(b_j^2/2)\exp(-b_j^2/2)$$
$$b_j^2/2 = 3.5(b_0^2/2)\exp(-j/5) \tag{18}$$

where $b_0^2/2$ has been presented before.

As we have pointed out at the beginning, it has been shown in three communications [9, 10, 12] that, by considering free–free transitions by a single atom, the effects due to the polarization of the He-target atom in its $1\,^1$S ground state are negligibly small under the experimental conditions of Wallbank and Holmes [4, 5]. Hence, to amplify our comparison of the experimental findings with the predictions of our above theory, we evaluated in addition the cross sections of free–free transitions for a single atom scattering event, using the cross section formula, derived by Zoller [15], for a chaotic field in the low frequency limit

$$\frac{\langle \mathrm{d}\sigma_n \rangle}{\mathrm{d}\sigma_{el}} = I_n(\lambda_0^2/2)\exp(-\lambda_0^2/2)$$
$$\lambda_0 = (\mu_0 c/\hbar\omega)(p - p_n' \cos\theta) \tag{19}$$

where the n-dependence of the outgoing electron, $p_n' = (2m(E+n\hbar\omega))^{1/2}$, has been retained, since for the small scattering angle, $\theta = 9°$, envisaged by Wallbank and Holmes [4, 5], it yields a considerable asymmetry of the cross sections for $n > 0$ versus $n < 0$ [17]. Employing the same averaging procedure over the laser pulse shape as before and using for the evaluation of λ_0 the following parameter values, $\mu_0 = 10^{-4}$, $\hbar\omega = 0.117$ eV, $E = 9.5$ eV and $\theta = 9°$, we find from (19) the corresponding averaged cross sections $\langle\langle \mathrm{d}\sigma_n \rangle\rangle/\mathrm{d}\sigma_{el}$ for a single atom scattering event.

The results of our calculations we present in figure 2. On the left-hand side of this figure, we plotted the laser-assisted signals in percent, i.e. $100 \times (\langle\langle \mathrm{d}\sigma_n \rangle\rangle - \mathrm{d}\sigma_{el})/\mathrm{d}\sigma_{el}$. The filled bars represent our predictions, evaluated from (18) and, for comparison, the empty bars show the averages over the experimental data (taken from figure 1 of [4]). On the right-hand side of figure 2, we present $\log_{10}$ of the averaged laser-assisted signals in percent, i.e. $\log_{10} |100 \times (\bar{\mathrm{d}\sigma}_n - \mathrm{d}\sigma_{el})/\mathrm{d}\sigma_{el}|$, where $\bar{\mathrm{d}\sigma}_n = (1/2)(\langle\langle \mathrm{d}\sigma_n \rangle\rangle + \langle\langle \mathrm{d}\sigma_{-n} \rangle\rangle)$. In particular, curve (a) represents the predicted cross sections of the theory based on the consideration of scattering by a single atom, evaluated from (19) for a chaotic CO_2-laser field and averaged over the laser pulse shape, curve (b) shows the corresponding data, obtained from our collective and coherent scattering model yielding (18) and curve (c) shows the experimental data (taken from figure 3 of [4]).

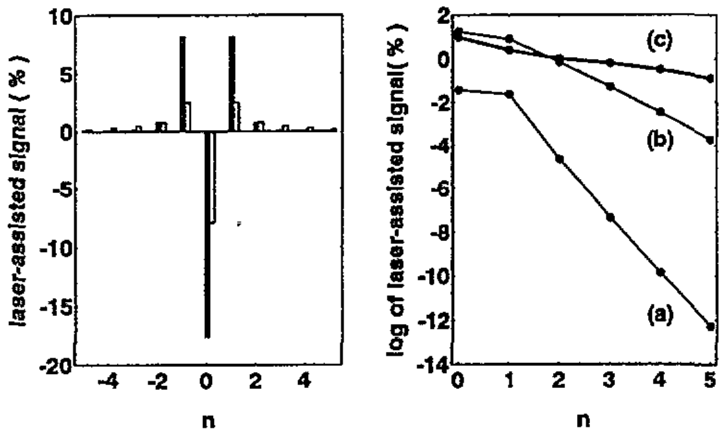

Figure 2. On the left-hand side of this figure, we compare the cross section data of our model calculations, obtained from (18), (filled bars) with the experimental data (empty bars), taken from [4], figure 1. On the right-hand side we compare the cross section data, evaluated from (19) for scattering by a single atom and averaged over the laser pulse, yielding the curve (a), with the corresponding results obtained from (18) of our collective model, represented by the curve (b), and with the experimental findings, curve (c), of Wallbank and Holmes (see figure 3 of [4]).

Summarizing, we may say that, despite the simplicity and apparent crudeness of our model, the agreement between our theory and experiment is surprisingly good, if compared with the predictions of the standard cross section formula (19). The main assumption of our model calculations, that the electrons are scattered by a collective and coherent quasi-macroscopic polarization potential, originating in the laser-induced dipole-potential of the nuclear Coulomb field of the He atoms, could be easily checked by varying the density n_a of the He atoms ejected from the nozzle of the atom gun. A drawback of our model appears to be that our cross section formula (18) does not depend on the energy of the scattered electrons. This is, however, not quite so, since the thickness d of the atomic dipole-layer (see figure 1) may be written $d = vt$, where v is the average electron velocity and t the passage time of the electrons. It may well be, however, that slower electrons feel on the average a stronger polarization potential, caused by a higher laser field intensity and, therefore, the argument b_0 of (16) will increase with decreasing electron velocity, yielding larger nonlinear cross sections. This conclusion appears to be confirmed by those experimental data (shown in figure 4 of [5]), where electrons of 8.2 eV are scattered. Finally, we should remark that, in the case where the electrons are scattered by the atoms at large angles, they go back into the region $z < -d/2$ from where they came in. In this case, the effect of the collective induced dipole-potential cancels out (there is no reflection in our low frequency approximation) and the nonlinear cross sections are governed by the scattering from a single atom; i.e. the Kroll–Watson formula describes the main contribution of free–free transitions. This explains why the Kroll–Watson formula works so well for large scattering angles and fails for small values of θ.

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References

[1] Mittleman M H 1993 *Introduction to the Theory of Laser Atom Interactions* 2nd edn (New York: Plenum)
[2] Faisal F H M 1987 *Theory of Multiphoton Processes* (New York: Plenum)
[3] Mason N J 1993 *Rep. Prog. Phys.* **56** 1275
[4] Wallbank B and Holmes J K 1993 *Phys. Rev.* A **48** R2515
[5] Wallbank B and Holmes J K 1994 *J. Phys. B: At. Mol. Opt. Phys.* **27** 1221
[6] Wallbank B and Holmes J K 1994 *J. Phys. B: At. Mol. Opt. Phys.* **27** 5405
[7] Kroll N M and Watson K M 1973 *Phys. Rev.* A **8** 808
[8] Francken P and Joachain C J 1990 *J. Opt. Soc. Am.* B **7** 554
[9] Rabadán I, Méndez L and Dickinson A S 1994 *J. Phys. B: At. Mol. Opt. Phys.* **27** L535
[10] Geltman S 1995 *Phys. Rev.* A **51** R34
[11] Radzig A A and Smirnov B M 1985 *Reference Data on Atoms, Molecules, and Ions* (Berlin: Springer) p 119
[12] Varró S and Ehlotzky F 1995 *Phys. Lett.* **203A** 203
[13] Wallbank B private communication
[14] Kramers H A 1956 *Collected Scientific Papers* (Amsterdam: North-Holland)
Henneberger W C 1968 *Phys. Rev. Lett.* **21** 838
[15] Zoller P 1980 *J. Phys. B: At. Mol. Phys.* **13** L249
[16] Daniele R and Ferrante G 1981 *J. Phys. B: At. Mol. Phys.* **14** L635
[17] Daniele R, Ferrante G, Morales F and Trombetta F 1986 *J. Phys. B: At. Mol. Phys.* **19** L133